This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

NYPL RESEARCH LIBRARIES
3 3433 06637302 2

SHOP MATHEMATICS

THE MACMILLAN COMPANY
NEW YORK · BOSTON · CHICAGO · DALLAS
ATLANTA · SAN FRANCISCO

MACMILLAN & CO., LIMITED
LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN CO. OF CANADA, LTD.
TORONTO

SHOP MATHEMATICS

U593

BY

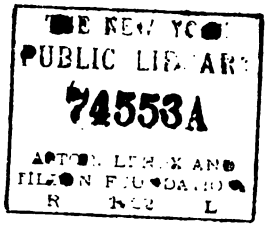
JOHN M. CHRISTMAN

FORMERLY SUPERVISOR OF MATHEMATICS,
FORD MOTOR APPRENTICE SCHOOL

New York
THE MACMILLAN COMPANY
1922

All rights reserved

PRINTED IN THE UNITED STATES OF AMERICA



COPYRIGHT, 1922,
BY THE MACMILLAN COMPANY.

Set up and electrotyped. Published November, 1922.

PREFACE

This treatise is intended for the person who has not the assistance of a teacher and who wishes to study mathematics as applied to shop problems, as well as for students in regular classes.

In the preparation of this book the author has had in view a threefold purpose: First, to furnish the apprentice, shop mechanic, and draftsman with a complete work on the mathematics used in modern shop practice. Second, to fill the long-felt need of a book for vocational training, which should take up the problems in such a direct and lucid manner that the student who has studied elementary mathematics can easily understand it. Third, to make original drawings that will teach mechanical principles and at the same time furnish a basis for the problems under consideration; that will require very little explanation to enable the student to obtain a clear conception; and that will teach him the value of drawings in the shop.

The methods used in explaining these problems are simple and original and are the ones that have given the very best results in the author's teaching for several years. Similar figures furnish a simple method of explaining many of the problems herein contained. The similar-figure method of teaching trigonometry, together with the method of breaking oblique triangles into right triangles, reduces the use of formulas to a minimum. By this means, a person having a limited mathematical knowledge can solve many unusual practical problems. The continued-fraction method used

Mar. DEC 21 1922

in finding the closest possible gear combinations for a given ratio is the result of much hard study and is offered for the first time in this book. The method by which the machines for backing off spiral-fluted hobs and cutting spiral gears are treated acquaints the student with the mechanism in such a way that he is able to apply the theory to any machines doing this work.

Many fundamental principles are explained in the first part of the book for the purpose of simplifying the more advanced problems in the latter part.

The tables of factors for use in gearing, the table of decimal equivalents, and the trigonometric tables, aside from their place as an important part of this text, will be valuable for reference in practical everyday work.

If the student makes careful study of this book there will be very few problems in actual shop practice that he will be unable to solve.

J. M. C.

August 11, 1922.

CONTENTS

CHAPTER		PAGE
I.	FACTORING	1
II.	SIGNS OF OPERATION	3
III.	FRACTIONS	6
	Reduction of fractions	7
	Least common denominator	9
	Addition of fractions	9
	Subtraction of fractions	10
	Multiplication of fractions	12
	Cancellation	13
	Complex fractions	16
	Reciprocals	17
	Division of fractions	18
IV.	DECIMALS	22
	Addition and subtraction of decimals	22
	Multiplication of decimals	23
	Division of decimals	23
	Reduction of decimals	24
V.	PROOFS OF ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION	28
	Proof of addition	28
	Proof of subtraction	29
	Proof of multiplication	29
	Proof of division	29
VI.	SHORT CUTS IN MULTIPLICATION AND DIVISION	31
VII.	PERCENTAGE	33
VIII.	GROUPING SYMBOLS	36
IX.	FORMULAS	38
X.	SIMPLE GEARING	41
XI.	COMPOUND GEARING	47

CHAPTER		PAGE
XII.	GEAR AND LEAD SCREW	54
	Gear and lead screw for lathe	55
	Micrometer	55
XIII.	RATIO AND PROPORTION	59
	Ratio	59
	Simple proportion	60
	Product of ratios	62
	Compound proportion	63
XIV.	GRADUATED DIALS	66
XV.	SQUARE ROOT	70
XVI.	CUTTING SPEEDS AND FEEDS	74
	To find R.P.M.	74
	To find feet per minute	75
	To find the outside diameter	75
XVII.	GEOMETRY AND MENSURATION	79
XVIII.	AREAS	89
XIX.	VOLUMES	96
XX.	PYTHAGOREAN THEOREM	100
	Relation of altitude line and midpoint of base	103
XXI.	ANGLES	106
XXII.	VERNIER MEASURING INSTRUMENTS	111
	Vernier calipers	111
	Vernier gear-tooth calipers	112
	Vernier protractors	113
XXIII.	SIMILAR FIGURES	116
XXIV.	TAPERS	122
XXV.	TRIGONOMETRY	126
	Functions of angles	130
	Use of trigonometric table	131
	Solution of right triangles when the angle and one side are given	134
	Solution of right triangles when any side is one and another side is given	139
	Solution of right triangles when two sides are given	142
	Perpendiculars of a triangle	148

CHAPTER		PAGE
	Solution of oblique triangles when three sides are given	150
	Solution of oblique triangles when two sides and one angle are given	156
	Solution of oblique triangles when two angles and one side are given	160
	To find the altitude of a triangle	165
	Practical problems	167
	Sine bar problems	173
	Gage problems	175
	Checking holes bored at an angle	178
XXVI.	SPUR GEAR AND RACK	179
	Checking gear and rack with gear-tooth vernier calipers	180
	Checking gear and rack with plugs	181
	Cutters for gear teeth	182
XXVII.	BEVEL GEARS	187
	Checking bevel gear teeth with gear-tooth vernier calipers	188
	Shafts forming an acute angle	191
XXVIII.	WORM GEARING	193
	Worm wheel	194
	Worm	194
	Checking worm wheel teeth with gear-tooth vernier calipers	195
	Checking worm threads with gear-tooth vernier calipers or plug	195
XXIX.	DIVIDING HEAD INDEXING	198
	Simple indexing	198
	Angular indexing	199
	Differential indexing	199
XXX.	CONTINUED FRACTIONS	204
	Continued fractions for simple gear ratios	206
	Uniting fractions	208
	Continued fractions for compound gearing	209
XXXI.	GEARING OF THE LATHE	217
	Simple gearing of the lathe	217
	Compound gearing of the lathe	220
	Gear box of a lathe	222

Chapter		Page
XXXII.	Gearing of the Milling Machine	226
	The lead of the spiral	226
	The change gears	228
XXXIII.	Hobs	230
	The lead of diametral pitch hobs	230
	The lead of the gash	232
	Gear ratios for backing off spiral-fluted hobs	235
XXXIV.	Spiral Gears	239
	Pitch diameter and lead of spiral gears	239
XXXV.	Hobbing Spiral Gears	242
	Change-gear ratios in automatic gear-hobbing machines	242
	Feed of the hob	243
	Hand of the gear and hob	243
	Feed spiral	243
	Theory of the intersecting points of spirals	245
	Calculations of change gears	250
	Trigonometric Table	257
	Factor Table	305
	Decimal Equivalents Table	316
	Index	317

SHOP MATHEMATICS

I

FACTORING

1. The **factors** of a number are the numbers whose product is that number.

For example, 6 and 5 are factors of 30; 7 and 2 are factors of 14.

2. A **prime number** is a number that has no integral factors, except itself and one. Thus, 2, 3, 5, 7, 11, 13, etc.

3. A **prime factor** is a factor that is a prime number. Thus, 6 and 4 are factors of 24, but 2, 2, 2, and 3 are prime factors of 24.

To find the prime factors of a number, try to divide the number by one of the prime numbers 2, 3, 5, 7, 11, 13, etc., in the order named, until a number is found that will exactly divide the given number; repeat the same process with each succeeding quotient until the last quotient is a prime number.

A number is divisible by 2, if the last digit is even; by 3, if the sum of the digits is divisible by 3; by 5, if it ends in 0 or 5.

The Table of Factors at the back of the book can be used to good advantage in factoring.

EXERCISE 1

Find the prime factors of the following:

1. 35	**3.** 120	**5.** 369
2. 58	**4.** 123	**6.** 420

7. If one factor of 1739 is 47, find the other.

Find two factors, between 25 and 100, of each of the following numbers. (Use factor table.)

8. 841	**10.** 1599	**12.** 7857
9. 1593	**11.** 8217	**13.** 9801

II

SIGNS OF OPERATION

4. The sign + is read "plus" and indicates addition. Thus, 6+4 equals 10.

5. The sign − is read "minus" and indicates subtraction. Thus, 6−2 equals 4.

6. The sign × is read "times" and indicates multiplication. Thus, 8×6 equals 48.

7. The signs ÷ , / , —, are read "divided by" and indicate division. Thus, 8÷2 equals 4; 12/4 equals 3; $\frac{6}{3}$ equals 2.

8. The sign = is read "equals" and means that the expression on one side of it is exactly equal to the expression on the other. Thus, 2=2; 6+4=10; 5×5=25; 9÷3=3.

9. An **exponent** is a small figure placed a little above and to the right of a number or literal quantity to indicate the number of times it is to be used as a factor. Thus, $8^2 = 8\times8$; $9^3 = 9\times9\times9$; $A^2 = A\times A$.

10. The **radical sign** $\sqrt{}$ indicates that the square root of the number following it is to be found. Thus, $\sqrt{9}=3$; $\sqrt{16}=4$.

11. If only + and − signs occur in an expression, the operations may be performed in any order, although it is a good method to add all the plus terms, then add all the minus terms, and finally find the difference between the two totals. If the sum of the plus terms is greater than the

sum of the minus terms, the remainder will have the + sign. If the sum of the plus terms is less than the sum of the minus terms, the remainder will have the − sign.

Thus, $12-6+4-2+9-18+5=?$

The sum of the plus terms equals $12+4+9+5=30$.
The sum of the minus terms equals $-6-2-18=-26$.
Then $30-26=4$, and $12-6+4-2+9-18+5=4$.

12. If the + and − signs occur together with × and ÷ signs, the × and ÷ operations *must* be performed first, and then the + and − operations may be performed in any order.

EXAMPLES:

$12\times8-6+4\times12=(12\times8)-6+(4\times12)=$
$96-6+48=144-6=138.$
$102\div6-6\times2+6\div3=(102\div6)-(6\times2)+(6\div3)=$
$17-12+2=19-12=7.$

NOTE.—That which is included within the parentheses ( ) is to be treated as a single number.

13. When the multiplication sign immediately follows the division sign, multiplication should be performed first. This rule is not always strictly followed; therefore, it is better to use parentheses in order to avoid ambiguity.

Thus, an expression like $10\div2\times5$ according to many authorities equals 1, while some claim that it equals 25.
By use of grouping symbols, $10\div(2\times5)=1$; and $(10\div2)\times5=25$.

14. When several numbers are to be multiplied together the product will be the same, regardless of the order in which the multiplications are performed.

Thus, $6\times35\times48$, $48\times6\times35$, $6\times48\times35$, and $35\times48\times6$,—all give the same product.

EXERCISE 2

1. $18-7+10-12-6=?$
2. $2-125-8+200-24=?$
3. $2+8\times3-7\times9+100\div2=?$
4. $12+8\times64-8\div4+3=?$
5. $100-25\times5+50=?$
6. $20\div(5\times2)=?$
7. $(20\div5)\times2=?$
8. $(8\times6)\div(3\times4)=?$

III

FRACTIONS

15. A **fraction** is an expression for one or more of the equal parts of a unit. Thus, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{12}$ are fractions and are read one half, two thirds, and five twelfths.

When a fraction is expressed by two numbers it is sometimes called a **common fraction.** Thus, $\frac{2}{7}$, $\frac{11}{4}$, etc., are called common fractions.

Two numbers are necessary to notate a fraction and are called **denominator** and **numerator.**

The **denominator** is written below the line and shows the number of parts into which the unit is divided.

The **numerator** is written above the line and shows how many parts are taken.

The **denominator** denominates, or names the parts; the **numerator** numbers the parts.

16. A **proper fraction** is one whose value is less than 1. Thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{8}$, etc.

An **improper fraction** is one whose value equals 1 or more than 1. Thus, $\frac{8}{8}$, $\frac{11}{4}$, $\frac{20}{3}$, etc.

17. A **mixed number** is a whole number and a fraction. Thus, $12\frac{1}{2}$, $2\frac{3}{4}$, $1\frac{7}{16}$.

18. A fraction may be considered as an indicated division. For example $\frac{45}{71}$ means $45 \div 71$.

From this it follows that to reduce an improper fraction to a whole or mixed number, we must divide the numerator by the denominator. The quotient will be the whole number, and the remainder, if any, will be the numerator of

the fractional part, of which the denominator is the same as the denominator of the improper fraction.

EXAMPLE: Reduce $\frac{21}{4}$ to a mixed number.

SOLUTION: 21 divided by 4 gives a quotient of 5 and a remainder of 1.

Thus, $$\frac{21}{4} = 21 \div 4 = 5\frac{1}{4}.$$

To reduce a mixed number to an improper fraction, multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the denominator under this sum.

EXAMPLE: Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION: $$8\frac{3}{4} = \frac{8 \times 4 + 3}{4} = \frac{35}{4}.$$

REDUCTION OF FRACTIONS

19. Multiplying or dividing both numerator and denominator of a fraction by the same number does not change the value of the fraction. This process is used in changing fractions to higher or lower terms.

EXAMPLES: $$\frac{12 \div 4}{52 \div 4} = \frac{3}{13}; \quad \frac{12 \times 4}{52 \times 4} = \frac{48}{208}; \quad \frac{12 \div 2}{52 \div 2} = \frac{6}{26}; \quad \frac{12 \times 2}{52 \times 2} = \frac{24}{104}.$$

The fractions $\frac{12}{52}$, $\frac{3}{13}$, $\frac{48}{208}$, $\frac{6}{26}$, and $\frac{24}{104}$ all have the same value.

EXAMPLE: Reduce $\frac{72}{120}$ to its lowest terms.

SOLUTION: $\frac{72 \div 8}{120 \div 8} = \frac{9}{15}$; $\frac{9 \div 3}{15 \div 3} = \frac{3}{5}$. Since the terms of $\frac{3}{5}$ are prime to each other, the lowest terms of $\frac{72}{120}$ are $\frac{3}{5}$.

20. When reducing a fraction to its lowest terms, it is sometimes difficult to find a number that will exactly divide both numerator and denominator.

RULE FOR ADDITION OF FRACTIONS: *Reduce the fractions to fractions having a common denominator, add their numerators, and write their sum over the common denominator.*

EXAMPLE: What is the sum of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{12}$?
The least common denominator is 24.

$$\frac{3\times6}{4\times6}=\frac{18}{24}$$
$$\frac{5\times3}{8\times3}=\frac{15}{24}$$
$$\frac{7\times2}{12\times2}=\frac{14}{24}$$

Then, $\frac{3}{4}+\frac{5}{8}+\frac{7}{12}=\frac{18}{24}+\frac{15}{24}+\frac{14}{24}=\frac{47}{24}=1\frac{23}{24}$.

When fractions, mixed numbers, and whole numbers are to be added together, add the fractions first, and, if their sum is an improper fraction, reduce it to a mixed number and add the whole number thus obtained to the other whole numbers.

EXAMPLE: What is the sum of $22\frac{3}{4}$, 14, $4\frac{1}{8}$, and $\frac{15}{16}$?
The least common denominator is 16.

$$22\tfrac{3}{4}=22\tfrac{12}{16}$$
$$14\ =14$$
$$4\tfrac{1}{8}=\ 4\tfrac{2}{16}$$
$$\tfrac{15}{16}=\ \ \tfrac{15}{16}$$
$$40+\tfrac{29}{16}=41\tfrac{13}{16}.$$

SUBTRACTION OF FRACTIONS

23. Subtraction of fractions is the process of finding the difference between two fractions. Fractions must have a common denominator before they can be subtracted.

RULE FOR SUBTRACTION OF FRACTIONS: *Reduce the fractions to fractions having a common denominator and write the difference of their numerators over the common denominator.*

EXAMPLE: Subtract $\frac{8}{15}$ from $\frac{5}{6}$.
The least common denominator is 30.

$$\tfrac{5}{6}-\tfrac{8}{15}=\tfrac{25}{30}-\tfrac{16}{30}=\tfrac{9}{30}=\tfrac{3}{10}.$$

When subtracting mixed numbers, subtract the fractions and whole numbers separately and add the two remainders.

EXAMPLE: Subtract $10\frac{7}{16}$ from $14\frac{7}{8}$.

The least common denominator is 16.

$$14\tfrac{7}{8}-10\tfrac{7}{16}=14\tfrac{14}{16}-10\tfrac{7}{16}=4\tfrac{7}{16}.$$

When the fractional part of the quantity to be subtracted is greater than the fractional part of the other quantity, it is necessary to borrow 1 from the whole number and add it to the fraction.

EXAMPLE: Subtract $6\frac{7}{8}$ from $14\frac{2}{9}$.

The least common denominator is 72.

$$14\tfrac{2}{9}-6\tfrac{7}{8}=14\tfrac{16}{72}-6\tfrac{63}{72}=13\tfrac{88}{72}-6\tfrac{63}{72}=7\tfrac{25}{72}.$$

The process of subtracting a fraction from a whole number is as follows:

EXAMPLE: Subtract $\frac{3}{4}$ from 7.

The least common denominator is 4.

$$7-\tfrac{3}{4}=6\tfrac{4}{4}-\tfrac{3}{4}=6\tfrac{1}{4}.$$

EXERCISE 4

1. What is the least common denominator of $\frac{7}{12}$, $\frac{2}{5}$, and $\frac{3}{4}$?
2. Add $\frac{3}{8}$, $\frac{5}{6}$, $\frac{7}{9}$, and $\frac{1}{4}$.
3. Add $\frac{7}{8}$, $2\frac{1}{2}$, $\frac{6}{5}$, $\frac{3}{2}$, and 17.
4. Add $\frac{1}{10}$, $\frac{1}{9}$, $\frac{1}{8}$, and $\frac{1}{7}$.
5. From $\frac{5}{12}$ subtract $\frac{3}{10}$.
6. From $\frac{1}{8}$ subtract $\frac{1}{9}$.
7. From the sum of $\frac{3}{4}$ and $\frac{3}{5}$ subtract the sum of $\frac{11}{30}$ and $\frac{1}{12}$.
8. Reduce $\frac{907383}{162747}$ to its lowest terms.
9. $\frac{1}{6}+\frac{1}{9}-9+16-\frac{1}{7}=?$

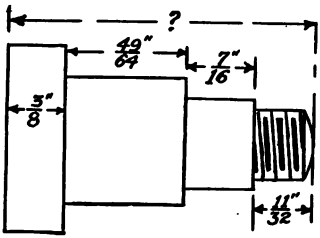


Fig. 2

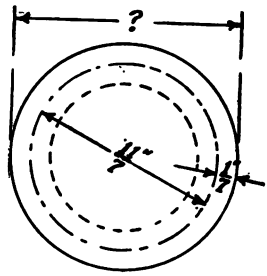


Fig. 3

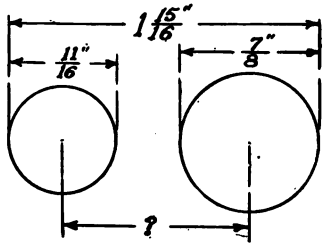


Fig. 4

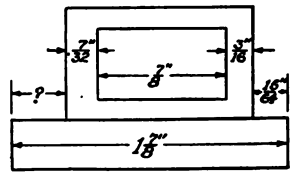


Fig. 5

10. In Fig. 2, find the value of ?.
11. In Fig. 3, find the value of ?.
12. In Fig. 4, find the value of ?.
13. In Fig. 5, find the value of ?.

Multiplication of Fractions

24. In the multiplication of fractions, it is not necessary to reduce the fractions to fractions having the same denominator.

To multiply a fraction by a fraction, multiply the numerators together for the numerator of the product and the denominators together for the denominator of the product.

Example: What is the product of $\frac{4}{5}$ and $\frac{7}{8}$?

$$\frac{4}{5}\times\frac{7}{8}=\frac{28}{40}=\frac{7}{10}.$$

Multiplying the numerator or dividing the denominator by a number multiplies the fraction by that number.

Thus, $\frac{7\times12}{24}=\frac{84}{24}=\frac{7}{2}=3\frac{1}{2}$; $\frac{7}{24\div12}=\frac{7}{2}=3\frac{1}{2}$.

When several fractions are to be multiplied together the numerators or denominators can be rearranged without changing the value of the expression.

Thus, $\frac{64}{62}\times\frac{72}{55}\times\frac{31}{30}$, $\frac{64}{30}\times\frac{31}{55}\times\frac{72}{62}$, $\frac{31}{62}\times\frac{64}{55}\times\frac{72}{30}$, and $\frac{72}{30}\times\frac{64}{55}\times\frac{31}{62}$ all have the same value.

If a number is multiplied by a fraction the value of the product will be the same whether or not the fraction is in its lowest terms.

Thus, the expressions, $18\times\frac{48}{36}$, $18\times\frac{16}{12}$, $18\times\frac{24}{18}$, and $18\times\frac{4}{3}$, all have the same value and are equal to 24.

Reducing a fraction to its lowest terms before multiplying shortens the operation in most cases.

Thus, $18\times\frac{48}{36}$, multiplied without reducing, becomes $18\times\frac{48}{36}=\frac{8\,64}{36}=24$, and $18\times\frac{48}{36}$, multiplied after reducing, becomes $18\times\frac{4}{3}=\frac{72}{3}=24$.

CANCELLATION

25. When several quantities are to be multiplied together, reducing them to lower terms will shorten the operation.

EXAMPLE:

$$\frac{42}{64}\times\frac{68}{72}\times\frac{20}{95}=\frac{21}{32}\times\frac{17}{18}\times\frac{4}{19}=\frac{21}{18}\times\frac{17}{19}\times\frac{4}{32}=\frac{7}{6}\times\frac{17}{19}\times\frac{1}{8}=\frac{119}{912}.$$

With a little logical reasoning it becomes evident that, if a numerator of one of the fractions is divided by a number, it is necessary to divide one of the denominators by that same number to keep the value the same.

This process is called **cancellation** and the solution of the above example would be as follows:

$$\frac{\cancel{42}^{\,7}}{\cancel{64}_{\,32}}\times\frac{\cancel{68}^{\,17}}{\cancel{72}_{\,18}}\times\frac{\cancel{20}^{\,1}}{\cancel{95}_{\,19}}=\frac{7\times17\times1}{8\times6\times19}=\frac{119}{912}.$$

26. If there are additions and subtractions in the numerator or the denominator of the fractional expression, cancellation cannot be performed. The operations of addition and subtraction above the line and those below the line are performed after the processes of multiplication or division have been performed; then the result above the line is divided by that below the line.

EXAMPLE:

$$\frac{8+325-17\times 2-4}{2\times 10-24+6+7}=\frac{8+325-34-4}{20-24+6+7}=\frac{333-38}{33-24}=\frac{295}{9}=32\tfrac{7}{9}.$$

27. To multiply a mixed number by a whole number, multiply the whole number of the mixed number and the fraction separately by the whole number and add these products.

EXAMPLE: Multiply $14\frac{7}{8}$ by 29.

$$\begin{array}{r} 14\frac{7}{8} \\ 29 \\ \hline 126 \\ 28 \\ \hline 406 \\ 25\frac{3}{8} \\ \hline 431\frac{3}{8} \end{array}$$

$$\tfrac{7}{8}\times\tfrac{29}{1}=\tfrac{203}{8}=25\tfrac{3}{8}$$

This example can also be solved by reducing the mixed number to an improper fraction and then multiplying the fractions.

Thus, $14\frac{7}{8}\times 29=\frac{119}{8}\times\frac{29}{1}=\frac{3451}{8}=431\frac{3}{8}$.

28. To multiply a fraction by a mixed number, reduce the mixed number to an improper fraction, and multiply the fractions.

EXAMPLE:

$$\frac{8}{15}\times 12\tfrac{3}{4}=\frac{\overset{2}{\cancel{8}}}{\underset{5}{\cancel{15}}}\times\frac{\overset{17}{\cancel{51}}}{\cancel{4}}=\frac{34}{5}=6\tfrac{4}{5}.$$

29. Multiplying or dividing a quantity by 1 does not change the quantity.

EXAMPLES: $124\times1=124$ $16\times1=16$ $\frac{7}{8}\times1=\frac{7}{8}$ $12\frac{1}{2}\times1=12\frac{1}{2}$

$124\div1=124$ $16\div1=16$ $\frac{7}{8}\div1=\frac{7}{8}$ $12\frac{1}{2}\div1=12\frac{1}{2}$

$\frac{124}{1}=124$ $\frac{16}{1}=16$ $\frac{\frac{7}{8}}{1}=\frac{7}{8}$ $\frac{12\frac{1}{2}}{1}=12\frac{1}{2}$

Thus, 12×2 is the same as $\frac{12}{1}\times\frac{2}{1}$; $15\times\frac{2}{3}$ is the same as $\frac{15}{1}\times\frac{2}{3}$.

EXERCISE 5

(Use cancellation where possible.)

1. Divide the product of 6, 8, 9, and 10 by the product of 8, 3, and 5.
2. $(48\times72\times8)\div(32\times24\times27)=?$
3. $\frac{96\times72\times16\times4\times3\times40\times13}{52\times6\times48\times34\times100\times2}=?$
4. $\frac{3}{4}\times\frac{9}{16}\times\frac{32}{18}\times\frac{4}{5}\times\frac{1}{8}\times\frac{1}{32}=?$
5. $\frac{93\times18\times14}{6\times3\times2}\times\frac{42\times6}{186\times9\times2}\times\frac{1}{7}=?$
6. $\frac{9\times6}{4}\times\frac{4}{9}\times\frac{6\times3\times9}{27}=?$
7. $\frac{12}{7}\times\frac{1}{4}\times\frac{8}{2}\times\frac{7}{6}\times\frac{4}{8}=?$
8. $\frac{7\times8-3+8}{8+3\times9-5}=?$
9. $\frac{7\times11-102+7\times4}{8+7\times3-17}=?$
10. From $19\frac{1}{2}$ subtract $12\frac{5}{6}$.
11. What is the product of $\frac{7}{12}$ and $9\frac{1}{4}$?
12. $\frac{7\times4+6-9\div3}{8\div2+4}=?$
13. $12\frac{2}{3}\times140=?$
14. $7\frac{5}{6}\times11\frac{1}{3}=?$

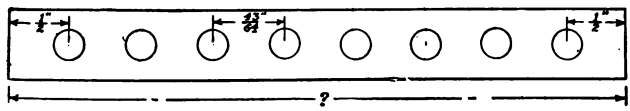


Fig. 6

15. In Fig. 6, the holes are equally spaced; find the measurement indicated by ?.

Fig. 7

16. In Fig. 7, a shaft is $\frac{9}{10}$ of a foot long; $\frac{5}{6}$ of it is turned. What is the length in inches of the unfinished part?

Complex Fractions

30. A complex fraction is a fraction that has for the numerator or denominator a fraction or a mixed number.

Thus, $\frac{2\frac{1}{7}}{\frac{1}{8}}, \frac{\frac{4}{5}}{2}, \frac{\frac{3}{4}}{\frac{1}{8}}, \frac{12\frac{1}{2}}{12}, \frac{6}{7\frac{1}{2}}$.

31. A **complex fraction** can be changed to a **common fraction** by multiplying both the numerator and the denominator of the complex fraction by a number that will contain the denominators of the fractions an exact number of times.

Example: $\frac{\frac{4}{7}}{\frac{3}{4}} = \frac{\frac{4}{7}\times 28}{\frac{3}{4}\times 28} = \frac{4\times 4}{3\times 7} = \frac{16}{21}$.

The denominators of the fractions are 4 and 7; 28 will contain 4 and 7 an exact number of times.

32. In a complex fraction all that is above the heavy line is called the numerator and is to be divided by that which is below the line, called the denominator. For example, $\frac{8}{\frac{2}{3}}$ means $8 \div \frac{2}{3}$; $\frac{\frac{3}{2}}{3}$ means $\frac{3}{2} \div 3$; $\frac{\frac{3}{4}}{\frac{5}{6}}$ means $\frac{3}{4} \div \frac{5}{6}$.

EXERCISE 6

1. What common fraction does $\frac{\frac{7}{8}}{\frac{3}{4}}$ equal?
2. What common fraction does $\frac{5\frac{1}{2}}{9}$ equal?
3. What common fraction does $\frac{15\frac{3}{4}}{16\frac{2}{3}}$ equal?
4. What mixed number does $\frac{8}{\frac{3}{7}}$ equal?

RECIPROCALS

33. The **reciprocal** of a number is 1 divided by that number. For example, the reciprocal of 12 is $1 \div 12$ or $\frac{1}{12}$; the reciprocal of 23 is $1 \div 23$ or $\frac{1}{23}$.

34. The reciprocal of a fraction is 1 divided by the fraction, which is the same as inverting the fraction. For example, the reciprocal of $\frac{3}{4}$ is $1 \div \frac{3}{4}$ or $\frac{1}{\frac{3}{4}}$, and according to Article 31 the complex fraction $\frac{1}{\frac{3}{4}}$ may farther be reduced to a common fraction. Thus, $\frac{1 \times 4}{\frac{3}{4} \times 4} = \frac{4}{3}$. Therefore, the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

35. The reciprocal of a mixed number is 1 divided by the mixed number; which is the same as reducing the mixed number to an improper fraction and then inverting the improper fraction. For example, the reciprocal of $12\frac{1}{4}$ is $1 \div 12\frac{1}{4}$ or $1 \div \frac{49}{4}$ or $\frac{4}{49}$.

36. A fractional expression like $\frac{42\times29}{16}$ is equivalent to $\frac{42\times29}{1\times16}$ or $\frac{42}{16}\times\frac{29}{1}$ or $\frac{42}{1}\times\frac{29}{16}$.

A fractional expression like $\frac{42}{83\times24}$ is equivalent to $\frac{42\times1}{83\times24}$ or $\frac{42}{83}\times\frac{1}{24}$ or $\frac{1}{83}\times\frac{42}{24}$.

37. A fractional expression like $\frac{\frac{4}{7}\times\frac{8}{5}}{\frac{6}{4}}$ is equivalent to $\frac{\frac{4}{7}}{1}\times\frac{\frac{8}{5}}{\frac{6}{4}}$ or $\frac{\frac{4}{7}}{\frac{6}{4}}\times\frac{\frac{8}{5}}{1}$.

A fractional expression like $\frac{\frac{3}{4}}{\frac{4}{7}\times\frac{2}{3}}$ is equivalent to $\frac{\frac{3}{4}}{\frac{4}{7}}\times\frac{1}{\frac{2}{3}}$ or $\frac{1}{\frac{4}{7}}\times\frac{\frac{3}{4}}{\frac{2}{3}}$.

Division of Fractions

38. Since a fraction may be considered an indicated division, it follows that $17\div42$ is equivalent to $\frac{17}{42}$. And, from Articles 29 and 36, $\frac{17}{42}$ is equivalent to $\frac{17\times1}{42}$. Therefore, $17\div42=\frac{17}{42}=17\times\frac{1}{42}$.

Instead of dividing 17 by 42 the same result may be obtained by multiplying by the reciprocal of 42, which is $\frac{1}{42}$.

39. Since in a complex fraction all that is above the heavy line is to be divided by that which is below, it follows that

$\frac{4}{9}\div\frac{6}{7}$ is equivalent to $\frac{\frac{4}{9}}{\frac{6}{7}}$; $\frac{\frac{4}{9}}{\frac{6}{7}}$ is equivalent to $\frac{\frac{4}{9}}{1}\times\frac{1}{\frac{6}{7}}$ or $\frac{4}{9}\times\frac{1}{\frac{6}{7}}$ or $\frac{4}{9}\times\frac{7}{6}$.

Note: $\frac{1}{\frac{6}{7}}=\frac{7}{6}$.

Instead of dividing $\frac{4}{9}$ by $\frac{6}{7}$, the same result may be obtained by multiplying by the reciprocal of $\frac{6}{7}$, which is $\frac{7}{6}$. Thus, to divide a fraction by a fraction, simply invert the fraction

which is the divisor and multiply the fraction which is the dividend by this inverted fraction.

It is evident also that, as in multiplication, it is not necessary to reduce fractions to a common denominator to perform the process of division.

40. Instead of dividing by a quantity, the same result may be obtained by multiplying by the reciprocal. This holds true for whole numbers, mixed numbers, and fractions.

EXAMPLE: $6\frac{1}{2} \div 2\frac{2}{3} = \frac{13}{2} \div \frac{8}{3} = \frac{13}{2} \times \frac{3}{8} = \frac{39}{16} = 2\frac{7}{16}$;
$8 \div \frac{2}{3} = 8 \times \frac{3}{2} = 12.$

41. Dividing the numerator or multiplying the denominator divides the fraction.

EXAMPLE: $\frac{24}{25} \div 6 = \frac{24 \div 6}{25} = \frac{4}{25}$, or $\frac{24}{25} \div 6 = \frac{24}{25 \times 6} = \frac{4}{25}$.

EXERCISE 7

1. What is the reciprocal of 72?
2. What is the reciprocal of $\frac{1}{6}$?
3. What is the reciprocal of $\frac{17}{82}$?
4. What is the reciprocal of $28\frac{1}{2}$?
5. What is the reciprocal of $\frac{\frac{2}{5}}{\frac{7}{12}}$?
6. $54 \div \frac{6}{7} = ?$
7. $\frac{7}{9} \div \frac{14}{6} = ?$
8. $34\frac{2}{5} \div 46\frac{2}{7} = ?$

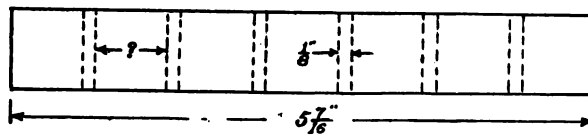


Fig. 8

9. In Fig. 8, find the measurement indicated by ?.

42. To find how many times one number contains another number, divide one of the numbers by the other, or simply designate the division by a fraction consisting of the two numbers. For example, $\frac{3}{4}$ is the number of times 3 contains 4; and $\frac{4}{3}$ is the number of times 4 contains 3.

43. A simple means is thus afforded (Article 42) for finding the fraction by which one number must be multiplied to make it equal another number.

EXAMPLE: By what fraction must 16 be multiplied to make it equal 23?

SOLUTION: $\frac{16}{1}\times\frac{23}{16}=23$. The fraction is $\frac{23}{16}$.

The numerator of the fraction is always the number to be obtained, and the denominator is always the given number.

Sometimes it is necessary to multiply one fraction by a complex fraction to make it equal another fraction.

EXAMPLE: By what complex fraction must $\frac{4}{5}$ be multiplied to obtain $\frac{7}{6}$?

SOLUTION: $\frac{\frac{4}{5}}{1}\times\frac{\frac{7}{6}}{\frac{4}{5}}=\frac{\frac{7}{6}}{1}=\frac{7}{6}$. The complex fraction is $\frac{\frac{7}{6}}{\frac{4}{5}}$.

The numerator of the complex fraction is always the fraction to be obtained, and the denominator is always the given fraction.

This method is extremely useful in solving a problem as shown below.

EXAMPLE: $\frac{3}{5}=\frac{x}{8}$. Find x.

SOLUTION: If 5 must be multiplied by $\frac{8}{5}$ to equal 8, then 3 must be multiplied by $\frac{8}{5}$ to equal x; therefore, $x=3\times\frac{8}{5}$ or $4\frac{4}{5}$.

44. Expressions like $\frac{\frac{7}{8}}{\frac{4}{5}}\times\frac{\frac{2}{9}}{\frac{4}{7}}$ can be solved best by reducing each complex fraction to a common fraction.

Thus, $\frac{\frac{7}{8}}{\frac{4}{5}}=\frac{7}{8}\times\frac{5}{4}=\frac{35}{32}$; $\frac{\frac{2}{9}}{\frac{4}{7}}=\frac{2}{9}\times\frac{7}{4}=\frac{7}{18}$.

Then, $\frac{\frac{7}{8}}{\frac{4}{5}}\times\frac{\frac{2}{9}}{\frac{4}{7}}$ equals $\frac{35}{32}\times\frac{7}{18}=\frac{245}{576}$.

EXERCISE 8

1. What improper fraction must 24 be multiplied by to obtain 35?
2. What common fraction must 25 be multiplied by to obtain 4?
3. What complex fraction must $\frac{5}{6}$ be multiplied by to obtain $\frac{9}{28\frac{1}{1}}$?
4. What complex fraction must $8\frac{2}{3}$ be multiplied by to obtain $11\frac{1}{2}$?
5. What complex fraction must $\frac{2}{3}$ be multiplied by to obtain $\frac{1}{4}\frac{7}{}$?
6. What complex fraction must 7 be multiplied by to obtain $\frac{4}{5}$?
7. What complex fraction must $\frac{5}{7}$ be multiplied by to obtain 6?
8. $\frac{2}{6}=\frac{x}{7}$. Find x.
9. What improper fraction does $\frac{\frac{3}{4}}{\frac{5}{6}}\times\frac{\frac{1}{1}\frac{1}{0}}{\frac{5}{12}}$ equal?
10. $\frac{78\frac{1}{3}}{\frac{9}{12}}\div\frac{2\frac{8}{9}}{3}=?$
11. $\frac{47\frac{2}{3}}{\frac{9}{14}}=\frac{x}{6\frac{2}{3}}$. What mixed number does x equal?

IV

DECIMALS

45. A **decimal fraction** is a fraction whose denominator (usually not expressed) is 10 or some power of 10.

The denominator, when omitted, is indicated by a point placed to the left of the numerator; as, $.3=\frac{3}{10}$; $.25=\frac{25}{100}$; $.123=\frac{123}{1000}$.

The denominator of a decimal fraction is always 1, with as many ciphers annexed as there are decimal places in the decimal. Thus, $.7=\frac{7}{10}$; $.003=\frac{3}{1000}$, etc.

.12 is read "12 hundredths"; .4 is read "4 tenths"; .011 is read "11 thousandths"; 122.123, which is a whole number and a decimal, is read "122 and 123 thousandths."

Adding a cipher to the right of a decimal fraction multiplies both numerator and denominator by 10; therefore the value of the decimal remains unchanged. Thus, $.2=\frac{2}{10}$; $.20=\frac{20}{100}$; $.200=\frac{200}{1000}$, etc.

Moving the decimal point one place to the right multiplies the decimal fraction by 10. Thus, $.123\times10=1.23$.

Moving the decimal point one place to the left divides the decimal fraction by 10. Thus, $.123\div10=.0123$.

Moving the decimal point the same number of places in both numerator and denominator of an expression like $\frac{2.642}{3.11}$ does not change its value.

$$\frac{2.642}{3.11}=\frac{26.42}{31.1}=\frac{264.2}{311}.$$

Addition and Subtraction of Decimals

46. Common fractions must have a common denominator before they can be added or subtracted; this is also

true of decimals. This is done by placing the decimals in a column so that the decimal points come one under the other.

	.120		
	24.037		12.7300
Addition:	.125	Subtraction:	3.0206
	1.022		9.7094
	25.304		

Multiplication of Decimals

47. Multiplication of decimals is performed in the same manner as the multiplication of integers. The number of decimal places in the product will be equal to the sum of the number of decimal places in the multiplicand and in the multiplier.

Multiplication:

$$\begin{array}{r} 12.46 \\ 1.032 \\ \hline 2492 \\ 3738 \\ 12460 \\ \hline 12.85872 \end{array}$$

In this example there are 2 decimal places in the multiplicand and 3 in the multiplier; therefore, the number of decimal places in the product must be 2+3 or 5.

Division of Decimals

48. To divide a decimal by a whole number, divide as in long division. To take care of the decimal point in the quotient, place it directly above the decimal point in the dividend.

Division:

$$\begin{array}{r} 00.0282+ \\ 873\overline{)24.672} \\ 1746 \\ \hline 7212 \\ 6984 \\ \hline 2280 \\ 1746 \\ \hline \end{array}$$

49. To divide a decimal by a decimal, indicate the division as a fraction, then move the decimal points in the numerator and denominator enough places to the right to make the denominator a whole number. The division is then performed as in the above example.

Thus, $18.73 \div 2.755 = \frac{18.73}{2.755} = \frac{18730}{2755} = 6.79+$

```
        6.79+                          6.79+
2755/18730.00                  2755./18730.00
   16530                          16530
    22000                          22000
    19285                          19285
     27150                          27150
     24795                          24795
```

The solution at the right shows how the division is actually performed. The decimal point in the divisor is moved to the right enough spaces to make the divisor a whole number, the new position is indicated by a larger decimal point. The decimal point in the dividend is then moved to the right the same number of places and its position is also indicated by a larger decimal point. The decimal point in the quotient is then placed directly above the larger point in the dividend.

Reduction of Decimals

50. To reduce a fraction to a decimal, it is only necessary to know that the line that separates the numerator and denominator indicates division. Divide the numerator by the denominator.

Thus, $\frac{7}{37} = 7 \div 37 = .1891+$.

51. To reduce a decimal to a fraction, express it as a fraction and reduce to its lowest terms.

Thus, $.0375 = \frac{375 \div 125}{10000 \div 125} = \frac{3}{80}$.

EXERCISE 9

1. Add 7, 6.342, .0098, 1.564, and 324.
2. Add .0004 and 4000.
3. $4.675+\frac{4}{10}+\frac{45}{1000}+.87+18\frac{11}{100}=?$
4. From 17.543 subtract 5.463.
5. $11.786-.00091=?$
6. $9\frac{5}{10}\frac{4}{000}\frac{2}{0}-8.976=?$
7. $86.54-176.43+124.092=?$

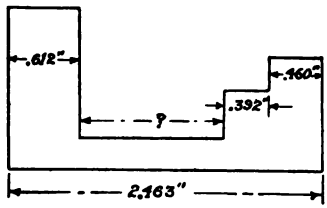


Fig. 9.

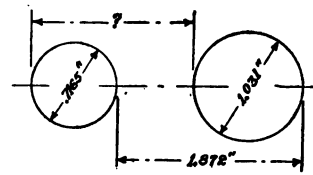


Fig. 10.

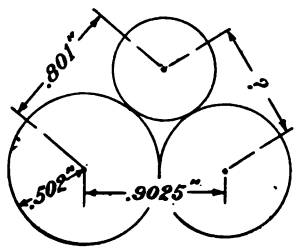


Fig. 11

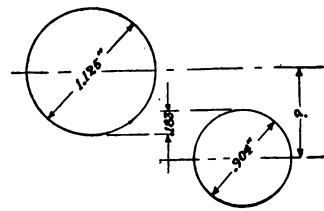


Fig. 12

8. In Fig. 9, find the measurement indicated by ?.
9. In Fig. 10, find the value of ?.
10. In Fig. 11, what length is indicated by ?.
11. In Fig. 12, find the value of ?.
12. In Fig. 13, what fraction of an inch does the unknown quantity represent ?
13. In Fig. 14, find value of ?.

14. In Fig. 15, find the measurement indicated by ?.

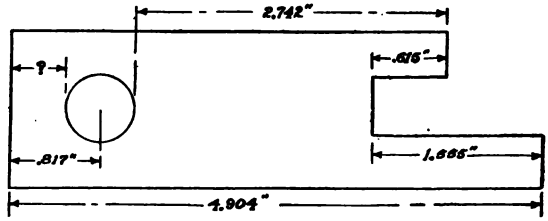


Fig. 13

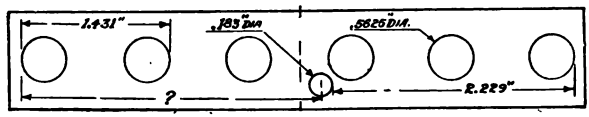


Fig. 14

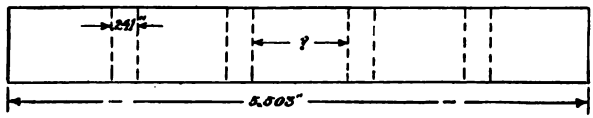


Fig. 15

15. In Fig. 16, if one revolution of A moves the slide .1726", how far will 8.34 revolutions move it?

16. In Fig. 16, if one revolution of A moves the slide .213", how many revolutions would be required to move it 4.3"?

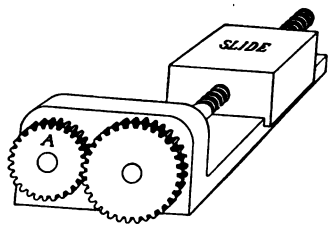


Fig. 16

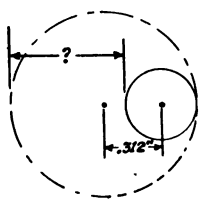


Fig. 17

17. $.5603 \times .17 \times .06 = ?$

18. $.8732 \div 9.76 = ?$

19. $21.7 \times 6.3 - 4.3 \times 18 = ?$

20. $18 + 1.4 \div 1.3 - .6 \times 4 = ?$

21. $\dfrac{6.6}{6.7} + \dfrac{87}{100} - .934 = ?$

22. $\dfrac{9.06}{.054} = \dfrac{13.2}{?}$

23. In Fig. 17, find value of ?.

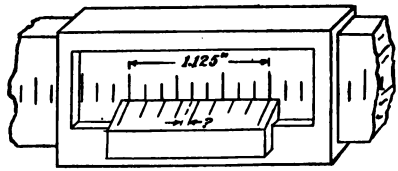


Fig. 18

24. In Fig. 18, find length indicated by ?.

25. Reduce $\frac{7}{15}$ to a decimal containing four decimal places.

26. Reduce .3875 to a common fraction in its lowest terms.

27. Reduce $\frac{18.6}{47.43}$ to a common fraction in its lowest terms.

28. From $\frac{7}{9}$ subtract .2321.

29. By what decimal must 8.674 be multiplied to make it equal 4.444?

30. What decimal does $\frac{17\frac{2}{3}}{23\frac{5}{7}}$ equal?

31. What is the decimal reciprocal of .4537?

32. What is the decimal reciprocal of $\frac{9}{15}$?

33. $\frac{14\times.8}{17.4\div6}=\frac{?}{18}$

34. In Fig. 19, find measurement indicated by ?.

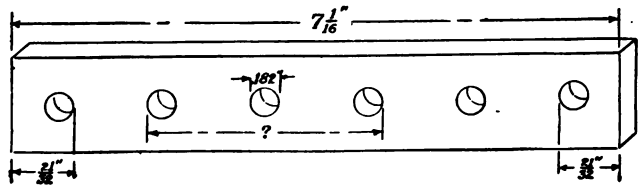


Fig. 19

V

PROOFS OF ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION

52. Any number divided by 9, has the same remainder as the sum of the digits divided by 9. For example, 9 is contained in 138 fifteen times with a remainder of 3. The sum of the digits 1, 3, and 8 is 12; 9 is contained in 12 once with a remainder of 3 also. The digits of 138 could be arranged in any order, but the remainder after dividing by 9 will always be the same. Thus, the remainders of 138, 1038, 831, 3018, 800301, etc., after dividing by 9, are 3 in each case.

The number that remains after casting out the 9's, or dividing by 9, is called the excess of 9's. These excesses of 9's are used in proving **addition, subtraction, multiplication,** and **division.**

The following is a good method for obtaining the excess of 9's.

EXAMPLE: Find the excess of 9's in 768945.

EXPLANATION: Begin at the left, $7+6=13$; dropping the $9=4$; $4+8=12$; dropping the $9=3$; $3+4+5=12$; dropping the $9=3$; the excess of 9's in 768945 is 3. The 9 in the number was not considered because adding 9 to a number and then subtracting it will not change the number. For the same reason it is not necessary to consider any group of digits that equals 9. Thus, the excess of 9's in 287915234 is 5. $\overbrace{287}\overbrace{9}\overbrace{15}\overbrace{234}$

PROOF OF ADDITION

53. The sum of the excesses of 9's in several numbers must equal the excess of 9's in their sum.

EXAMPLE: The excesses in the numbers in the following addition are 8, 2, 4, and 3, and the excess of 9's in the sum of these excesses,

17, is 8. The excess of 9's in the sum, 26729, is likewise 8. Therefore, the excesses being the same, the work is correct.

7352	8	
5834	2	
6241	4	9
7302	3	
26729	8	

Proof of Subtraction

54. The excess of 9's in the minuend must equal the sum of the excesses of 9's in the subtrahend and remainder.

Example: As the minuend is the sum of the subtrahend and remainder, the reason for this proof is seen from the proof for addition.

Minuend	7640	8
Subtrahend	1234	1
Remainder	6406	7

Proof of Multiplication

55. Find the excesses of 9's in the factors and in the product. The excess of 9's in the product of the excesses of the factors should equal the excess in the product of the factors themselves.

Example: Multiply 7234 by 8615. The product is 62320910. The excess in the multiplicand is 7, in the multiplier 2, and in the product 5; the two former multiplied give 14; and the excess in 14 is also 5. Hence the multiplication is correct.

7234	7
8615	2
36170	14–5
7234	
43404	
57872	
62320910	5

Proof of Division

56. Find the excess of 9's in each of the terms. To the

excess of 9's in the product of the excesses in the divisor and quotient, add the excess in the remainder; the excess in the sum should equal the excess in the dividend.

EXAMPLE: Divide 131885 by 317. The quotient is 416, and the remainder, 13. The excess of 9's in the divisor is 2; in the quotient, 2. Their product is 4, the excess of which is 4. The excess in the remainder is 4. $4+4=8$, of which the excess is 8. The excess of 9's in the dividend is also 8. Hence the division is correct.

```
 2   8    2
317)131885(416
   1268          2×2=4
    508            4
    317            8
   1915
   1902
     13-4
```

VI

SHORT CUTS IN MULTIPLICATION AND DIVISION

57. Process of multiplying when one part taken as a unit in the multiplier is a factor of another part so taken:

EXAMPLE: Multiply 387295 by 216324. In this problem 216324 is divided into three groups 216, 3, and 24. First multiply by 3 and obtain 1161885; then multiply this product by 8, which is the same as multiplying the multiplicand by 24, because $3\times8=24$, and 9295080 is obtained. This last product is then multiplied by 9, which is the same as multiplying the multiplicand by 216, because $9\times24=216$, and the product 83655720 is obtained.

387 2 95
216 3 24
116188 5
9295 0 80
83655720
83781203 5 80

58. Process of multiplying and dividing by aliquot parts:

An **aliquot part** of a number is such a part as will exactly divide that number. Thus, .125 is an aliquot part of 1, 1.25 is an aliquot part of 10, 125 is an aliquot part of 1000, .025 is an aliquot part of .1.

The reciprocals of aliquot parts can always be written completely without a remainder, and very often, when the aliquot part is a long decimal, the operations can be shortened considerably by using the reciprocal and dividing instead of multiplying or vice versa.

EXAMPLE: Since $.0625=\frac{1}{16}$, $8848\times.0625=8848\times\frac{1}{16}=\frac{8848}{16}=553$.

EXAMPLE: Since $.03125=\frac{1}{32}$, $1742\div.03125=1742\div\frac{1}{32}=1742\times32=55744$.

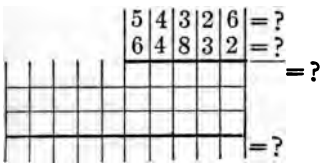
59. Process of multiplying when the multiplier is a little less than 100, 500, 1000, 5000, 10000:

EXAMPLE: $3046 \times 997 = 3046 \times (1000 - 3) = 3046000 - 9138 = 3036862$.

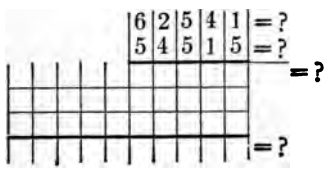
EXERCISE 10

Use the short cuts in the following exercises and prove the results by casting out the 9's.

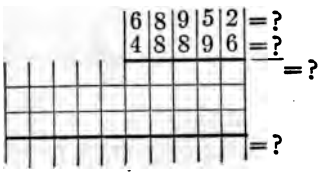
1. 54326×64832



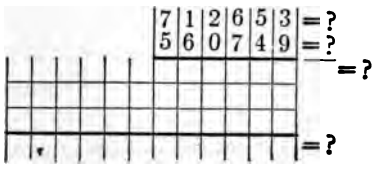
2. 62541×54515



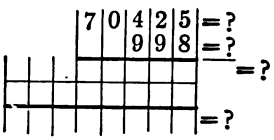
3. 68952×48896



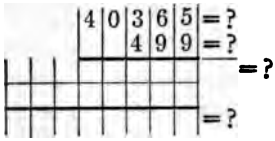
4. 712653×560749



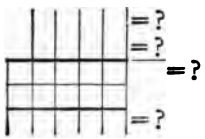
5. 70425×998



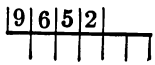
6. 40365×499



7. $7241 \div 3125$



8. 9652×25



VII

PERCENTAGE

60. Percentage is a term applied to computations in which 100 is used as a fixed measure or standard.

The sign % is read "per cent." Thus, 6% is read "six per cent."

Three per cent, 3%, $\frac{3}{100}$, and .03 are equivalent expressions.

Five per cent, or 5%, means 5 out of every hundred, or $\frac{5}{100}$ of the entire amount; the 5 stands for the numerator, and the % sign for the denominator 100.

Thus, 5% of 500 means 5 of every hundred, which would be 25; or 5% of 500 means $\frac{5}{100}$ of 500, or $\frac{5}{100} \times 500$, or 25.

61. A careful study of the following examples should be made to obtain a clear conception of percentage.

EXAMPLE 1: How many hundredths of a number is 7%?

SOLUTION: If $1\% = \frac{1}{100}$ of the number, then 7% equals $\frac{7}{100}$ of the number.

EXAMPLE 2: What per cent of a number is $\frac{4}{100}$ of it?

SOLUTION: $\frac{1}{100} = 1\%$; then $\frac{4}{100}$ equals 4%.

EXAMPLE 3: What per cent of a number is $\frac{2}{7}$ of it?

SOLUTION: $\frac{1}{100} = 1\%$; then $\frac{2}{7}$ equals $\frac{2}{7} \div \frac{1}{100}$ or $\frac{2}{7} \times \frac{100}{1} = \frac{200}{7}$ or $28\frac{4}{7}\%$.

EXAMPLE 4: What per cent of a number is .017 of it?

SOLUTION: $.01 = 1\%$; then .017 equals $.017 \div .01$, or 1.7%.

EXAMPLE 5: What part of a number is 20% of it?

SOLUTION: $1\% = \frac{1}{100}$; then 20% equals $\frac{20}{100}$, or $\frac{1}{5}$.

EXAMPLE 6: 22 is 3% of what number?

SOLUTION: If $3\% = 22$, then $1\% = 22 \div 3$ or $\frac{22}{3}$, and 100% equals $100 \times \frac{22}{3} = \frac{2200}{3}$, or $733\frac{1}{3}$.

EXAMPLE 7: 27 is what per cent of 837?

SOLUTION: 100%=837; then 1%=8.37. Hence, 27 will be $\frac{27}{8.37}$ or 3.225+% of 837.

EXAMPLE 8: 10% of 84 is 19% of what number?

SOLUTION: 10% of 84 equals .1 of 84 or 8.4. This problem now reads 8.4 is 19% of what number? If 19%=8.4, $1\%=\frac{8.4}{19}$. Then $100\%=100\times\frac{8.4}{19}=\frac{840}{19}=44\frac{4}{19}$.

EXAMPLE 9: 40 is 125% of what number?

SOLUTION: If 40=125%, $1\%=\frac{40}{125}=\frac{8}{25}$, and $100\%=100\times\frac{8}{25}=\frac{800}{25}$ or 32.

EXAMPLE 10: What number diminished by 20% of itself will equal 40?

SOLUTION: After diminishing a number by 20%, the part that remains must equal 80%; therefore, 40 equals 80%; then $1\%=\frac{40}{80}$, and the original number is 100% or 50.

EXERCISE 11

1. What per cent of 136 is 17?

2. A certain bronze casting weighing 345 pounds contains 75% of copper and 25% of tin. Find weight of copper in casting.

3. 1500 pounds of manganese steel contains 6 pounds of manganese. What is the per cent of manganese in the steel?

4. Out of a lot of 640 castings, 5% were found defective from molding and 2% of the remainder were machined under size. What number were scrapped?

5. A grade of pig iron weighing .26 of a pound per cubic inch, contained 2.8% of silicon, .4% of sulphur, 1.5% of phosphorus, and 3.6% of carbon. The remaining part was pure iron. How many pounds of pure iron would be used in 7 cubic feet of this metal?

6. The coke used in melting iron in a cupola contains $1\frac{1}{2}\%$ of sulphur. If $\frac{1}{4}$ of the sulphur in the coke goes into the iron during the melting, how many pounds of sulphur does the iron absorb from 2200 pounds of coke?

7. A pound of steel cost \$0.95 two years ago but now sells at \$2.25. What is the per cent of increase in price?

8. In a certain department the production was 2500 parts in 24 hours. This was increased to 1500 in 10 hours. What per cent increase was accomplished?

9. $12\frac{1}{2}\%$ of 96 is $33\frac{1}{3}\%$ of what number?

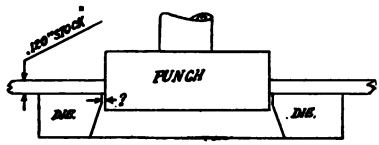


Fig. 20

10. In Fig. 20, the clearance on each side of a punch and die for a certain metal was 8% of the thickness of the stock. What was the clearance on one side of the die?

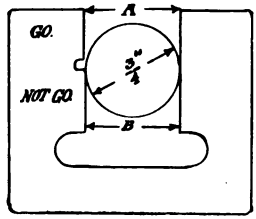


Fig. 21

11. In Fig. 21, a snap guage for checking $\frac{3}{4}''$ shafts had an allowance of .05% over or under size. Find A and B.

VIII

GROUPING SYMBOLS

62. In a series of operations the parenthesis ( ) and other grouping symbols, the bracket [ ], the brace { }, and the vinculum —, may be used to indicate the order in which certain additions, subtractions, multiplications, and divisions are to be performed. What is within the grouping symbols is to be treated as a single number.

63. In arithmetic, the grouping symbols can always be removed by completing the operations within them. If one or more pairs are included within another pair of grouping symbols, it is usual to remove the innermost pair or pairs first.

Thus: $9+(8-6)=9+2=11.$
$19-(8-6)=19-2=17.$
$8\times(4+2)=8\times6=48.$
$9\div(3+2)=9\div5=1\frac{4}{5}.$
$102-[16+8-2(4+2)]=102-[16+8-2\times6]=$
$102-[16+8-12]=102-12=90.$
$(8-3)\times(5+14)=5\times19=95.$

64. In algebraic expressions, very frequently the quantities within the grouping symbols cannot be worked out. In order to remove them the following rules must be applied.

RULE 1: *If the grouping symbols are immediately preceded by a + sign they can be removed without making any change.*

EXAMPLE: $A+(B-C)=A+B-C$
$6+(7-2)=6+7-2.$

RULE 2: *If the grouping symbols are immediately preceded by a − sign it is necessary to change the signs of all quantities within them before they can be removed.*

EXAMPLE: $A-(B-D+X)=A-B+D-X.$
$10-(8-6+2)+6=10-8+6-2+6.$

RULE 3: *If the grouping symbols are immediately preceded by a number or quantity, multiplication is implied and each quantity within the grouping symbols must be multiplied by that number or quantity; then rule 1 or 2 is applied.*

EXAMPLE:

$$A-2(B-C+H)=A-(2B-2C+2H)=A-2B+2C-2H.$$
$$9-2(4-2)=9-(8-4)=9-8+4.$$

RULE 4: *If the grouping symbols are immediately followed by a ÷ sign, divide every quantity within the grouping symbols by the quantity following the division sign; then rule 1 or 2 is applied.*

EXAMPLE: $D-(B-C)\div A=D-\left(\frac{B}{A}-\frac{C}{A}\right)=D-\frac{B}{A}+\frac{C}{A}.$

$$10-(18+12)\div 3=10-(6+4)=10-6-4.$$

65. When two sets of grouping symbols are needed, the parenthesis and bracket are used.

EXAMPLE: $9\times[(20-5)\div 3]$ means that the difference between 20 and 5 is to be divided by 3, and this result multiplied by 9.

Thus, $9\times[(20-5)\div 3]=9\times[15\div 3]=9\times 5=45.$

$$D[(A-B)\div C]=D\left[\frac{A}{C}-\frac{B}{C}\right]=\frac{AD}{C}-\frac{BD}{C}.$$

EXERCISE 12

1. $42-(9+6)+2=?$
2. $12-(7-3)=?$
3. $2+2(8-6)-2+6(4+3)=?$
4. $2+7[6+2(2+4)-3]=?$
5. $(16-2)(17+9)=?$
6. $2\sqrt{10+6}-2(4-2)=?$
7. $5\left(\frac{15-6}{2(4+3)}\right)-\frac{34-2}{16}=?$
8. $2+\{14-(6+2)\}\{(6+2)(18+2)-90\}-6\div 2=?$

IX

FORMULAS

66. A **formula** is a rule expressed in algebraic symbols. Formulas are much more convenient than rules and require less space. The symbols in a formula represent numbers and, to apply a formula, it is only necessary to substitute the given values in place of these letters or symbols.

A rule for finding the area of a triangle is as follows: *From the half sum of the three sides subtract each side separately. This will give three differences. The square root of the continued product of the three differences and the half sum equals the area.*

This rule expressed as a formula is:

$A = \sqrt{S(S-A)(S-B)(S-C)}$, where A, B, and C represent the lengths of the sides, and S the half sum of the three sides.

EXAMPLE: If the three sides of a triangle are 3, 4, and 5; then $S = (3+4+5) \div 2$ or 6.

$\text{Area} = \sqrt{S(S-A)(S-B)(S-C)} = \sqrt{6(6-3)(6-4)(6-5)} = \sqrt{6 \times 3 \times 2 \times 1} = \sqrt{36} = 6.$

67. The multiplication sign (which is likely to be mistaken for the letter X) is usually omitted between two numbers or literal quantities or grouping symbols and, if needed, is indicated by a dot (·).

Thus, 6×8 is written $6 \cdot 8$; $A \times B = .286 \times P$ is written $AB = .286P$.

It is also customary to omit the multiplication sign in expressions similar to the following: $S\times(A+C)\times(D-F)$ is written $S(A+C)(D-F)$.

An expression like $3B^2=3BB$; $(5B)^3=5\cdot5\cdot5BBB=5^3B^3$; $(A+B)^2=(A+B)(A+B)$.

An expression like $\sqrt{\frac{A}{B}}=\frac{\sqrt{A}}{\sqrt{B}}$; $\sqrt{\frac{9}{16}}=\frac{\sqrt{9}}{\sqrt{16}}=\frac{3}{4}$.

NOTE: $\sqrt{A}+B$ does not mean $\sqrt{A+B}$; $\sqrt{A}B$ does not mean $\sqrt{AB}$.

EXERCISE 13

1. If A = 6, B = 2, C = 8, and D = A + B, find the value of $\frac{3(2+B)+ABC-3}{CD-(BC-A)+3B}$.

Sharp V Thread

Fig. 22

2. In Fig. 22, if A = .866P and P = .125″, find A.

U. S. Form Thread

Fig. 24

4. In Fig. 24, if A = .6495P and P = $\frac{1}{12}$″, find A.

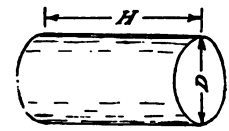
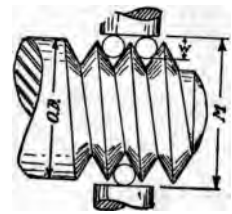


Fig. 23

3. In Fig. 23, if volume equals .7854D²H, find the volume when H = 10″ and D = 6″.



Sharp V Thread

Fig. 25

5. In Fig. 25, if O.D. = 1.125″, N = 5, and W = .130″, find M, using the formula, $M = O.D. - \frac{1.732}{N} + 3W$.

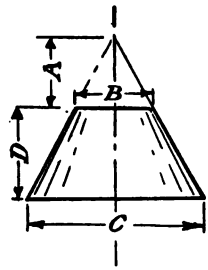


Fig. 26

6. In Fig. 26, if $A=\frac{BD}{C-B}$ and if B=2, C=5, and D=6, find A.

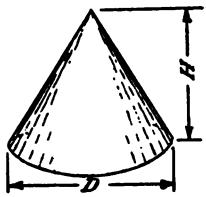


Fig. 28

8. In Fig. 28, if volume= 1.0472 R^2H and D=2R, find the volume when H=6″ and D=7″.

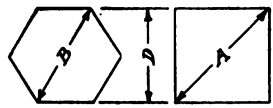


Fig. 30

10. In Fig. 30, if A= 1.4142D and D=1.625″, find A.

11. In Fig. 30, if B= 1.1547D and D=2.034″, find B.

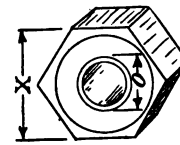


Fig. 27

7. In Fig. 27, if $X=1\frac{1}{2}O+\frac{1}{8}$″, and if $O=\frac{7}{8}$″, find X.

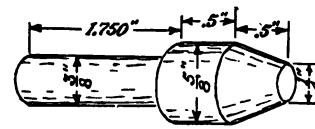


Fig. 29

9. In Fig. 29, from dimensions given, find the volume.

Fig. 31

12. In Fig. 31, since A (addendum) $=\frac{1}{P}$ (P being diametral pitch), if gear shown is 10 diametral pitch, what does A equal?

X

SIMPLE GEARING

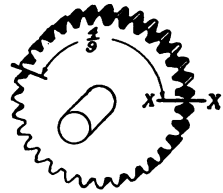


Fig. 32

68. If the gear shown has 32 teeth and is revolved one tooth, it will cause the shaft to make $\frac{1}{32}$ of a revolution. Then 5 teeth will cause the shaft to revolve $5 \times \frac{1}{32}$ or $\frac{5}{32}$ of a revolution. If the gear is revolved 75 teeth, it will cause the shaft to revolve $75 \times \frac{1}{32}$ or $\frac{75}{32}$ revolutions.

When the gear is revolved $\frac{1}{32}$ of a revolution, it will cause one tooth to pass the line XX. If the gear is revolved $\frac{4}{9}$ of a revolution, it will cause $\frac{4}{9} \div \frac{1}{32}$ or $14\frac{2}{9}$ teeth to pass the line XX.

69. If one tooth in gear A is moved across the line XX, it will cause one tooth in gear B to cross the line XX. If

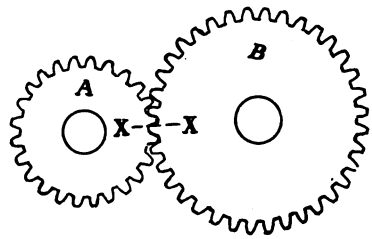


Fig. 33

$16\frac{1}{2}$ teeth in gear A are moved across the line XX, then $16\frac{1}{2}$ teeth in gear B will also cross the line XX.

From this it can be seen that if gear A has 23 teeth and is given one complete revolution, it will cause 23 teeth in gear B (which has 36 teeth) to pass the line XX. If one tooth will cause gear B to make $\frac{1}{36}$ of a revolution, then 23 teeth will cause it to make $\frac{23}{36}$ of a revolution. Therefore, while gear A makes one revolution, gear B will make $\frac{23}{36}$ of a revolution; and while gear B makes one revolution, gear A will make $\frac{36}{23}$ revolutions.

The numbers of revolutions of two gears are to each other inversely as their numbers of teeth. That is, if gear A has 23 teeth and gear B has 36 teeth, gear B will make 23 revolutions while gear A makes 36 revolutions.

PROOF: If 23 teeth pass line XX in one revolution of gear A, then 23×36 teeth will pass the line in 36 revolutions. This will cause 23×36 teeth in gear B also to cross the line XX. If 36 teeth crossing line XX causes gear B to make one revolution, then 23×36 teeth will cause gear B to make $23 \times 36 \div 36$, or 23 revolutions.

The following problem shows how to ascertain the number of revolutions one gear will make while the other makes a given number of revolutions.

PROBLEM: While B makes 12 revolutions, how many will A make, if B has 36 teeth and A has 23 teeth?

SOLUTION: One revolution of B will cause A to make $\frac{36}{23}$ revolutions. Therefore, 12 revolutions of B will cause A to make $12 \times \frac{36}{23}$ or $18\frac{18}{23}$ revolutions.

70. A set of gears resembles a fraction. That is, multiplying or dividing a set by the same number will not change the ratio.

EXAMPLE: If a set of gears has 11 and 15 teeth, as in Fig. 34, but a set with a greater number of teeth would be more convenient, this more suitable set could be found by multiplying both gears by the same number.

(11 to 15) multiplied by 4 equals (44 to 60) shown at B.
(11 " 15) " " 2 " (22 " 30) " " C.
(11 " 15) " " 3 " (33 " 45) " " D or E.

The ratio of X to Y is the same in sets A, B, C, D, and E.

Idler gears (marked O) are used to transmit power and reverse motion.

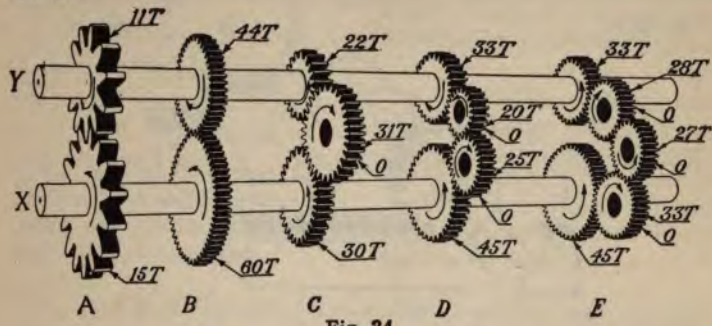


Fig. 34

The number of teeth in an idler will not change the ratio; that is, the ratio of X and Y is the same in sets C, D, and E.

71. To connect two shafts with suitable gears when the revolutions that each shaft must make are given, it is only necessary to invert the given revolutions and call them gear teeth.

EXAMPLE 1: In Fig. 35, if A makes 24 revolutions while B makes 19 revolutions, then A must have 19 teeth and B 24 teeth.

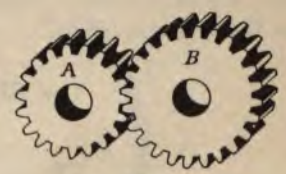


Fig. 35

EXAMPLE 2: In Fig. 35, if A makes $\frac{2}{3}$ of a revolution while B makes 1.7 revolutions, then A must have 1.7 teeth and B $\frac{2}{3}$ of a tooth. It is impossible to construct a gear with a fractional part of a tooth; therefore, these last numbers of gear teeth are multiplied by some number to clear of fractions.

Thus, $A = 1.7 \times 30 = 51$ teeth, and $B = \frac{2}{3} \times 30 = 20$ teeth.

Since it is impossible to construct a gear with a fractional part of a tooth, it would be better to say A must have $1.7 \times N$ teeth and B, $\frac{2}{3} \times N$ teeth. Then let N equal some number that will cause both gears to have a whole number of teeth after the multiplication has been performed. In the above case $N = 30$.

72. To determine the distance a gear will move on a rack, it is necessary to know the distance from tooth to tooth on the rack.

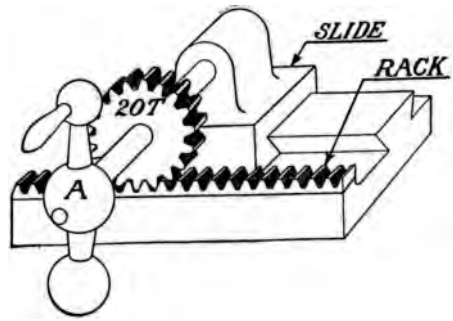


Fig. 36

EXAMPLE: In Fig. 36, if the distance from tooth to tooth on the rack is .400 of an inch, how far will the slide move while A makes 4 revolutions?

SOLUTION: While A makes one revolution, the slide will move $20 \times .400$ or 8 inches. Therefore, 4 revolutions will move it 4×8 or 32 inches.

EXERCISE 14

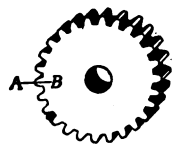


Fig. 37

1. In Fig. 37, if a gear with 26 teeth is revolved $\frac{4}{7}$ of a revolution, how many teeth pass line AB?

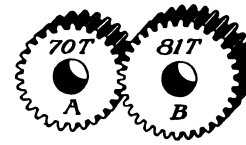


Fig. 38

2. In Fig. 38, while A makes 7 revolutions, how many revolutions will B make?

3. In Fig. 38, while B makes $\frac{4}{9}$ of a revolution, what part of a revolution will A make?

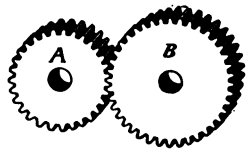


Fig. 39

4. In Fig. 39, how many teeth must A and B have so that while A makes 55 revolutions B will make 37 revolutions?

5. In Fig. 39, how many teeth must A and B have so that while A makes $\frac{4}{7}$ of a revolution B will make $\frac{1}{18}$ of a revolution?

6. In Fig. 39, according to a gear calculation the gear on shaft A was to have 11 teeth, and the gear on shaft B 13 teeth. If there were a gear with 66 teeth, what other gear would have to be made?

7. In Fig. 40, according to a certain gear calculation, the gear on shaft A was to have 1537 teeth and the gear on shaft B was to have 2597 teeth. What two gears would answer the same purpose and have teeth numbering between 20 and 100?

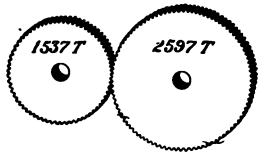


Fig. 40

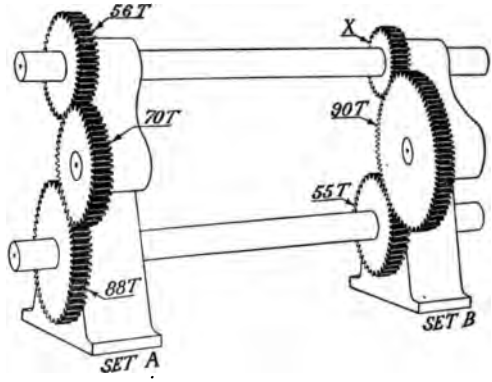


Fig. 41

8. In Fig. 41, how many teeth must the gear marked X have in set B, so that the ratio will be the same as in set A?

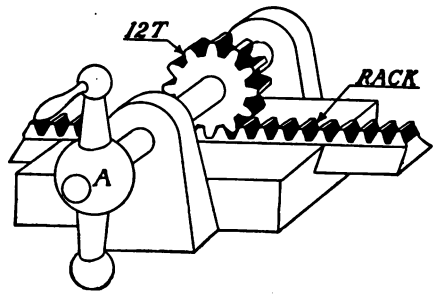


Fig. 42

9. In Fig. 42, if the distance from tooth to tooth in the rack is .200 of an inch, how far will 3 revolutions of the handle A cause the rack to move?

XI

COMPOUND GEARING

73. For ordinary purposes the number of teeth in a gear is between the limits of 12 and 100, and, if the numbers of teeth in the simple gears are unusually large, it is impossible to use them. In such cases they are treated as **imaginary simple gears,** and, if the numbers of teeth in both of the imaginary simple gears can be factored, the factors can be used in **compound gearing** in place of the simple gears.

EXAMPLE: In Fig. 43, if A makes 713 revolutions while B makes 1189 revolutions, then A must have 1189 teeth and B 713 teeth. Factoring 1189 into 29×41 and 713 into 23×31 and using the factors in compound gearing, as shown in Fig. 44, will cause the ratio between A′ and B′ to be the same as between A and B.

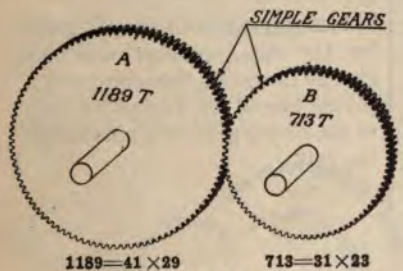


Fig. 43

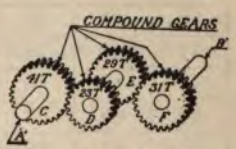


Fig. 44

RULE FOR COMPOUND GEARING:

First Step: Factor the number of teeth in one of the large simple gears and place one of the factored gears on the shaft that was to receive this simple gear.

Second Step: Factor the number of teeth in the other simple gear and place one of these on the shaft that was to receive this simple gear.

The two remaining factored gears are keyed so that they revolve together and are placed in mesh so that the factors of the number of teeth in one of the simple gears will not mesh together.

CAUTION: The 23-tooth gear can not mesh with the 31-tooth gear because 23 and 31 are the factors of the simple gear 713.

If one of the factors of A is multiplied by a number, it is necessary to multiply a factor of B by the same number in order to keep the ratio the same.

Thus, any of the following sets of compound gears could be used and the ratio of A to B would be the same as the ratio of A′ to B′.

Factors of A.		Factors of B.	
C	E	D	F
41	29	23	31
82	29	46	31
41	58	23	62
41	58	46	31
82	29	23	62
41	87	69	31
41	87	23	93

Multiplying both simple gears by the same number will not change the ratio; therefore, multiplying one of the factors of each of the simple gears will not change the ratio.

74. Example of a gear problem where factors have to be multiplied to obtain proper numbers of teeth in the compound gears:

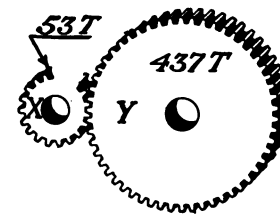


Fig. 45

PROBLEM: In Fig. 45, what compound gears as in Fig. 46 could be used in place of the simple gears 53 and 437? Keep gears between the limits of 20 and 100.

SOLUTION: Factor the number of teeth in the simple gears. Thus, $53=53\times1$, and $437=19\times23$. If 19 is multiplied by 5, it is necessary to multiply 1 by 5 to keep the ratio the same. The factors will then be 53×5 and 95×23. Then, if 23 is multiplied by 4 it is necessary to multiply 5 by 4 to keep the ratio the same. The factors will then be 53×20 and 95×92, which are between the required limits. Placing in proper compound order (Fig. 46), A = 53, B = 92, C = 20, and D = 95.

53×1	19×23
53×5	95×23
53×20	95×92

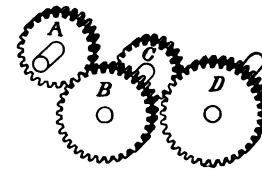


Fig. 46

Condensed form.

$$\frac{53}{437}=\frac{53\times1}{19\times23}=\frac{53\times(1\times5)}{(5\times19)\times23}=\frac{53\times(1\times4\times5)}{(5\times19)\times(23\times4)}=\frac{53\times20}{95\times92}.$$

75. Method of determining the number of revolutions one shaft will make, while the other shaft makes a given number of revolutions:

PROBLEM: In Fig. 47, while A makes 10 revolutions, how many revolutions will D make?

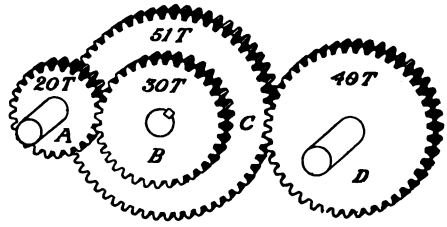


Fig. 47

SOLUTION: While A makes 10 revolutions, B and C will make $10\times\frac{20}{60}$, or $\frac{20}{3}$ revolutions. While C makes $\frac{20}{3}$ revolutions, D will make $\frac{20}{3}\times\frac{51}{40}$, or 8.5 revolutions.

Therefore, while A makes 10 revolutions, D will make $10\times\frac{20}{60}\times\frac{51}{40}$ or 8.5 revolutions.

76. To connect two shafts with suitable compound gears when the revolutions that each shaft must make are given, it is only necessary to invert the given revolutions to obtain the number of teeth in the imaginary simple gears. The number of teeth in the imaginary simple gears are then factored, and the factors used in compound gearing. When the numbers cannot be factored they are reduced to other approximate numbers by the principle of continued fractions. (See Article 240.)

PROBLEM: In Fig. 48, how many teeth must A, B, C, and D have so that, while A makes 483 revolutions, D will make 837 revolutions.

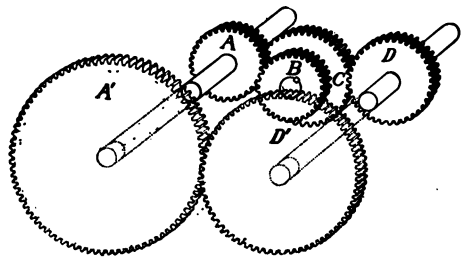


Fig. 48

SOLUTION: By knowing the number of revolutions the imaginary simple gears (A′ and D′) must make, the number of teeth in them is obtained by inverting the revolutions. Thus, if A′ makes 483 revolutions while D′ makes 837 revolutions, A′ must have 837 teeth and D′ 483 teeth. After factoring ($837 = 27 \times 31$; $483 = 21 \times 23$) and placing in proper compound order, A = 27 teeth, B = 21 teeth, C = 31 teeth, and D = 23 teeth. (See Article 73.)

77. Example of a problem in which compound gearing and a rack are used:

PROBLEM: In Fig. 49, how many teeth must gear marked X have so that, while rack rises $\frac{1}{4}$ of an inch, A will make one revolution?

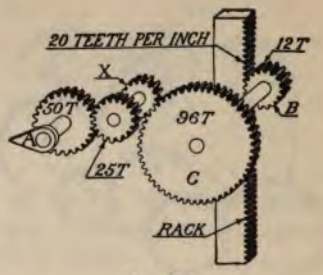


Fig. 49

SOLUTION: While A makes one revolution, X will make 2 revolutions; while the rack moves $\frac{1}{4}$ of an inch, it will move 5 teeth which will cause B and C to make $\frac{5}{12}$ of a revolution. To find the number of teeth X and C must have so that, while X makes 2 revolutions, B will make $\frac{5}{12}$ of a revolution, it is only necessary to invert the revolutions and call them gear teeth. Thus, $X=\frac{5}{12}$ teeth and $C=2$ teeth. Since we know that multiplying both gears by the same number will not change the ratio, 2 is multiplied by 48 to equal 96; therefore, $\frac{5}{12}$ must be multiplied by 48 also to make it equal to the number of teeth in X. Thus, $C=2\times48=96$ and $X=\frac{5}{12}\times48=20$.

EXERCISE 15

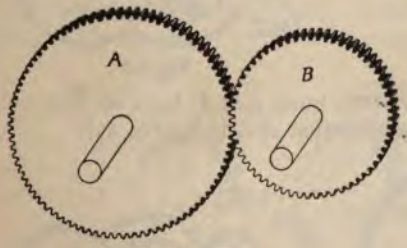


Fig. 50

Fig. 51

1. In Fig. 50, if gear A has 1739 teeth and B 1541 teeth, how many teeth should C, D, E, and F have so that the ratio of A′ to B′ (Fig. 51) will be the same as A to B?

2. In Fig. 50, if gear A has 4753 teeth, B 1333 teeth, and in Fig. 51, if C has 49 teeth, and F 43 teeth, how many teeth must D and E have so that the ratio of A′ to B′ will be the same as the ratio of A to B?

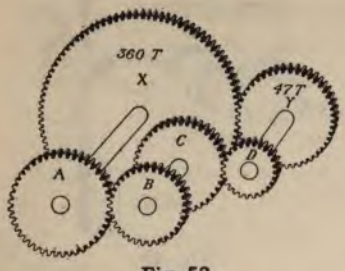


Fig. 52

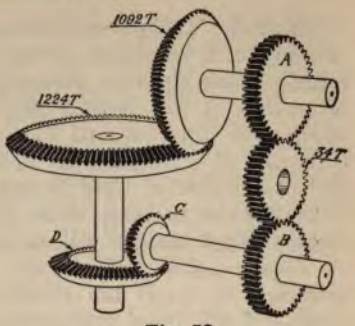


Fig. 53

3. In Fig. 52, how many teeth each must the gears A, B, C, and D have so that the ratio will be the same as that of X to Y?

4. In Fig. 53, find the number of teeth in each of the gears, A, B, C, and D.

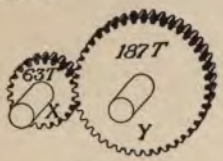


Fig. 54

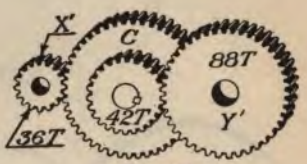


Fig. 55

5. In Fig. 55, how many teeth must gear C have so that the ratio of X′ to Y′ will be the same as that of X to Y (Fig. 54)?

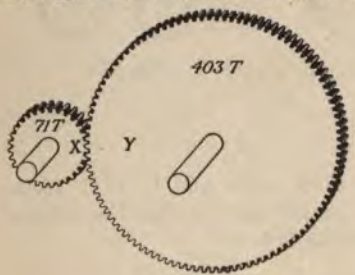


Fig. 56

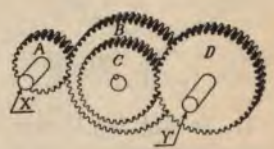


Fig. 57

6. In Fig. 57, how many teeth between the limits of 20 and 100 must the gears A, B, C, and D have so that the ratio of X′ to Y′ will be the same as that of X to Y (Fig. 56)?

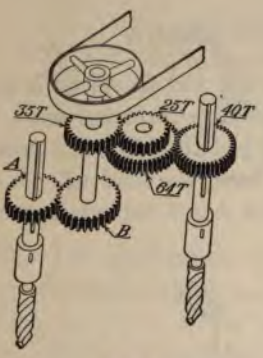


Fig. 58

7. In Fig. 58, how many teeth between the limits of 20 and 100 must gears A and B have so that both drills will have the same speed?

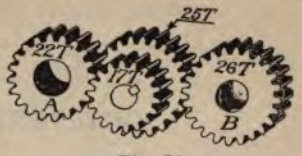


Fig. 59

8. In Fig. 59, while A makes 6.455 revolutions, how many will B make?

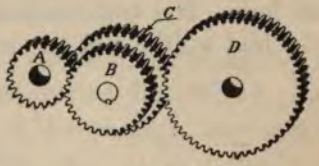


Fig. 60

9. In Fig. 60, how many teeth must A, B, C, and D have so that while A makes 203 revolutions, D will make 121 revolutions?

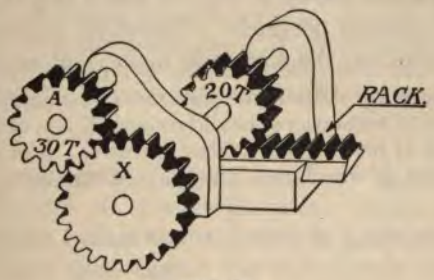


Fig. 61

10. In Fig. 61, if the distance from tooth to tooth in the rack is .200 of an inch, how many teeth must X have so that 4 revolutions of A will move the rack 8 inches?

XII

GEAR AND LEAD SCREW

78. The **lead** is the distance a nut will travel on a screw in one complete revolution. If a single-threaded screw has 5 threads per inch, the lead will be $\frac{1}{5}$ of an inch. If a double-threaded screw has 5 threads per inch, the lead will be $\frac{2}{5}$ of an inch. If a triple-threaded screw has 5 threads per inch, the lead will be $\frac{3}{5}$ of an inch.

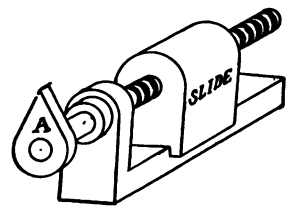


Fig. 62

79. The screw shown in Fig. 62 has a single thread, and there are 7 threads per inch. Therefore, if the screw A is given 1 revolution, the slide will move $\frac{1}{7}$ of an inch.

If A is given 2 revolutions the slide moves $2\times\frac{1}{7}$, or $\frac{2}{7}$ of an inch.

If A is given 27 revolutions the slide moves $27\times\frac{1}{7}$, or $3\frac{6}{7}$ inches.

If A is given $4\frac{1}{2}$ revolutions the slide moves $4\frac{1}{2}\times\frac{1}{7}$, or $\frac{9}{14}$ of an inch.

If A is given $\frac{3}{6}\frac{1}{0}$ revolution the slide moves $\frac{3}{6}\frac{1}{0}\times\frac{1}{7}$, or $\frac{3}{4}\frac{1}{2}\frac{1}{0}$ of an inch.

To move the slide $\frac{1}{7}$ of an inch A must make 1 revolution.

To move the slide $\frac{2}{7}$ of an inch A must make $\frac{2}{7}\div\frac{1}{7}$, or 2 revolutions.

To move the slide $\frac{3}{4}\frac{7}{9}$ of an inch A must make $\frac{3}{4}\frac{7}{9}\div\frac{1}{7}$, or $5\frac{2}{7}$ revolutions.

To move the slide $4\frac{1}{2}$ inches A must make $4\frac{1}{2}\div\frac{1}{7}$, or $31\frac{1}{2}$ revolutions.

Gear and Lead Screw for a Lathe

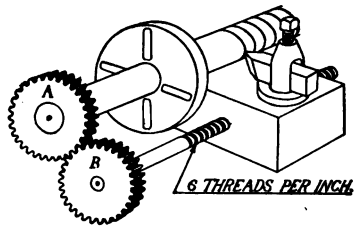


Fig. 63

80. The illustration (Fig. 63) shows the gear and lead screw arrangement on a lathe for cutting threads.

Problem 1: If A = 60 teeth and B = 50 teeth, how many threads per inch will the lathe cut?

Solution: While A makes 1 revolution, the screw will make $\frac{60}{50}$ revolutions. If 1 revolution of the screw moves the tool $\frac{1}{6}$ inch, $\frac{60}{50}$ revolutions will move it $\frac{60}{50} \times \frac{1}{6}$ or $\frac{1}{5}$ inch. Therefore, the lathe will cut a $\frac{1}{5}$-inch lead or 5 threads per inch.

Problem 2: How many teeth must the gears A and B have so that the lathe will cut 13 threads per inch?

Solution: 13 threads per inch is a $\frac{1}{13}$-inch lead. To move the tool $\frac{1}{13}$ inch, B must make $\frac{1}{13} \div \frac{1}{6}$ or $\frac{6}{13}$ revolution. While B makes $\frac{6}{13}$ revolution, A must make 1 revolution; therefore A must have $\frac{6}{13} \times N$ teeth and B, $1 \times N$ teeth. If N = 52, A = 24 teeth and B = 52 teeth.

81. The **micrometer screw,** for the English system of measurement, has 40 threads per inch. From this it may be seen that one turn of the **thimble** will advance the **spindle** $\frac{1}{40}$ or .025 inch. (Fig. 64.)

If we know the distance 1 revolution of the thimble moves the spindle, the number of revolutions required for any distance is readily obtained by dividing the distance by .025. Thus, to move the spindle .207 inch, the thimble must make $.207 \div .025$ or $8\frac{7}{25}$ revolutions.

The distance a given number of revolutions of the thimble will move the spindle is readily obtained by multiplying the distance by .025. Thus, $9\frac{4}{25}$ revolutions of the thimble will move the spindle $9\frac{4}{25} \times .025$ or .229 inch.

The thimble of a micrometer is divided into 25 parts, so that to turn it one part moves the spindle $\frac{1}{25}$ of .025 inch or .001 inch.

MICROMETER

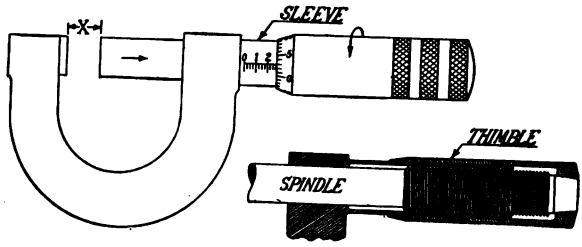


Fig. 64

The divisions on the **sleeve** are .025 inch apart; therefore, it is evident that one more mark will be visible for each turn of the thimble, when it is being screwed out. The numbers opposite every fourth division are .100 inch apart.

In the illustration (Fig. 64) the distance X, or the micrometer reading, equals $.200+(3\times.025)+.003$ or .278 inch.

82. Example of compound gears and lead screw:

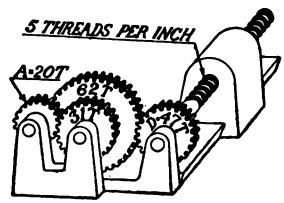


Fig. 65

PROBLEM: In Fig. 65, if A makes one revolution, how far will the slide move?

SOLUTION: While A makes one revolution, D will make $\frac{20}{81}\times\frac{62}{47}$ of a revolution. If one revolution of D moves the slide $\frac{1}{5}$ of an inch, $\frac{20}{81}\times\frac{62}{47}$ revolution will move it $\frac{20}{81}\times\frac{62}{47}\times\frac{1}{5}$ or .17021 inch.

EXERCISE 16

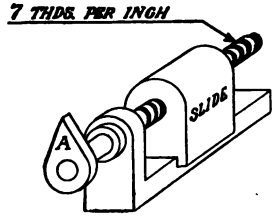


Fig. 66

1. In Fig. 66, while A makes $4\frac{2}{3}$ revolutions how far will the slide move?

2. In Fig. 66, while A makes 8.5 revolutions, how far will the slide move?

3. In Fig. 66, how many revolutions must A make to cause the slide to move 1.703 inches?

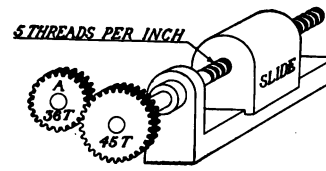


Fig. 67

4. In Fig. 67, while A makes $5\frac{1}{5}$ revolutions, how far does the slide move?

5. In Fig. 67, how many revolutions must A make to cause the slide to move 4.362 inches?

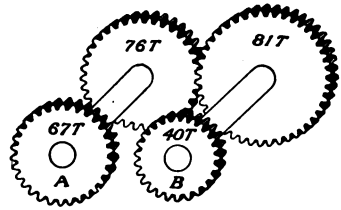


Fig. 68

6. In Fig. 68, while A makes 1 revolution, how many revolutions will B make?

7. In Fig. 69, while A makes 2 revolutions, how many revolutions will B make?

8. In Fig. 69, while B makes 2 revolutions, how many revolutions will A make?

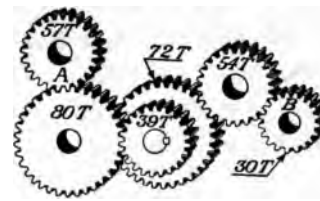


Fig. 69

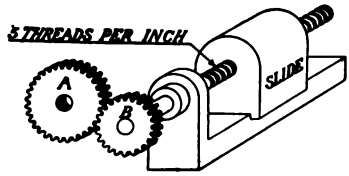


Fig. 70

9. In Fig. 70, how many teeth must A and B have so that, while A makes 1 revolution, the slide will move .355 of an inch?

10. In Fig. 71, if the crank is turned 1 revolution and 5 holes, what part of a revolution will the worm wheel make?

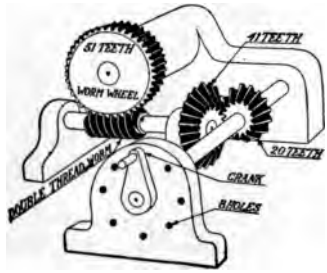


Fig. 71

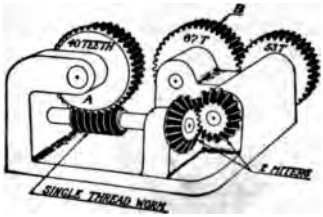


Fig. 72

11. In Fig. 72, how many revolutions must B make to cause A to make one complete revolution?

XIII

RATIO AND PROPORTION

Ratio

83. The **ratio** of two numbers is the number of times one of the numbers contains the other. It may be either a whole number or a fraction.

EXAMPLE: 12 contains 6 two times; therefore the ratio of 12 to 6 is 2. 6 contains 12 one-half time; therefore the ratio of 6 to 12 is $\frac{1}{2}$.

The nature of the problem will indicate whether 2 or $\frac{1}{2}$ is the ratio to be taken into consideration, but you should remember that $\frac{1}{2}$ is the reciprocal of 2.

The ratio of two numbers is represented usually by placing one of the numbers above the other in the form of a fraction.

EXAMPLES: The ratio of 12 to 6 is $\frac{12}{6}$.

The ratio of 6 to 12 is $\frac{6}{12}$.

The ratio of 12.653 to 7.42 is $\frac{12.653}{7.42}$.

The ratio of 7.42 to 12.653 is $\frac{7.42}{12.653}$.

The ratio of $\frac{3}{4}$ to $\frac{1}{2}$ is $\frac{\frac{3}{4}}{\frac{1}{2}}$.

The ratio of $\frac{1}{2}$ to $\frac{3}{4}$ is $\frac{\frac{1}{2}}{\frac{3}{4}}$.

84. The reciprocal of a ratio is the ratio inverted.

If the value of the ratio is more than one, the reciprocal of the ratio will be less than one.

EXAMPLE: If $\frac{47}{16}$ is the ratio, the reciprocal of the ratio is $\frac{16}{47}$.

If the value of the ratio is less than one, the reciprocal of the ratio will be more than one.

EXAMPLE: If $\frac{17}{23}$ is the ratio, the reciprocal of the ratio is $\frac{23}{17}$.

85. The following explanations are very helpful in the solving of problems in proportion. That is, they show why it is not necessary to use the terms "inverse" and "direct" proportion.

If a quantity is multiplied by a ratio whose value is more than one, the product will be more than the original quantity. For example, $12 \times \frac{4}{3} = 16$.

The ratio $\frac{4}{3}$ is more than one; therefore, the product 16 is more than 12.

If a quantity is multiplied by a ratio whose value is less than one, the product will be less than the original quantity. For example, $12 \times \frac{3}{4} = 9$.

The ratio $\frac{3}{4}$ is less than one; therefore, the product 9 is less than 12.

Simple Proportion

86. A **proportion** is an expression of equality between two ratios, or fractions. Proportion is used in solving problems, where the ratios (which must be between two numbers of the same denomination) are equal.

Example: If 6 drills cost \$1.44, 18 drills must cost \$4.32.

$$\frac{18}{6} = \frac{4.32}{1.44}$$

The ratio of the numbers of drills, $\frac{18}{6}$, must equal the ratio of the respective costs $\frac{4.32}{1.44}$.

87. Various methods can be used in solving problems in proportion and they can be explained best by an example.

Problem: If 8 men build a machine in 24 hours, how long will it take 16 men to build it?

Solution: Let X = the time 16 men require. Then the ratio of hours $\left(\frac{24}{X}\right)$ must equal the ratio of the men $\left(\frac{16}{8}\right)$. Thus, $\frac{24}{X} = \frac{16}{8}$.

Multiplying both numerator and denominator of a fraction by the same quantity will not change its value. Then, if 16 must be multiplied by $\frac{24}{16}$ to equal 24, 8 must be multiplied by $\frac{24}{16}$ to equal X.

Thus, $X=8\times\frac{24}{16}=12$. *Ans.* 12 hours.

88. A better solution for such problems is as follows:

RULE: *Multiply the given number which is of the denomination that the answer is to be by the given ratio.*

PROBLEM 1: If 8 men build a machine in 24 hours, how long will it take 16 men to build it?

SOLUTION: 24 hours is of the same denomination as the required answer, and $\frac{8}{16}$ is the given ratio which is less than one, because the answer must be less than 24.

Therefore, $24\times\frac{8}{16}=12$. *Ans.* 12 hours.

PROBLEM 2: In Fig. 73, if 6 revolutions of A move the slide 2.203 inches, how far will 7 revolutions move it?

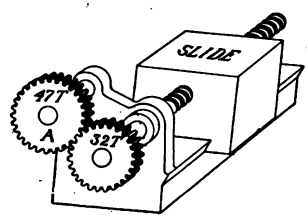


Fig. 73

SOLUTION: 2.203 inches is of the same denomination as the required answer, and $\frac{7}{6}$ is the given ratio which is more than one, because the answer must be more than 2.203.

Therefore, $2.203''\times\frac{7}{6}=2.570''$. *Ans.* 2.570 inches.

EXERCISE 17

1. What is the ratio of 8.432 to 7.67? Answer to be carried to four decimal places.

2. What is the ratio of 5.6 to .2? Answer to be a whole number.

3. What is the ratio of $\frac{1}{3}$ to $\frac{8}{18}$? Answer to be a common fraction.

4. If 9 drills cost \$11.43, what will 4 drills cost?

5. If 23 inches of tool steel cost \$4.56, how many inches can be bought for \$3.52?

6. What common fraction must 64 be multiplied by to obtain 17?

7. In Fig. 74, if shaft A makes 100 revolutions, how many revolutions will shaft B make?

8. In Fig. 75, while A makes 12 revolutions, what number of teeth must B have to make 18 revolutions?

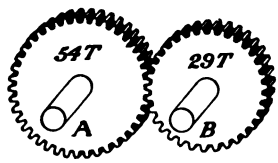


Fig. 74

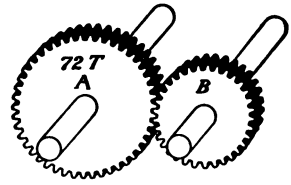


Fig. 75

9. In Fig. 76, if $\frac{4}{5}$ of a revolution of A moves the slide .236 inch, how far will $\frac{7}{8}$ of a revolution move it?

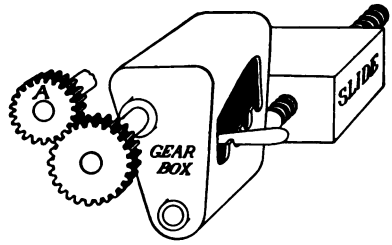


Fig. 76

Product of Ratios

89. A compounded ratio is the product of two or more simple ratios.

For example, the simple ratios $\frac{7}{4}$, $\frac{9}{6}$, and $\frac{2}{3}$ used as a compounded ratio has a value of $\frac{7}{4}\times\frac{9}{6}\times\frac{2}{3}$ or $\frac{7}{4}$.

Example 1: If a man rated at 60 cents per hour, works 10 hours a day, 6 days a week, and his son, rated at 30 cents per hour,

works 4 hours a day, 4 days a week, the ratio of the man's to the son's weekly pay will be the product of the simple ratios.

SOLUTION:

$$\underbrace{\frac{60}{30}}_{\text{Ratio of cents}} \times \underbrace{\frac{10}{4}}_{\text{Ratio of hours}} \times \underbrace{\frac{6}{4}}_{\text{Ratio of days}} = 7\tfrac{1}{2}$$

The ratio of the man's to the son's weekly pay is $7\frac{1}{2}$, or the man earns $7\frac{1}{2}$ times as much as the son earns.

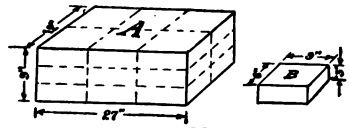


Fig. 77

EXAMPLE 2: In Fig. 77, block A contains block B 18 times. Therefore the ratio of A to B is 18.

$$\underbrace{\frac{27}{9}}_{\text{Ratio of length}} \times \underbrace{\frac{12}{6}}_{\text{Ratio of width}} \times \underbrace{\frac{9}{3}}_{\text{Ratio of height}} = 18$$

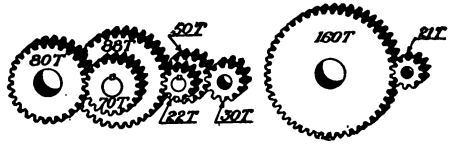


Fig. 78

EXAMPLE 3: In Fig. 78, the product of the ratios of the compound gears shown is equal to the ratio of the simple gears.

Thus, $\frac{80}{70} \times \frac{88}{22} \times \frac{50}{30} = \frac{160}{21}$.

COMPOUND PROPORTION

90. A **compound proportion** is an equality of ratios, one or more of which are compounded.

EXAMPLE: If 12 men mill 150 jigs in 20 days working 6 hours a day, how many days will it take 15 men working 10 hours a day to mill 250 jigs?

By the unit method, the solution would be as follows:

12 men mill 150 jigs, working 6 hours a day, in 20 days.

1 man mills 150 jigs, working 6 hours a day, in 20×12 days.

15 men mill 150 jigs, working 6 hours a day, in $20\times12\div15$ or $\frac{20\times12}{15}$ days.

15 men mill 1 jig, working 6 hours a day, in $\frac{20\times12}{15}\div150$ or $\frac{20\times12}{15\times150}$ days.

15 men mill 250 jigs, working 6 hours a day, in $\frac{20\times12}{15\times150}\times250$ or $\frac{20\times12\times250}{15\times150}$ days.

15 men mill 250 jigs, working 1 hour a day, in $\frac{20\times12\times250}{15\times150}\times6$ or $\frac{20\times12\times250\times6}{15\times150}$ days.

15 men mill 250 jigs, working 10 hours a day, in $\frac{20\times12\times250\times6}{15\times150}\div10$ or $\frac{20\times12\times250\times6}{15\times150\times10}$ or 16 days.

91. A better method is the compound proportion method.

RULE OF COMPOUND PROPORTION: *Multiply that number which is of the denomination that the answer is to be, by the product of the given ratios or ratios inverted. The nature of the problem indicates which ratios are to be inverted.*

NOTE.—The ratios must be between terms of the same denomination.

Days same denomination as answer.

Ratio of men to be less than one because there are more men; therefore the number of days should be less.

Ratio of jigs to be more than one because there are more jigs; therefore the number of days should be more.

$20\times\frac{12}{15}\times\frac{250}{150}\times\frac{6}{10}=16$ days.

Ratio of hours to be less than one because there are more hours; therefore the number of days should be less.

EXERCISE 18

1. If four bars of steel 5 feet long cost $12.50, what will seven bars 6 feet long cost?

2. If four men do 26 pieces of work in 7 hours, how long will it take twelve men to do 40 pieces?

3. If 45 pounds of steel are required for 250 pins 3 inches long, how many pounds would be required for 112 pins 7 inches long?

4. If a block of steel 8 feet long, 3 feet wide, and 2 feet thick, weighs 20539 pounds, what will be the weight of a block of the same kind of steel 10 feet long, 4 feet wide, and 3 feet thick?

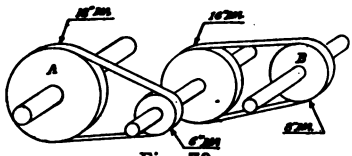


Fig. 79

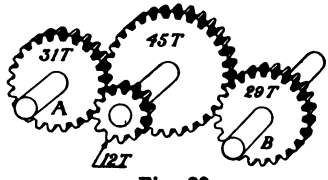


Fig. 80

5. In Fig. 79, if shaft A makes 1000 revolutions, how many revolutions will B make?

6. In Fig. 80, if A makes 100 revolutions, how many revolutions will B make?

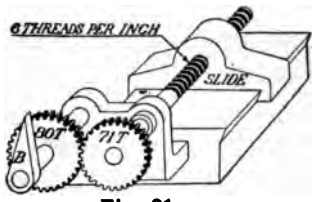


Fig. 81

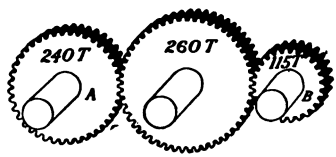


Fig. 82

7. In Fig. 81, if B makes 5 revolutions, how far will the slide move?

8. In Fig. 82, if A makes 50 revolutions, how many revolutions will B make?

9. In Fig. 83, if A makes 6 revolutions, how many revolutions will B make?

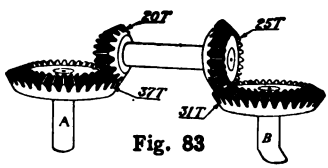


Fig. 83

XIV

GRADUATED DIALS

92. When it is desirable to revolve a lead screw or some other form of shafting a definite amount, **graduated dials** are used as indicators. The dial is permanently fixed to the shafting, and a pointer arrangement is used to keep tab of the number of divisions the dial is being revolved (Fig. 84).

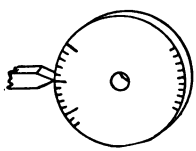


Fig. 84

For example, a dial with 100 marks is divided into 100 equal divisions, and if a shaft is to be revolved $\frac{6}{100}$ of a revolution the dial will be revolved through 6 divisions.

How many divisions would this dial be revolved for 1/200 of a revolution? 1/300 of a revolution? 1/50 of a revolution? 1/160 of a revolution?

93. A shaft can be revolved any part of a revolution, and the part determined accurately if the dial has a suitable number of divisions. For example, to revolve a shaft $\frac{4}{45}$ of a revolution if the dial had 45 divisions, it would be moved through 4 divisions.

94. Proportion can be used to good advantage in solving problems involving graduated dials.

PROBLEM 1: If a dial has 170 divisions, through how many divisions would it be revolved to cause it to make $\frac{1}{4}$ of a revolution?

SOLUTION: If one division equals $\frac{1}{170}$ of a revolution, $\frac{1}{4}$ of a revolution will equal $1 \times \frac{\frac{1}{4}}{\frac{1}{170}}$ or $42\frac{1}{2}$ divisions.

PROBLEM 2: If one revolution of a dial moves a slide .125″, how many divisions should dial have so that each division will move it .005″?

SOLUTION: If one revolution of the dial moves a slide .125″, to move it .005″ would require $1 \times \frac{.005}{.125}$ or $\frac{1}{25}$ of a revolution.

Therefore, the dial should have 25 divisions.

PROBLEM 3: In Fig. 85, how many divisions must A have so that each division will move slide .001″?

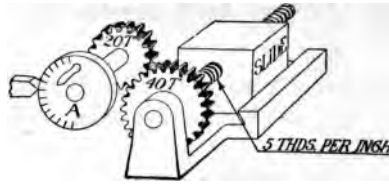


Fig. 85

SOLUTION: First find how far one revolution of the dial will move the slide. Thus, when A makes one revolution the screw will make $\frac{1}{2}$ of a revolution. If one revolution of the screw moves the slide .200″, then $\frac{1}{2}$ of a revolution will move it $\frac{1}{2}$ of .200″ or .100″; therefore, one revolution of the dial will move the slide .100″.

Then since one revolution of the dial moves the slide .100″, to move it .001″ would require $1 \times \frac{.001}{.100}$ or $\frac{1}{100}$ of a revolution.

Therefore, the dial must have 100 divisions.

95. When it is desired to move a slide a very small amount, the worm and worm wheel are frequently used in connection with a screw or a rack.

A rack, worm, and worm wheel construction is shown in Fig. 86.

PROBLEM: Let it be required to find the number of divisions on the dial so that each division will move the slide .0001″.

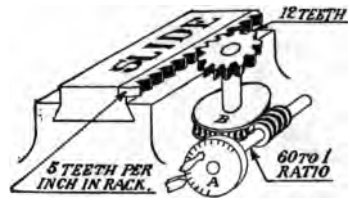


Fig. 86

SOLUTION: First find how far one revolution of the dial A will cause the slide to move. Thus, while A makes one revolution, B

will make $\frac{1}{60}$ of a revolution. If one revolution of B moves the slide the distance of 12 teeth or $\frac{12}{5}$ of an inch, $\frac{1}{60}$ of a revolution will move the slide $\frac{12}{5} \times \frac{\frac{1}{60}}{1}$ or .040″; therefore, one revolution of A will move the slide .040″.

Since one revolution of A moves the slide .040″, to move it .0001″ would require $1 \times \frac{.0001}{.040}$ or $\frac{1}{400}$ of a revolution. Therefore, the dial would have 400 divisions.

EXERCISE 19

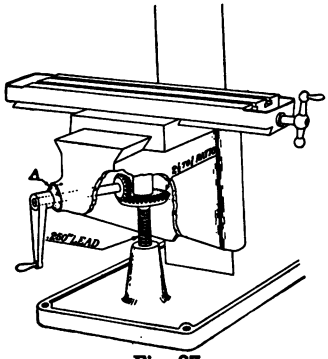


Fig. 87

1. In Fig. 87, how many divisions should A have so that each division will elevate the table .001″?

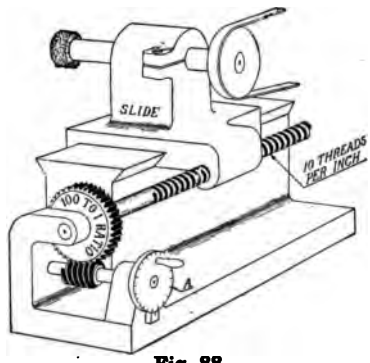


Fig. 88

2. In Fig. 88, how many divisions should A have so that each division will move the slide .0001″?

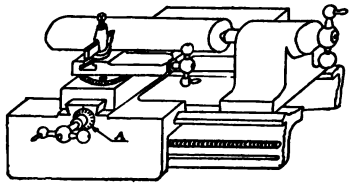


Fig. 89

3. In Fig. 89, the cross-slide screw on the lathe shown has 4 threads per inch and there are 125 divisions on the collar A.

How many divisions should the collar turn to reduce the diameter of the shaft to be turned .018″?

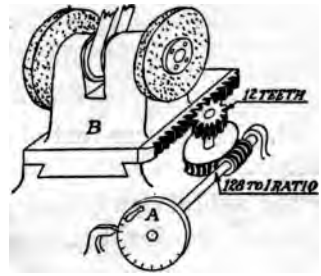


Fig. 90

4. In Fig. 90, if the rack has $2\frac{1}{2}$ teeth per inch, how many divisions should A have so that each division will move B .0005″?

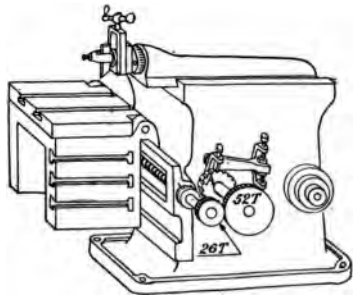


Fig. 91

5. In Fig. 91, if wheel A has 50 notches and the lead screw has $2\frac{1}{2}$ threads per inch, how far will 3 notches of the wheel feed the table?

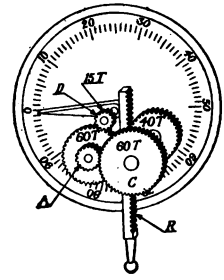


Fig. 92

6. In Fig. 92, if the rack R has 40 teeth per inch and if each division on the dial indicator moves the rack .001″, what number of teeth must A have?

XV

SQUARE ROOT

96. The **square root** of a number is one of its two **equal** factors.

The **radical sign** $\sqrt{}$ indicates that the square root of the number following it is to be found. For example, $\sqrt{9}=3$.

The **vinculum** is used with the radical sign. Thus, $\sqrt{4\times16}=8$. This grouping symbol indicates that the expression under it is to be treated as one quantity.

The square of		$1=1$	The square of		$6=36$
"	"	$2=4$	"	"	$7=49$
"	"	$3=9$	"	"	$8=64$
"	"	$4=16$	"	"	$9=81$
"	"	$5=25$			

97. To extract the square root of a number:

PROBLEM: What is the square root of 87.654?

$$
\begin{array}{r}
9\ 3\ .\ 6\ 2\ 3 \\
\hline
8\,7\,.\,6\,5\,4\,0\,0\,0\,0 \\
8\ 1 \\
\hline
183 \mid\ 6\ 6\ 5 \\
5\ 4\ 9 \\
\hline
1866 \mid 1\ 1\ 6\ 4\ 0 \\
1\ 1\ 1\ 9\ 6 \\
\hline
18722 \mid\ 4\ 4\ 4\ 0\ 0 \\
3\ 7\ 4\ 4\ 4 \\
\hline
187243 \mid\ 6\ 9\ 5\ 6\ 0\ 0 \\
5\ 6\ 1\ 7\ 2\ 9 \\
\hline
1\ 3\ 3\ 8\ 7\ 1
\end{array}
$$

SOLUTION: Point off the given number into groups of two figures, starting at the decimal point. Should there be only one figure in the last group, place a cipher to the right of it.

Thus, 87.654 equals 87.|65|40|

1. Find the largest single number whose square is less than or equal to the first group of figures to the left. Write this number directly over the first group of figures as the first figure of the root. The square of this number is then subtracted as in long division. Thus, $87-81=6$.

2. Bring down and annex to the first remainder the next group of figures to form the new dividend. Double the first figure of the root and place it to the left of the first dividend to form part of the new divisor. Thus $9\times2=18$. Omitting the last figure of this dividend, see how many times the partial divisor is contained into it and place this figure (which cannot exceed nine) as a probable second figure of the root. This number helps to form the complete divisor, being placed to the right of the number or numbers there.

3. This complete divisor is then multiplied by the second figure of the root and the product is placed under the dividend, and subtracted but if this product is larger than the dividend, the second figure of the root is made one smaller and carried through as before.

4. Bring down and annex to this second remainder the next group of figures to form a new dividend. Double the first figures now in the root and place them to the left of this new dividend to form part of the new divisor. Thus, $93\times2=186$. Omitting the last figure of this new dividend, see how many times this partial divisor is contained in it and place this figure as a probable third figure of the root. This number helps to form the complete divisor, being placed to the right of the numbers there. This complete divisor is then multiplied by the third figure of the root, the product placed under the dividend, and then subtracted.

This process is continued until the required root is obtained.

After all the groups of figures have been used, bring down successively two ciphers till the required number of decimals are obtained.

To take care of the decimal point place the decimal points of the root directly above the other decimal point.

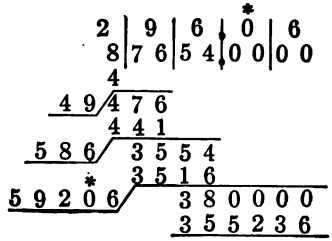
To prove square root, multiply the root by itself and add the last remainder. This should equal the original number.

Thus, $9.3623\times9.3623+.00133871=87.65400000$.

98. Other illustrations of the method of extracting square root.

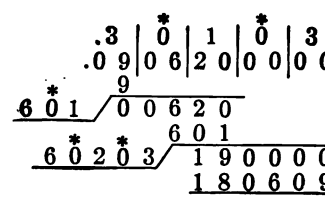
PROBLEM: Extract the square root of 87654.

SOLUTION:



PROBLEM: Extract the square root of .090620.

SOLUTION:



If the partial divisor is not contained in the dividend, the next group of figures is brought down and annexed to the dividend. Then add a cipher to both the divisor and root. This is shown at *.

99. The root of a fraction can be found in two different ways.

EXAMPLE: Extract the square root of $\frac{16.703}{25}$.

FIRST SOLUTION: Extract the square root of both the numerator and the denominator; then reduce the resulting fraction to a decimal.

Thus, $$\sqrt{\frac{16.703}{25}} = \frac{\sqrt{16.703}}{\sqrt{25}} = \frac{4.0869}{5} = .81738$$

SECOND SOLUTION: Reduce the fraction to a decimal; then extract the square root.

Thus, $$\sqrt{\frac{16.703}{25}} = \sqrt{.66812} = .81738.$$

EXERCISE 20

1. Extract the square root of 45369.
2. Extract the square root of 1020100.
3. Extract the square root of $\frac{9}{16}$.

Find the values of:

4. $\sqrt{2222}$.
5. $\sqrt{.0036}$.
6. $\sqrt{9.0001}$.
7. $\sqrt{814.34}$.
8. $\sqrt{\frac{32.1}{16.07}}$.

XVI

CUTTING SPEEDS AND FEEDS

100. Tools of various kinds of steel are used in machining metals. Some tools can be run at higher speed than others, and likewise some metals can be machined at a greater speed than others.

The makers of these steels usually specify at what speed they are to be run. The speed of a tool or work is given in feet per minute. Therefore, in circular work it becomes necessary to calculate the number of revolutions per minute the tool or work must have, so that the proper peripheral speed may be obtained.

The peripheral speed of drills, shafts, cutters, emery wheels, etc. are calculated on the outside diameter.

To Find R. P. M.

101. The outside diameter in inches and the feet per minute the tool or work is to travel being given, the number of **revolutions per minute** can be obtained by the following formula:

$$\frac{\text{feet per minute}}{\text{outside diameter}} \times \frac{42}{11} = \text{R. P. M.}$$

Problem: How many R.P.M. must a 6-inch shaft be given so that it will have a peripheral speed of 48 feet per minute?

Solution: $\frac{48}{6} \times \frac{42}{11} = \frac{336}{11} = 30.5+$ *Ans.*

Problem: How many R.P.M. must a 10-inch emery wheel be given so that it will have a peripheral speed of 5500 feet per minute?

Solution: $\frac{5500}{10} \times \frac{42}{11} = 2100$ *Ans.*

To Find Feet per Minute

102. The R. P. M. and diameter of the shaft being given, the peripheral speed can be found by the formula:

R. P. M. $\times$ outside diameter $\times \frac{11}{42}$ = feet per minute.

Problem: What is the peripheral speed of a 6-inch shaft running at 30.5 R.P.M.?

Solution: $30.5 \times 6 \times \frac{11}{42} = \frac{335}{7} = 47.9+$ feet per minute.

Problem: What is the peripheral speed of a 10-inch emery wheel running at 2100 R.P.M.?

Solution: $2100 \times 10 \times \frac{11}{42} = 5500$ feet per minute.

To Find the Outside Diameter

103. The R. P. M. and the peripheral speed being given, the diameter of wheel or shaft in inches can be found by the following formula:

$$\frac{\text{feet per minute}}{\text{R. P. M.}} \times \frac{42}{11} = \text{outside diameter in inches.}$$

Problem: What diameter of wheel should be used on a spindle running at 2100 R.P.M., so that the peripheral speed will be 5500 feet per minute?

Solution: $\frac{5500}{2100} \times \frac{42}{11} = 10$ inches diameter.

104. To determine the time required to drill a hole when cutting speed of drill and dimensions are given:

Problem: If a cutting speed of 60 feet per minute is used, how long will it take to drill a two-inch hole 4 inches deep if the feed equals .020 inch per revolution?

Solution: Drill must make $\frac{4}{.020}$ or 200 revolutions to advance 4 inches. At 60 feet per minute the drill must make $\frac{60}{2} \times \frac{42}{11}$ or 114.5 R.P.M. At 114.5 R.P.M. it would take 1.74 minutes for 200 revolutions.

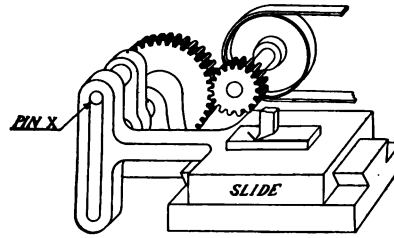


Fig. 93

105. In Fig. 93 the maximum speed of the tool is the same as the peripheral speed of pin X. For example, if the pin X has a peripheral speed of 70 feet per minute, the maximum speed of the slide will be 70 feet per minute.

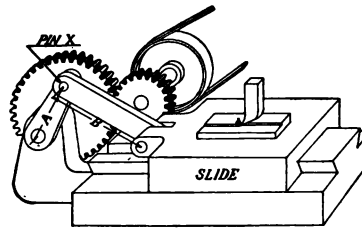


Fig. 94

106. In Fig. 94 the maximum speed of the tool equals (the peripheral speed of pin) $\times \frac{\sqrt{A^2+B^2}}{B}$. For example, if $A=4''$, and $B=10''$, and the peripheral speed of pin equals 70 feet per minute, the maximum speed of the slide equals $70 \times \frac{\sqrt{4^2+10^2}}{10} = 75.4$ feet per minute.

EXERCISE 21

1. At 400 R.P.M., how long will it take a shaft to make 70201 revolutions?

2. How many R.P.M. should a $\frac{1}{4}''$ drill be given to have a cutting speed of 45 feet per minute?

3. How many revolutions are required to drill a hole 3″ deep, if a feed of .025″ is used?

4. How many R.P.M. should an 8″ cutter for a milling machine be given in order to produce a cutting speed of 45 feet per minute?

5. Find the surface speed of an emery wheel 17″ in diameter making 1200 R.P.M.

6. What cutting speed is required to drill a 3″ hole 2″ deep in 4 minutes, if a feed of .025″ is used?

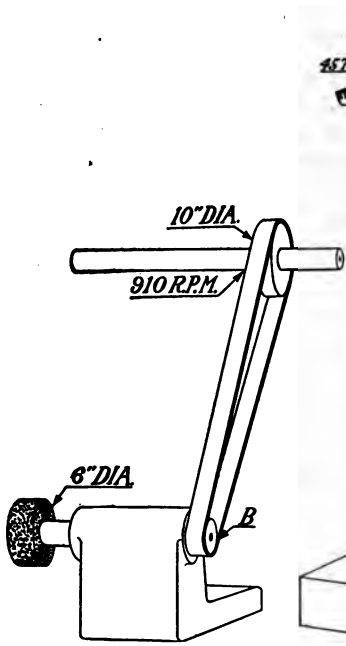


Fig. 95

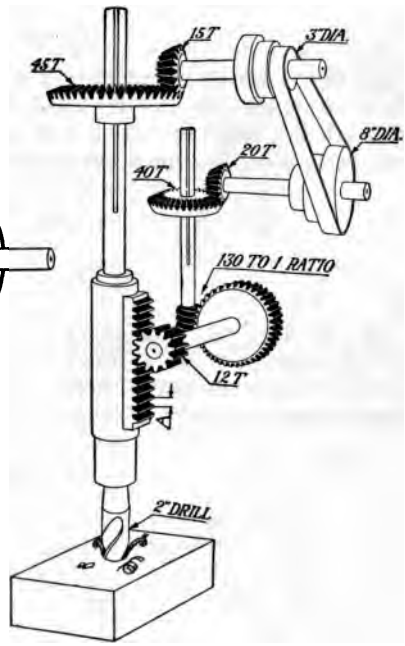


Fig. 96

7. In Fig. 95, find the diameter of pulley B so that the peripheral speed of the emery wheel will be 6000 feet per minute.

8. In Fig. 96, if the distance from tooth to tooth A equals .3142″ and if a cutting speed of 60 feet per minute is used, how long would it take to drill a hole 5 inches deep?

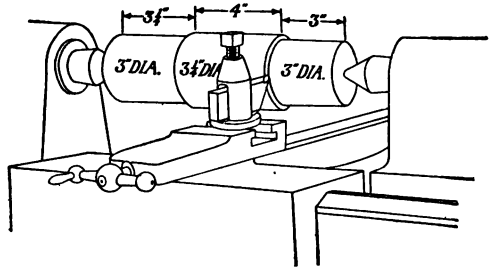


Fig. 97

9. In Fig. 97, if the rough stock is $3\frac{3}{4}'' \times 10\frac{1}{4}''$ and the speeds of the lathe are 20−28−44−68−100 R.P.M., how long would it take to rough this shaft by taking $\frac{1}{8}$ of an inch cuts and using .030" feed, with a running speed not exceeding 60 feet per minute?

10. In Fig. 98, how many R.P.M. must A make to cause the tool to have a maximum speed of 45 feet per minute?

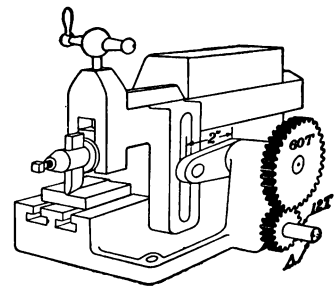


Fig. 98

XVII

GEOMETRY AND MENSURATION

107. Parallel lines are lines that continue the same distance apart (Fig. 99).

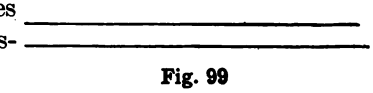


Fig. 99

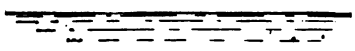


Fig. 100

108. A **horizontal line** is a line that runs parallel with the water level (Fig. 100).

109. An **angle** is the opening between two intersecting lines and its unit of measure is the degree (Fig. 101).

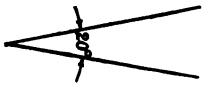


Fig. 101

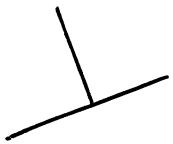


Fig. 102

110. A **perpendicular line** is a line that joins or crosses another line so that the angle on one side equals the angle on the other side (Fig. 102).

111. A **vertical line** is a line that is perpendicular to a horizontal line (Fig. 103).

Fig. 103

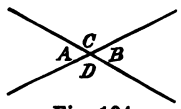


Fig. 104

112. If two lines intersect each other, the **opposite angles** formed will be equal (Fig. 104).

Angle A = angle B.
Angle C = angle D.

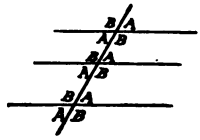
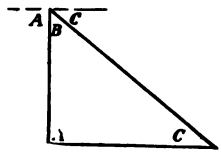


Fig. 105

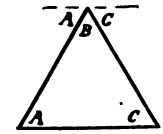
113. If a number of parallel lines are cut by a transversal, the **corresponding angles** will be equal (Fig. 105).

114. A **triangle** is a figure bounded by three straight lines. The sum of the three interior angles of any triangle is always equal to 180°. If the sum of two given angles of a triangle be subtracted from 180°, the remainder will be the third angle.



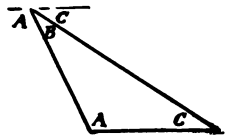
Right Triangle

Fig. 106



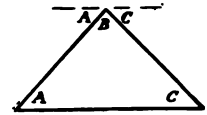
Equilateral Triangle

Fig. 107



Scalene Triangle

Fig. 108



Isosceles Triangle

Fig. 109

115. An **oblique triangle** is one that has no right angle.

The sum of any two interior angles of a triangle is always equal to the complement of the third angle (Fig. 110).

A+B=D; B+C=F; A+C=E.

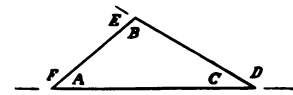


Fig. 110

EXERCISE 22

In the following figures, find the value in degrees of the angle indicated by ?.

1.

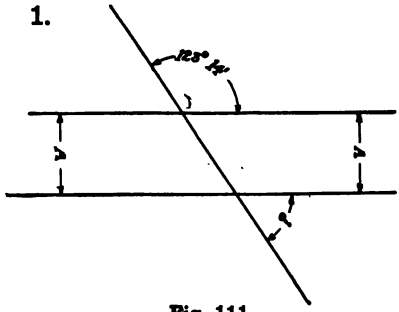


Fig. 111

2.

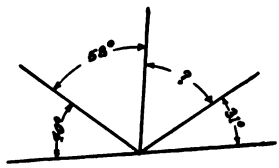


Fig. 112

3.

Fig. 113

4.

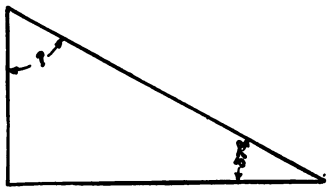


Fig. 114

5.

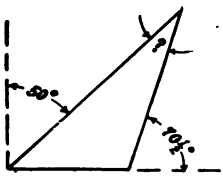


Fig. 115

6.

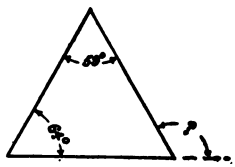
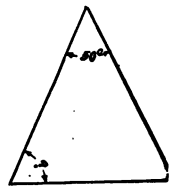


Fig. 116

7.



Isosceles Triangle

Fig. 117

8.

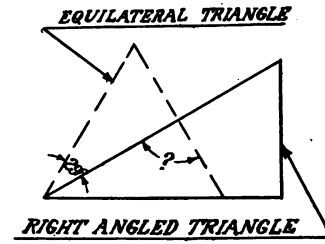


Fig. 118

116. A **circle** is a plane figure bounded by a uniformly curved line called the **circumference,** every part of which is an equal distance from a point called the **center** (Fig. 119).

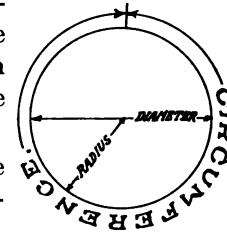


Fig. 119

The **radius** of a circle is a straight line from the center to any point in the circumference.

The **diameter** of a circle is the line passing through the center and terminating at two opposite points in the circumference.

$$A = \text{diameter.} \qquad C = \text{circumference.}$$

$$3.1416 = \pi = \text{Pi.}$$

$$A = \frac{C}{3.1416}. \qquad C = A \times 3.1416.$$

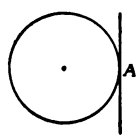


Fig. 120

117. A straight line is a **tangent** to a circle when it touches it at one point only.

The point at which the straight line meets the circumference is called the **point of tangency** (A in Fig. 120).

118. If a line is tangent to a circle, it is also perpendicular to a line drawn from the center of the circle to the point of tangency (Fig. 121).

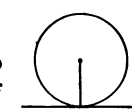


Fig. 121

119. If two circles are tangent to each other, their centers and the point of tangency are in the same straight line (Fig. 122).

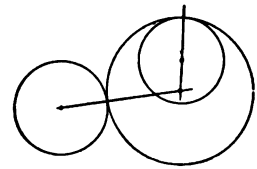


Fig. 122

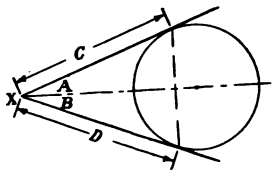


Fig. 123

120. If, from a point without a circle (Fig. 123), two tangents are drawn to a circle, then a line drawn from the **vertex** X through the center of the circle will bisect the angle formed. Thus, angle A = angle B.

The distances from the vertex of the angle to the points of tangency will be equal. Thus, C = D.

If a line is drawn through the points of tangency, it will be perpendicular to the line drawn from the vertex to the center of the circle.

EXERCISE 23

1. What is the circumference of a circle 5.674 inches in diameter?

In Figs. 124, 125, and 126, find the values of the angles indicated by ?.

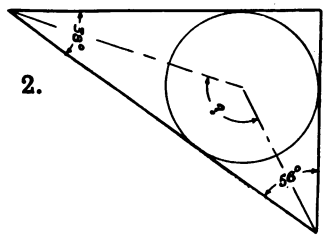


Fig. 124

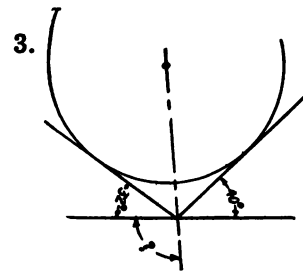


Fig. 125

4.

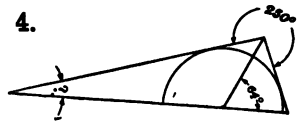


Fig. 126

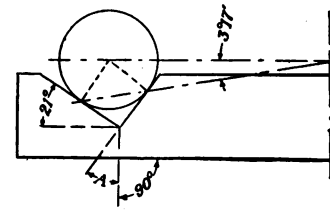


Fig. 127

5. In Fig. 127, what is value of angle A?

121. The **square** has four equal sides and four equal angles.

In Fig. 128, A equals the length of a side or the diameter of the inscribed circle.

C equals the diameter of the circumscribed circle.

$$A=\frac{C}{1.4142}. \qquad C=A\times 1.4142.$$

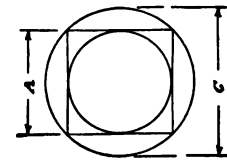


Fig. 128

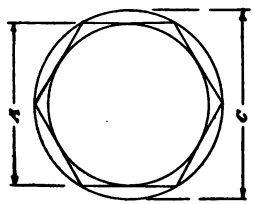


Fig. 129

122. A **regular hexagon** has six equal sides and six equal angles.

In Fig. 129, A equals the diameter of the inscribed circle.

C equals the diameter of the circumscribed circle.

$$A=\frac{C}{1.1547}. \qquad C=A\times 1.1547.$$

123. An **arc** is any portion of the circumference (Fig. 130).

A **chord** is a straight line joining the two extremities of an arc.

A **segment** is a plane figure bounded by a chord and its arc.

A **sector** is a plane figure bounded by two radii and the arc between them.

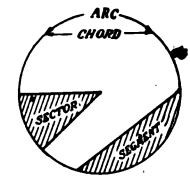


Fig. 130

124. A **polygon** is a plane figure bounded by three or more straight lines.

The sum of the interior angles of a polygon is equal to the product of 180 degrees and the number of sides minus two.

Fig. 131

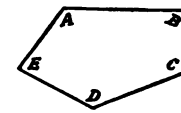


Fig. 132

EXAMPLES:

In Fig. 131, $(4-2)(180)=360$ degrees.
$A+B+C+D=360$ degrees.

In Fig. 132, $(5-2)(180)=540$ degrees.
$A+B+C+D+E=540$ degrees.

EXERCISE 24

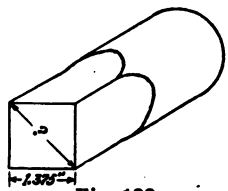


Fig. 133

1. What is the smallest diameter that a shaft could have, so that a perfect 1.375″ square can be milled on it? (Fig. 133.)

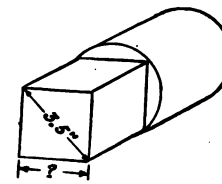


Fig. 134

2. What is the largest perfect square that can be milled on a shaft 3.5″ in diameter? (Fig. 134.)

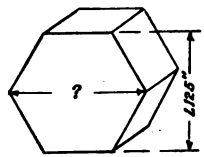


Fig. 135

3. In Fig. 135, find the value of ?.

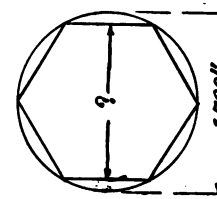


Fig. 136

4. In Fig. 136, find the value of ?.

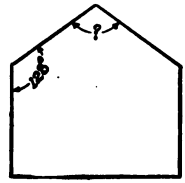


Fig. 137

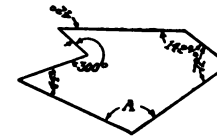


Fig. 138

5. In Fig. 137, find the value of the angle indicated by ?.

6. In Fig. 138, what is the value of the angle A?

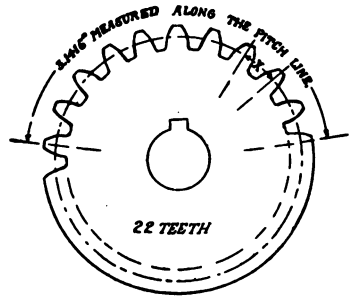


Fig. 139

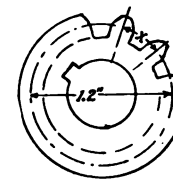


Fig. 140

7. In Figs. 139 and 140, if the gears are in the same pitch, how many teeth will the small gear have, X being the distance from one tooth to the next, measured along the pitch line?

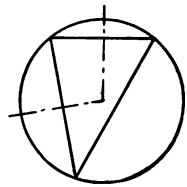


Fig. 141

125. The center of a circle circumscribed about a triangle will be at the point of intersection of perpendiculars erected at the middle points of any two sides (Fig. 141).

126. To inscribe a circle in a given triangle: Bisect two of the angles. The intersection of the two bisectors will be equally distant from the three sides (Fig. 142).

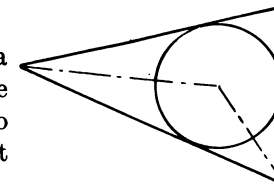


Fig. 142

127. In any right triangle, the sum of the two legs is equal to the sum of the hypotenuse and the diameter of the inscribed circle (Fig. 143).

Thus, A+B=C+D.

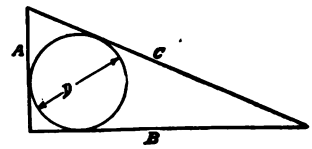


Fig. 143

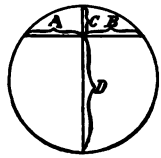


Fig. 144

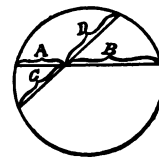


Fig. 145

128. If two chords intersect each other in a circle, the product of the segments of the one chord equals the product of the segments of the other (Figs. 144 and 145).

Thus, A×B=C×D.

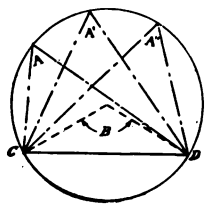


Fig. 146

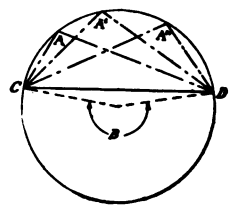


Fig. 147

129. All angles having their vertices in the periphery of a circle and subtended by the same chord are equal (Figs. 146 and 147).

Angles A, A′, and A″ are all subtended by the same chord CD and are equal.

If an angle at the circumference of a circle, between two chords, is subtended by the same arc as the angle at the

center between two radii, then the angle at the circumference is equal to one half of the angle at the center (Figs. 146 and 147).

Thus, angle A = ½ angle B.

EXERCISE 25

In the following figures, solve for ?.

1.

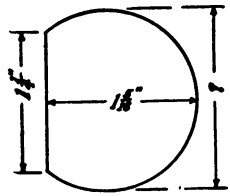


Fig. 148

2.

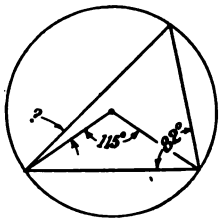


Fig. 149

3.

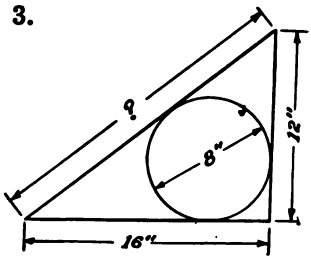


Fig. 150

4.

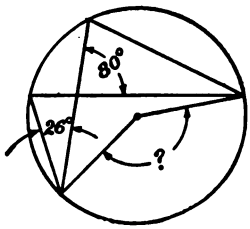


Fig. 151

XVIII

AREAS

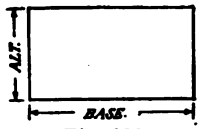


Fig. 152

130. The **area of a rectangle** equals the product of the base and the altitude (Fig. 152). For example, the area of a rectangle 4 inches long and 3 inches wide is 3×4 or 12 square inches.

If the area and one side of a rectangle are given, the other side can be found by dividing the area by the given side. For example, the area of a rectangle equals 12 square inches and one side is 3 inches, then the other side equals $\frac{12}{3}$ or 4 inches.

One side squared equals the **area of a square.** For example, the area of a square whose sides are 5 inches equals 5^2 or 25 square inches.

If the area of a square is given the side can be found by extracting the square root of the area. For example, the area of a square is 16 square inches, then one side equals $\sqrt{16}$ or 4 inches.

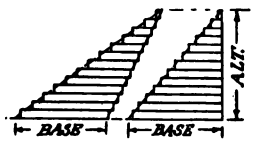


Fig. 153

131. The **area of a triangle** equals one half the product of the base and the altitude (Fig. 153). Therefore, triangles that have equal bases and altitudes have equal areas. For example, if the base equals 5 inches and the altitude 8 inches, then the area $= \frac{1}{2} \times 5 \times 8 = 20$ square inches.

Fig. 154

132. The **area of a parallelogram** equals the product of the base and the altitude (Fig. 154). Therefore, parallelograms that have equal bases and altitudes have equal areas. For example, if the base equals 6 inches and the altitude 5 inches, then the area equals 6×5 or 30 square inches.

133. The **area of a trapezoid** equals the product of the altitude and one half of the sum of the base and the top (Fig. 155). Therefore, trapezoids that have equal bases, tops, and altitudes have equal areas. For example, if the top equals 4 inches, base 7 inches, and the altitude 6 inches, then the area equals $\frac{1}{2}(4+7) \times 6$ or 33 square inches.

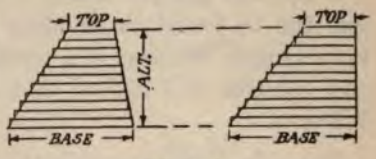


Fig. 155

134. The **area of a circle** equals the radius squared multiplied by 3.1416 or $A = \pi R^2$.

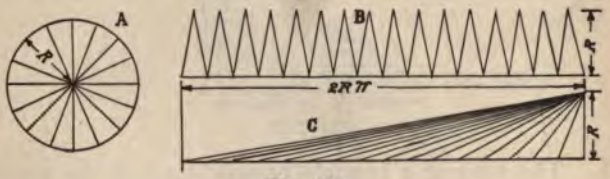


Fig. 156

In Fig. 156, the areas A, B, and C are equal.

Area $C = \frac{1}{2}(2R\pi)(R) = \pi R^2$.

For example, if the radius equals 6 inches, then the area equals $3.1416 \times 6^2 = 113.1$ square inches.

135. The ratio of the area of a circle that can be inscribed in a square to the square equals .7854; therefore, the diameter squared and multiplied by .7854 also equals the **area of a circle.**

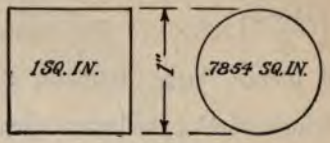


Fig. 157

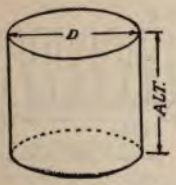


Fig. 158

136. The **lateral area of a cylinder** equals the product of the diameter of the base, the altitude, and 3.1416; or area $=D\times A\times\pi$. For example, if the diameter is 10 inches and the altitude is 11 inches, then the lateral area of the cylinder equals $3.1416\times10\times11=345.576$ square inches.

137. The **lateral area of a cone** equals one half the product of the diameter, the slant height, and 3.1416; or area $=\frac{1}{2}D\times S\times\pi$. For example, if the diameter is 5 inches and the slant height is 8 inches, then the lateral area equals $\frac{1}{2}\times5\times3.1416\times8=62.832$ square inches.

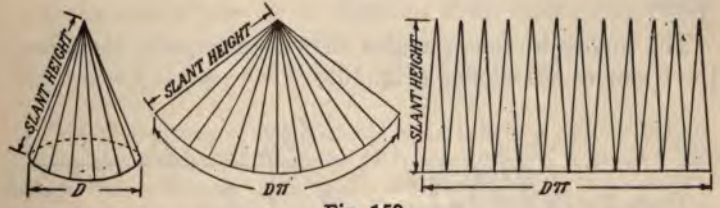


Fig. 159

In Fig. 159, the areas of the three figures are equal.

138. The **lateral area of the frustum of a cone** equals the product of one half the sum of the diameters of the base

and the top, the slant height, and 3.1416; or area = $\left(\frac{D+d}{2}\right) \times S \times \pi$. For example, if the small diameter is 4 inches, large diameter 6 inches, and the slant height 5 inches, then the lateral area equals $\frac{6+4}{2} \times 3.1416 \times 5 = 78.54$ square inches.

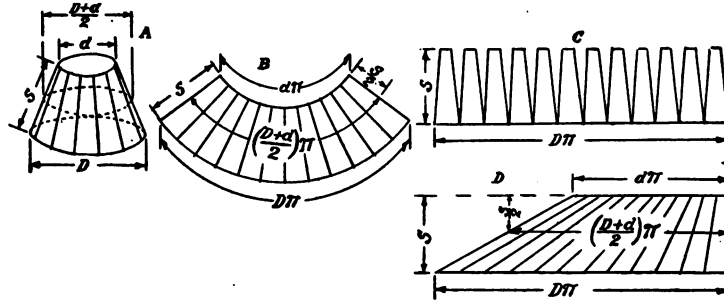


Fig. 160

In Fig. 160, the areas A, B, C, and D are equal.

139. The **area of a sphere** equals $4\,\pi\,R^2$ or $\pi\,D^2$.

A thorough conception of **similar figures** is necessary before taking up the proof of the formula for the area of a sphere.

The angles in the triangles shown are equal; therefore, the triangles are similar (Fig. 161).

Then, $Z = (R)\left(\frac{\frac{R}{8}}{X}\right) = \frac{R^2}{8X}$

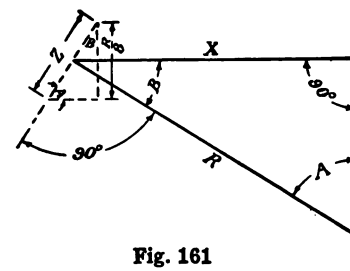


Fig. 161

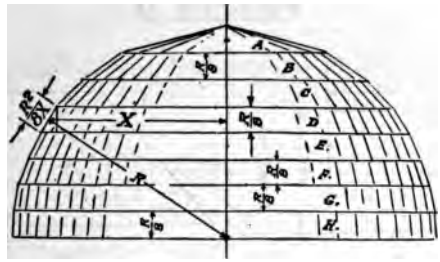


Fig. 162

Each thickness of the frustums of the cones, shown in Fig. 162, is equal to $\frac{R}{8}$.

The lateral area of the frustum of a cone equals $\left(\frac{D+d}{2}\right)\times S\times \pi$. (See Article 138.) In Fig. 162, 2X is equal to $\frac{D+d}{2}$, and $\frac{R^2}{8X}$ is equal to S; therefore, the area of the frustum of cone D equals $2X\left(\frac{R^2}{8X}\right)\pi$ or $\frac{2\pi R^2}{8}$.

The areas of the frustums of the cones A, B, C, E, F, G, and H can be proven in the same manner to equal $\frac{2\pi R^2}{8}$.

If the area of one of the frustums of the cones equals $\frac{2\pi R^2}{8}$, the area of 8 of them will equal $8\times\frac{2\pi R^2}{8}$ or $2\pi R^2$.

These 8 frustums of cones do not form half of a perfect sphere, but as the number of frustums of cones increases a perfect half sphere will be formed. That is, by using an infinitely large number of frustums of cones the area of each being the same, their sum will equal $2\pi R^2$.

If half of the area of a sphere equals $2\pi R^2$, the whole area must equal $2(2\pi R^2)$ or $4\pi R^2$.

For example, if the radius of a sphere is 6 inches, then the area equals $4\times 3.1416\times 6\times 6 = 452.39$ square inches.

EXERCISE 26

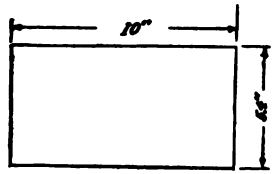


Fig. 163

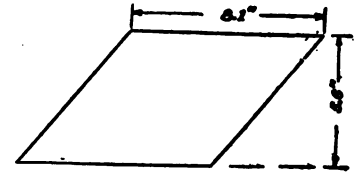


Fig. 164

1. In Fig. 163, what is the area of the rectangle?
2. In Fig. 164, what is the area of the parallelogram?

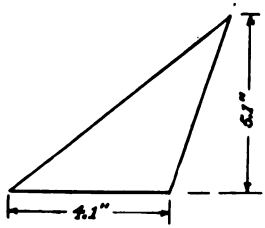


Fig. 165

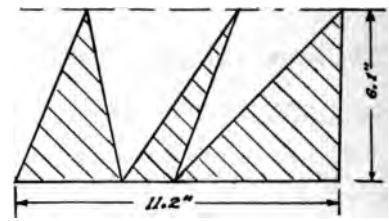


Fig. 166

3. In Fig. 165, what is the area of the triangle?
4. In Fig. 166, how many square inches in the shaded figure?

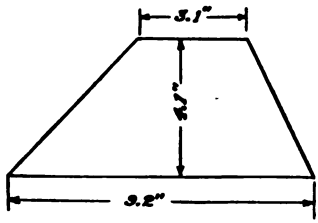


Fig. 167

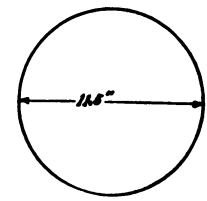


Fig. 168

5. Find the area of the quadrilateral in Fig. 167.
6. What is the area of the circle in Fig. 168?

7. If the area of a circle is 12.76 square inches, what is the diameter?

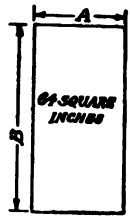


Fig. 169

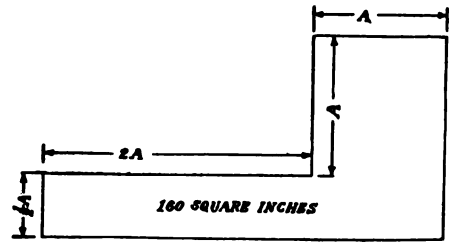


Fig. 170

8. In Fig. 169, if B = 2A and the area = 64 square inches, find the length of B.

9. In Fig. 170, find the length of A.

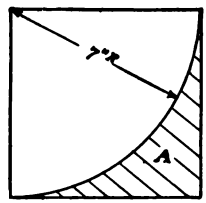


Fig. 171

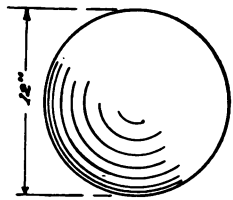


Fig. 172

10. In Fig. 171, find the area of the shaded portion A.

11. In Fig. 172, find the area of the surface of the sphere.

XIX

VOLUMES

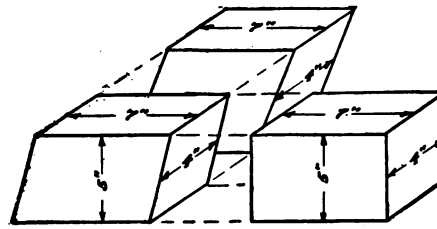


Fig. 173

140. The **volume of a parallelepiped** equals the product of the area of the base and the altitude (Fig. 173). Therefore, parallelepipeds that have equal bases and the same altitudes have equal volumes. For example, volume equals $7 \times 4 \times 5 = 140$ cubic inches.

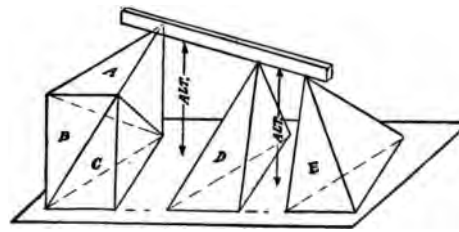


Fig. 174

141. The **volume of a pyramid** equals one third of the product of the area of the base and the altitude. Therefore, pyramids that have equal bases and the same altitudes have equal volumes. For example, if the area of the base equals 6

square inches and the altitude 8 inches, then the volume equals $\frac{1}{3}\times6\times8=16$ cubic inches.

In Fig. 174, the volumes of pyramids A, B, C, D, and E are equal.

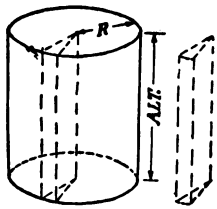


Fig. 175

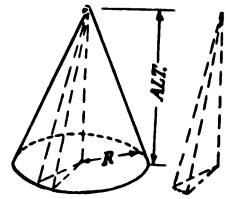


Fig. 176

142. The **volume of a cylinder** equals the product of the area of the base and the altitude, or $\pi R^2 \times$ altitude (Fig. 175). For example, if the radius is 5 inches and the altitude 8 inches, then the volume equals $3.1416\times5^2\times8=628.32$ cubic inches.

143. The **volume of a cone** equals one third the product of the area of the base and the altitude, or $1.0472\ R^2\times$ altitude (Fig. 176). For example, if the radius equals 5 inches and the altitude 8 inches, then the volume equals $1.0472\times5^2\times8=209.44$ cubic inches.

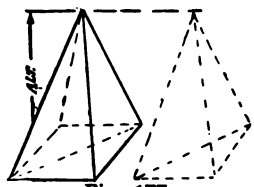


Fig. 177

144. The **volume of a rectangular pyramid** equals one third of the product of the area of the base and the altitude (Fig. 177). For example, if the area of the base is 9 square inches and the altitude 8 inches, then the volume equals $\frac{1}{3}\times9\times8=24$ cubic inches.

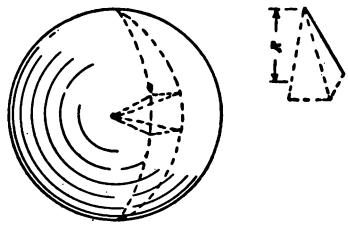


Fig. 178

145. The **volume of a sphere** equals one third of the product of the area and the radius, or $\frac{4}{3}\pi R^3$ (Fig. 178). For example, if the radius is 6 inches, the volume equals $\frac{4}{3} \times 3.1416 \times 6^3 = 904.78$ cubic inches.

EXERCISE 27

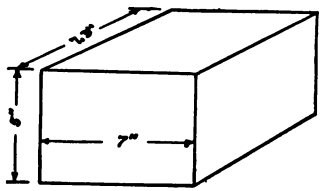


Fig. 179

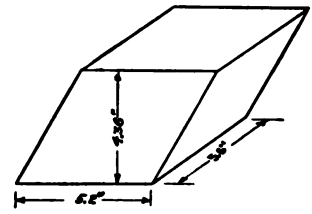


Fig. 180

1. Find the volume of the rectangular parallelepiped in Fig. 179.
2. What is the volume of the oblique parallelepiped in Fig. 180?

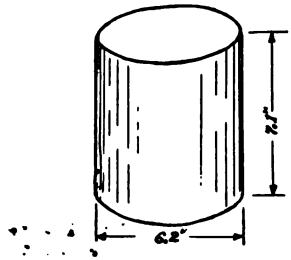


Fig. 181

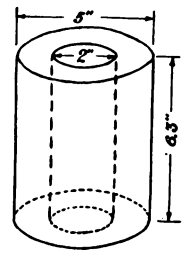


Fig. 182

3. Find the volume of the cylinder in Fig. 181.
4. Find the cubical contents of the large cylinder exclusive of the core, in Fig. 182.

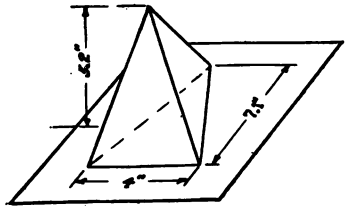


Fig. 183

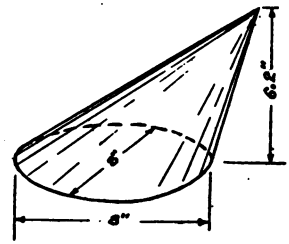


Fig. 184

5. What is the volume of the triangular pyramid in Fig. 183?
6. In Fig. 184, what is the volume of the circular cone?

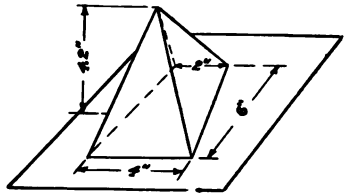


Fig. 185

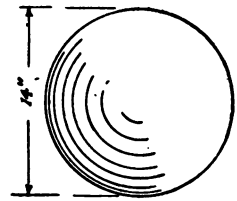


Fig. 186

7. Find the volume of the quadrilateral pyramid in Fig. 185.
8. What is the volume of the sphere given in Fig. 186?

74553A

XX

PYTHAGOREAN THEOREM

146. The following is the proof that the sum of the squares of the two sides of a right triangle is equal to the square of the hypotenuse. This is called the **Pythagorean theorem.**

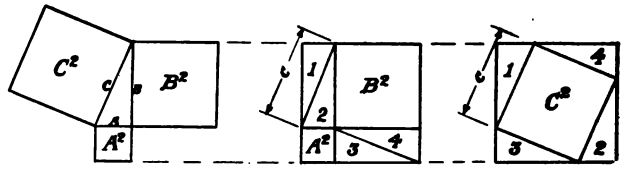


Fig. 187

In Fig. 187, the total area of Fig. 1 is equal to the total area of Fig. 2.

If, in Fig. 1, the area of the pieces 1, 2, 3, and 4 be subtracted from the total area, the parts that remain will be A^2+B^2.

If, in Fig. 2, the area of the pieces 1, 2, 3, and 4 be subtracted from the total area, the part that remains will be C^2.

In both figures the pieces 1, 2, 3, and 4 all have the same area.

The total areas in Fig. 1 and Fig. 2 are equal, and equal areas have been subtracted from both figures; therefore, the parts that remain must be equal.

With a little logical reasoning it becomes evident that $A^2+B^2=C^2$.

147. From the above proof the following rules are derived:

RULE 1: *When the lengths of the two sides of a right triangle are known, the hypotenuse can be found by adding the squares of the lengths of the two sides and extracting the square root of this sum.*

PROBLEM: If A = 5 inches and B = 6 inches, what is the length of the hypotenuse C?

SOLUTION: $5^2=25$; $6^2=36$.

$25+36=61$.

$C=\sqrt{61}=7.81$ inches.

RULE 2: *When the hypotenuse and one side are known, the other side can be found by subtracting the square of the known side from the square of the hypotenuse and extracting the square root of the difference.*

PROBLEM: If A = 7 inches and C = 10 inches, what is the length of the other side B?

SOLUTION: $10^2 = 100$; $7^2 = 49$.

$100 - 49 = 51$.

$B = \sqrt{51} = 7.141$ inches.

EXERCISE 28

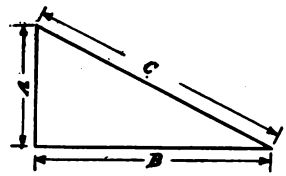


Fig. 188

Use Fig. 188 for reference:

1. If A = 10.11 and B = 6.72, what does C equal?
2. If A = 11 and B = 14, what does C equal?
3. If B = 6.4 and C = 7.2, what does A equal?
4. If A = .312 and C = 8.04, what does B equal?
5. If B = 5.661 and C = 10.7, what does A equal?

In the following figures, solve for the unknown quantity indicated by ?.

6.

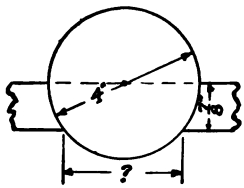


Fig. 189

7.

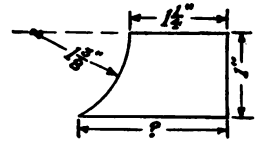
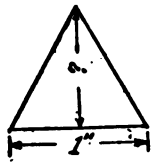


Fig. 190

8.



Equilateral Triangle

Fig. 191

9.

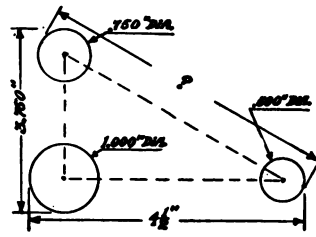


Fig. 192

10.

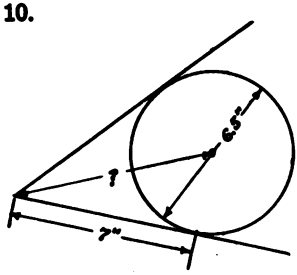


Fig. 193

11.

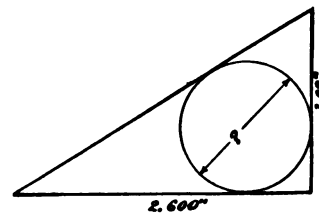


Fig. 194

12.

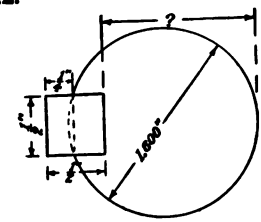


Fig. 195

13.

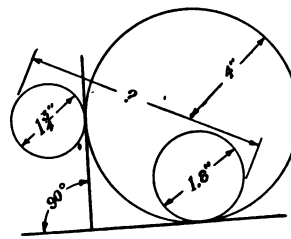


Fig. 196

14.

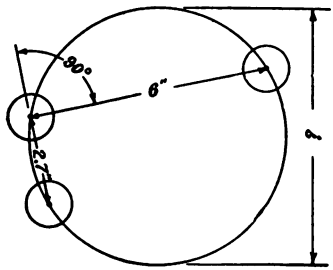


Fig. 197

15.

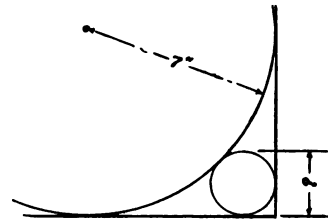


Fig. 198

Relation of Altitude Line and Midpoint of Base

148. The **altitude of a triangle** is a line drawn from the vertex of any angle perpendicular to the opposite side or side extended.

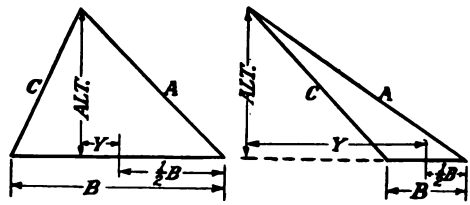


Fig. 199

In Fig. 199, Y equals the distance between the foot of the altitude line and a point in the middle of the base. If B represents the base and A the longer of the two remaining sides, then $Y = \frac{A^2 - C^2}{2B}$.

$$\text{Altitude} = \sqrt{A^2 - (Y + \tfrac{1}{2}B)^2} \text{ or } \sqrt{C^2 - (Y - \tfrac{1}{2}B)^2}.$$

$$\text{Area} = (\tfrac{1}{2}B)(\text{altitude}).$$

149. The proof that $Y=\frac{A^2-C^2}{2B}$:

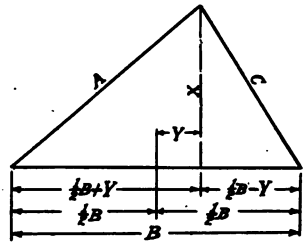


Fig. 200

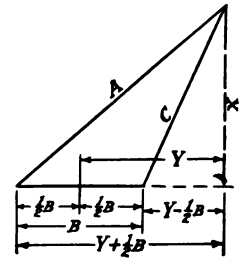


Fig. 201

From Fig. 200:

$A^2-(\frac{1}{2}B+Y)^2=X^2$;
$C^2-(\frac{1}{2}B-Y)^2=X^2$.
$C^2-(\frac{1}{2}B-Y)^2=A^2-(\frac{1}{2}B+Y)^2$.
$C^2-(\frac{1}{4}B^2-BY+Y^2)=$
$A^2-(\frac{1}{4}B^2+BY+Y^2)$.
$C^2-\frac{1}{4}B^2+BY-Y^2=$
$A^2-\frac{1}{4}B^2-BY-Y^2$.
$2BY=A^2-C^2$.
$Y=\frac{A^2-C^2}{2B}$.

From Fig. 201:

$A^2-(Y+\frac{1}{2}B)^2=X^2$;
$C^2-(Y-\frac{1}{2}B)^2=X^2$.
$C^2-(Y-\frac{1}{2}B)^2=A^2-(Y+\frac{1}{2}B)^2$.
$C^2-(Y^2-BY+\frac{1}{4}B^2)=$
$A^2-(Y^2+BY+\frac{1}{4}B^2)$.
$C^2-Y^2+BY-\frac{1}{4}B^2=$
$A^2-Y^2-BY-\frac{1}{4}B^2$.
$2BY=A^2-C^2$.
$Y=\frac{A^2-C^2}{2B}$.

EXERCISE 29

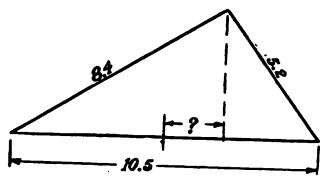


Fig. 202

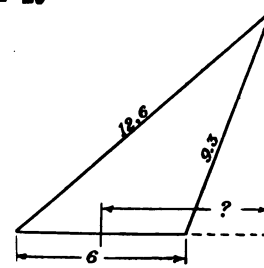


Fig. 203

1. In Fig. 202, find the distance from the foot of the altitude to the midpoint of the base.

2. In Fig. 203, in which the altitude cuts the line of the base extended, find ?.

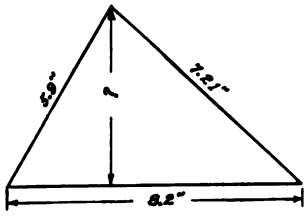


Fig. 204

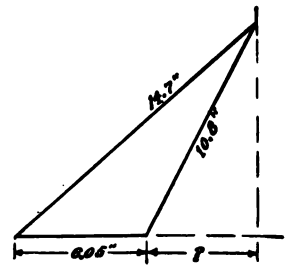


Fig. 205

3. In Fig. 204, with the three sides given, find the altitude.

4. In Fig. 205, with three sides given and the altitude falling outside the base, find ?.

XXI

ANGLES

150. If a line revolves about one of its extremities an **angle** is generated, the magnitude of the angle depending on the amount of the revolution.

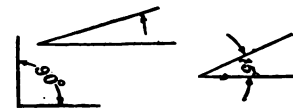


Fig. 206

The magnitude of the angle is usually measured in degrees, minutes, and seconds (Fig. 206).

151. A **degree** is $\frac{1}{360}$ of a circle (Fig. 207).

One degree is equal to 60 minutes.

A **minute** is $\frac{1}{21600}$ of a circle.

One minute is equal to 60 seconds.

A **second** is $\frac{1}{1296000}$ of a circle.

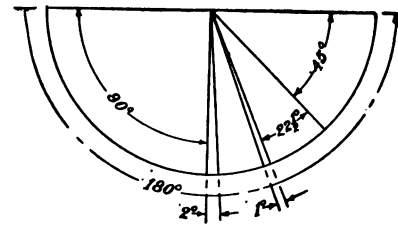


Fig. 207

152. One degree equals 60 minutes or 3600 seconds. Degrees, minutes and seconds are indicated by the marks ° ′ ″. Thus, 36° 20′ 25″ is read "36 degrees, 20 minutes, and 25 seconds."

153. For a given angle the ratio of the length of its arc to the length of the radius is constant; in other words, for a fixed angle the ratio (**arc** to **radius**) is the same for any length of the radius.

Thus, $\frac{8\frac{1}{3}}{5}=\frac{5}{3}$ (Fig. 208).

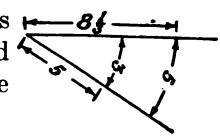


Fig. 208

154. Example of **addition of angles.**

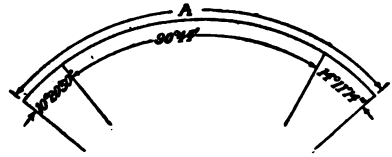


Fig. 209

PROBLEM: In Fig. 209, what angle does A equal?

SOLUTION:
$$\begin{array}{r} 10^\circ\ 20'\ 50'' \\ 90^\circ\ 44' \\ 14^\circ\ 11'\ 14'' \\ \hline 114^\circ\ 75'\ 64'' = 115^\circ\ 16'\ 4''\ \textit{Ans.} \end{array}$$

155. Example of **subtraction of angles.**

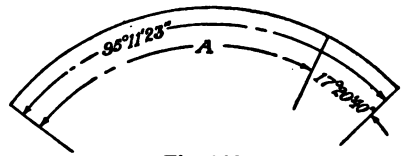


Fig. 210

PROBLEM: In Fig. 210, what angle does A equal?

SOLUTION:
$$\begin{array}{rl} 95^\circ\ 11'\ 23'' = & 94^\circ\ 70'\ 83'' \\ & 17^\circ\ 20'\ 40'' \\ \hline & 77^\circ\ 50'\ 43''\ \textit{Ans.} \end{array}$$

156. Example of **multiplying an angle** by a number.

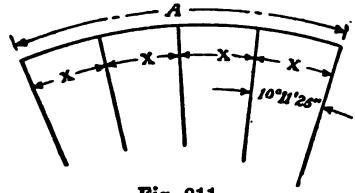


Fig. 211

PROBLEM: In Fig. 211, what angle does A equal?

SOLUTION:
$$\begin{array}{r} 10^\circ\ 11'\ 25'' \\ \times\ \ 4 \\ \hline 40^\circ\ 44'\ 100'' = 40^\circ\ 45'\ 40''.\ \textit{Ans.} \end{array}$$

157. Example of **dividing an angle** by a number.

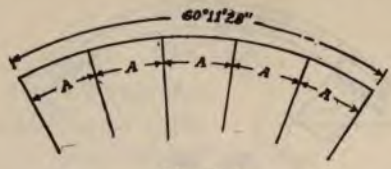


Fig. 212

PROBLEM: In Fig. 212, what angle does A equal?

SOLUTION: $A=\frac{60°\ 11'\ 28''}{5}=\frac{60°\ 10'\ 88''}{5}=12°\ 2'\ 17.6''$. *Ans.*

158. A **decimal degree** can be reduced to degrees, minutes, and seconds, by multiplying the part of a degree by 60 to reduce it to minutes; then multiplying the part of a minute by 60 to reduce it to seconds.

EXAMPLE: Reduce 18.743° to degrees, minutes, and seconds.

SOLUTION: .743° equals .743×60 or 44.58 minutes; .58 minutes equals .58×60 or 34.8 seconds. Then, 18.743° equals 18° 44′ 34.8″.

159. Degrees, minutes, and seconds may be reduced to a decimal degree by dividing the seconds by 60 to reduce them to a decimal part of a minute; then add this to the given minutes and divide this sum by 60 to reduce it to a decimal degree.

EXAMPLE: Reduce 14° 17′ 35″ to a decimal degree.

SOLUTION: 35 seconds equals 35÷60 or .583 minutes. Therefore 14° 17′ 35″ = 14° 17.583′. 17.583 minutes equals 17.583÷60 or .29305 degrees. Then, 14° 17′ 35″ = 14.29305°.

160. To find the angle equal to a certain part of a circle, multiply the part of a circle by 360°.

EXAMPLE: What angle is equal to .2465 of a circle?

SOLUTION: .2465×360° = 88.74° = 88° 44.4′ = 88° 44′ 24″.

161. To find the part of a circle that is equal to a given angle, divide the given angle by 360°.

EXAMPLE: What part of a circle is equal to 17° 42′ 11″?

SOLUTION: 17° 42′ 11″ = 17° 42.183′ = 17.7003°.
17.7003° ÷ 360° = .04916 of a circle.

162. To find the diameter when the arc and angle are given: Find what part of the complete circle the angle equals. Then, the arc divided by this part of the circle equals the circumference.

The circumference divided by 3.1416 equals the diameter.

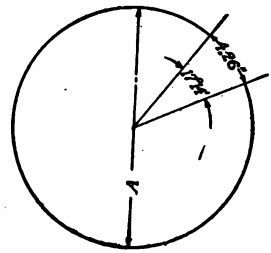


Fig. 213

EXAMPLE: In Fig. 213, find A.

SOLUTION: 17° 14′ = 17.2333°.

17.2333° ÷ 360° = .04787 of the complete circle.

4.26 ÷ .04787 = 88.991, which is the circumference.

88.991 ÷ 3.1416 = 28.326, which is the diameter.

EXERCISE 30

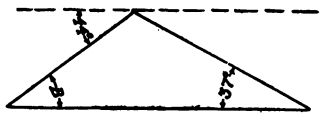


Fig. 214

1. In Fig. 214, how many degrees in angle B?

3. Reduce 17.945° to degrees, minutes, and seconds.

4. Add 18° 47′ 13″, 26° 53′, 17° 42′, and 16° 0′ 32″.

5. Reduce 26° 37′ 23″ to a decimal degree.

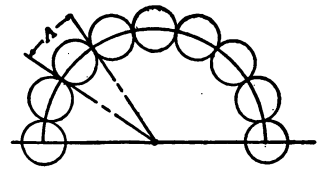


Fig. 215

2. In Fig. 215, what angle does A equal?

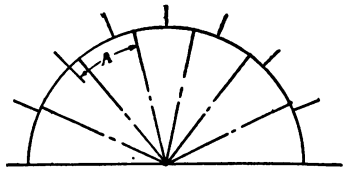


Fig. 216

6. In Fig. 216, what angle does A equal?

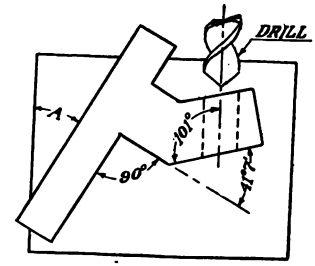


Fig. 217

7. Find the value of angle A in Fig. 217.

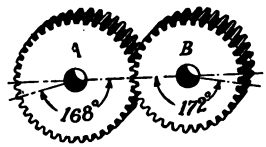


Fig. 218

8. In Fig. 218, how many teeth must A and B have so that, while A revolves 168°, B will revolve 172°?

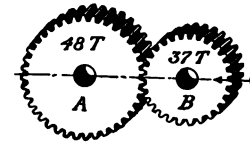


Fig. 219

9. In Fig. 219, while A revolves 123°, how many degrees, minutes, and seconds in gear B pass the line?

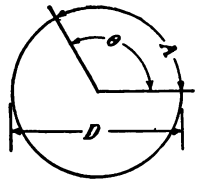


Fig. 220

10. In Fig. 220, if D = 6.35″, and A = 3.271″, what angle does θ equal?

11. In Fig. 220, if A = 1.865″, and θ = 62° 14′, what does D equal?

XXII

VERNIER MEASURING INSTRUMENTS

Vernier Calipers

163. Scales of different descriptions are used for determining linear measurements. When more accurate dimensions are to be measured, the **vernier** is used.

164. The number of divisions on a vernier is governed by the distance between the divisions on the scale and the distance that the vernier must move to cause the next set of lines to match.

Rule: *To obtain the number of divisions on a vernier, divide the distance between each two divisions on the scale by the distance the vernier must move to cause the next set of lines to match.*

Fig. 221

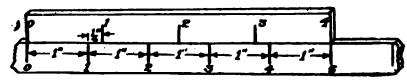


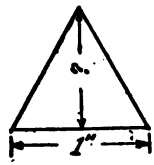
Fig. 222

Example: What number of divisions must the vernier have that, when the next set of lines matches, the vernier will move $\frac{1}{4}$ of an inch, the distance between divisions on the scale being equal to 1 inch.

Solution: The number of divisions the vernier must have equals $1 \div \frac{1}{4}$ or 4.

To function properly these 4 divisions must occupy the same space as 1 more or 1 less divisions on the scale. Fig. 221 shows the 4 divisions of the vernier occupying the same space as 3 divisions on the scale, and Fig. 222 shows the 4 divisions of the vernier occupying the same space as 5 divisions on the scale. In either

8.



Equilateral Triangle

Fig. 191

9.

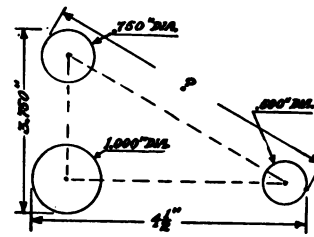


Fig. 192

10.

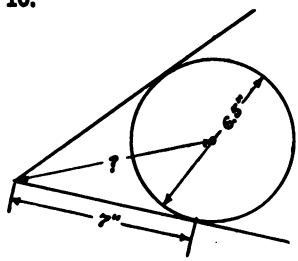


Fig. 193

11.

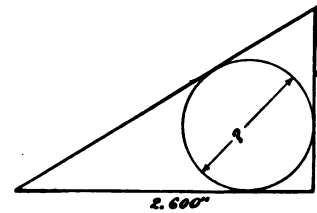


Fig. 194

12.

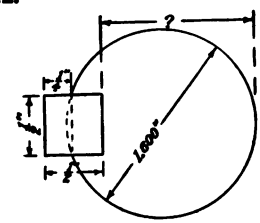


Fig. 195

13.

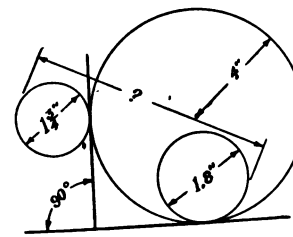


Fig. 196

14.

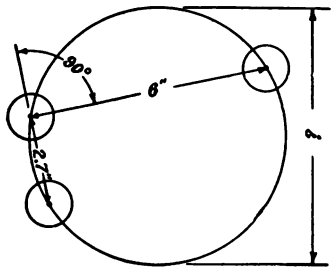


Fig. 197

15.

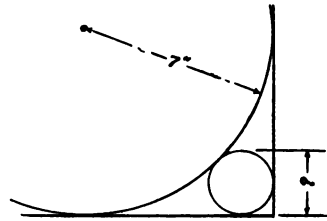


Fig. 198

RELATION OF ALTITUDE LINE AND MIDPOINT OF BASE

148. The **altitude of a triangle** is a line drawn from the vertex of any angle perpendicular to the opposite side or side extended.

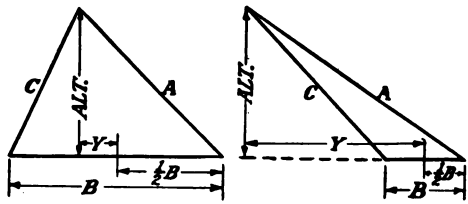


Fig. 199

In Fig. 199, Y equals the distance between the foot of the altitude line and a point in the middle of the base. If B represents the base and A the longer of the two remaining sides, then $Y = \frac{A^2 - C^2}{2B}$.

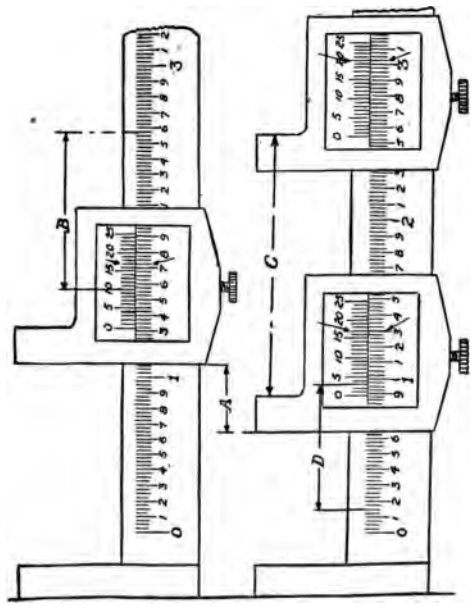
Altitude $= \sqrt{A^2 - (Y + \frac{1}{2}B)^2}$ or $\sqrt{C^2 - (Y - \frac{1}{2}B)^2}$.
Area $= (\frac{1}{2}B)$(altitude).

Then, 30 equals the number of divisions the vernier must have, and these 30 divisions must occupy the same space as 29 divisions or 29° on the protractor.

EXERCISE 31

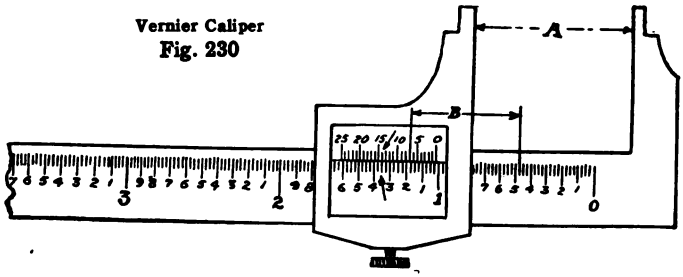
In Figures 229, 230, 231, and 232, coinciding lines being indicated by arrows, determine the values of the unknown quantities indicated by the letters A, B, C, and D.

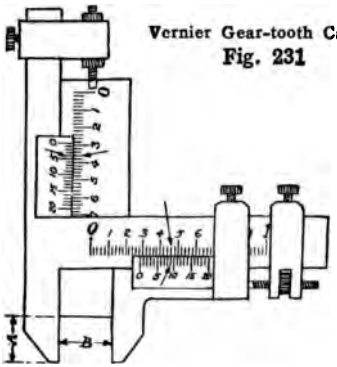
Fig. 229	Fig. 230	Fig. 231	Fig. 232
1. A.	**5.** A.	**7.** A.	**9.** A.
2. B.	**6.** B.	**8.** B.	**10.** B.
3. C.			**11.** C.
4. D.			**12.** D.



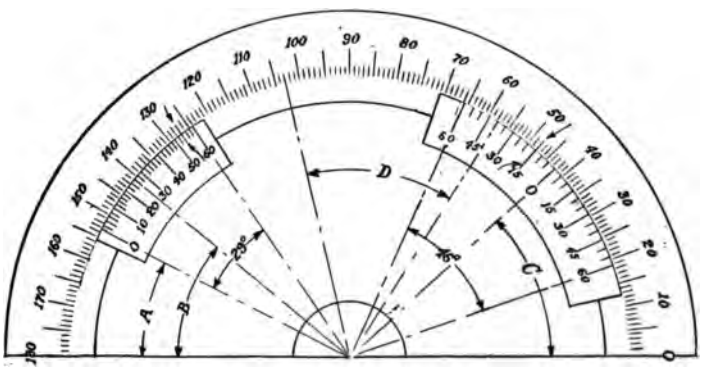
Vernier Height Gauge
Fig. 229

Vernier Caliper
Fig. 230





Vernier Gear-tooth Caliper
Fig. 231



Vernier Protractor
Fig. 232

XXIII

SIMILAR FIGURES

170. Similar figures are figures such that one may be obtained from the other by proportional enlargement or reduction. Fig. 233 shows a series of similar figures.

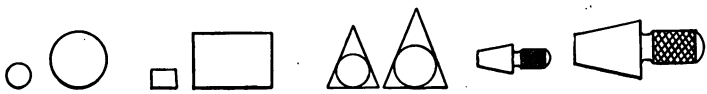


Fig. 233

171. The ratios of the corresponding sides of similar figures are equal.

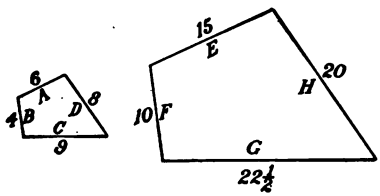


Fig. 234

Thus, in Fig. 234,

E is $2\frac{1}{2}$ times A,
F " " " B,
G " " " C,
H " " " D,

A and E, B and F, C and G, and D and H being corresponding sides.

172. If all the sides of a figure be multiplied by the same number, and these products used to construct a second figure, the two figures will be similar.

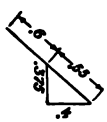


Fig. 235

In this way Fig. 235 may be enlarged to Fig. 235A by using 1.4 for such a multiplier.

Thus, $.6 \times 1.4 = .84$,
$.55 \times 1.4 = .77$,
$.4 \times 1.4 = .56$,
$.375 \times 1.4 = .525$.

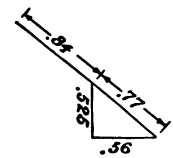


Fig. 235A

173. If all the sides of a figure are divided by the same number, and these quotients used to construct a second figure, the two figures will be similar.

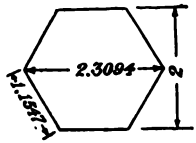


Fig. 236

In this way Fig. 236 may be reduced to Fig. 236A by using 1.5 for such a divisor.

Thus,

$2 \div 1.5 = 1.3333,$
$2.3094 \div 1.5 = 1.5396,$
$1.1547 \div 1.5 = .7698.$

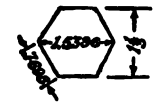


Fig. 236A

174. All the dimensions of a figure, and one side of a similar figure being given, to find the other sides: Find what fraction the corresponding side must be multiplied by to make it equal the given side in the second figure; then multiply successively the given dimensions to find the corresponding dimensions of the similar figure.

EXAMPLE: What is the length of A, B, and C, in Fig. 237A?

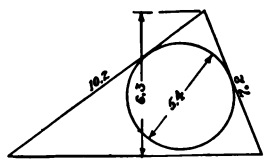


Fig. 237

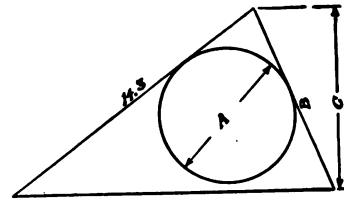


Fig. 237A

SOLUTION: To find what fraction one number must be multiplied by to obtain another given number, let the denominator be the one given number and the numerator the given number to be obtained.

In the above problem 10.2 in Fig. 237 must be multiplied by $\frac{14.3}{10.2}$ to obtain 14.3, the length of the corresponding side in Fig. 237A. Thus, $10.2 \times \frac{14.3}{10.2} = 14.3$. Then the other sides in the first figure

must be multiplied by $\frac{14.3}{10.2}$ to equal the corresponding sides in the second figure.

Thus, A equals $5.4 \times \frac{14.3}{10.2} = 5.4 \times 1.402 = 7.5708$.
B equals $7.2 \times \frac{14.3}{10.2} = 7.2 \times 1.402 = 10.0944$.
C equals $6.3 \times \frac{14.3}{10.2} = 6.3 \times 1.402 = 8.8326$.

175. Many problems that seem to require algebra for their solutions may be solved by constructing similar figures.

PROBLEM 1: Find X as indicated in Fig. 238.

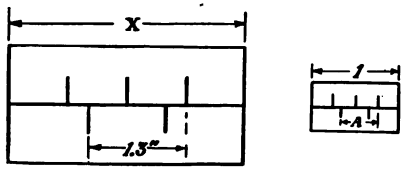


Fig. 238

SOLUTION: Construct a similar figure as indicated, making the side, which corresponds to the unknown side, 1. In the second figure $A = 1 - \frac{1}{4} - \frac{1}{3} = \frac{5}{12}$. Then, by similar figures, $X = 1 \times \frac{1.3}{A} = 1 \times \frac{1.3}{\frac{5}{12}} = 3.12$.

PROBLEM 2: If the hypotenuse of a triangle is 12 inches, and the altitude 3 times as long as the base, find the base (Fig. 239).

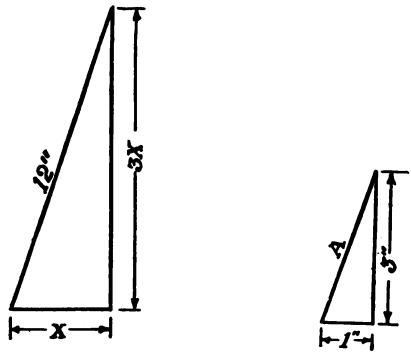


Fig. 239

SOLUTION: Construct a similar figure as indicated, making the side, which corresponds to the unknown side, 1. In the second figure $A = \sqrt{3^2+1^2} = 3.162''$. Then, by similar figures, $X = 1 \times \frac{12}{A} = 1 \times \frac{12}{3.162} = 3.795''$.

EXERCISE 32

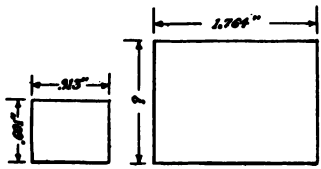


Fig. 240

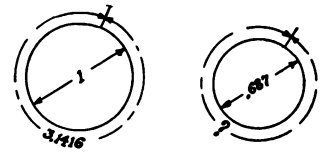


Fig. 241

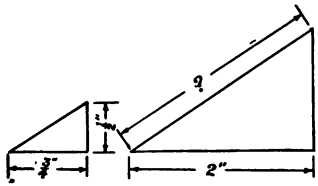


Fig. 242

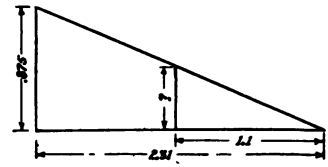


Fig. 243

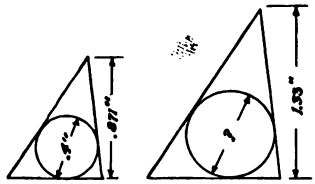


Fig. 244

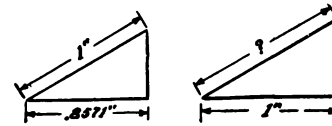


Fig. 245

1. In Fig. 240, find the measurement indicated by ?.
2. In Fig. 241, find the measurement indicated by ?.
3. In Fig. 242, find the measurement indicated by ?.
4. In Fig. 243, find the measurement indicated by ?.
5. In Fig. 244, find the measurement indicated by ?.
6. In Fig. 245, find the measurement indicated by ?.

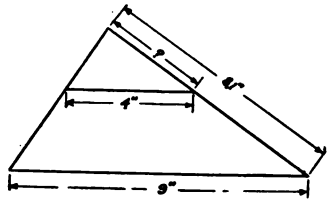


Fig. 246

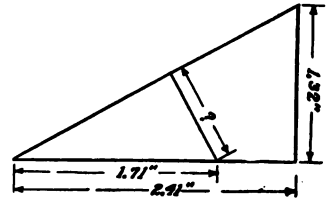


Fig. 247

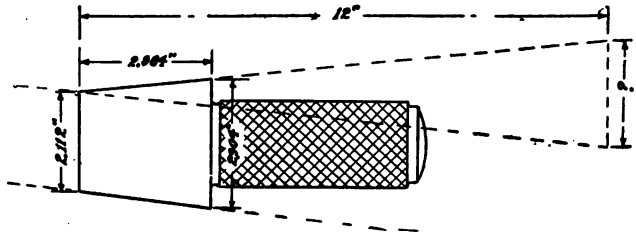


Fig. 248

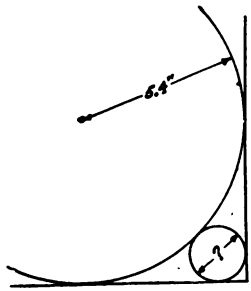


Fig. 249

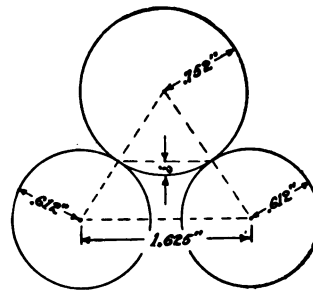


Fig. 250

7. In Fig. 246, find the measurement indicated by ?.
8. In Fig. 247, find the measurement indicated by ?.
9. In Fig. 248, find the measurement indicated by ?.
10. In Fig. 249, find the measurement indicated by ?.
11. In Fig. 250, find the measurement indicated by ?.

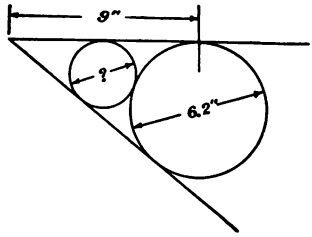


Fig. 251

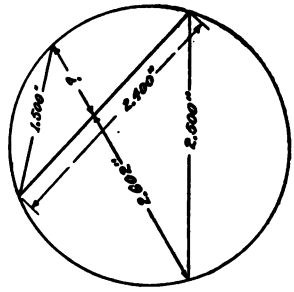


Fig. 252

12. In Fig. 251, find the measurement indicated by ?.

13. In Fig. 252, find the measurement indicated by ?.

XXIV

TAPERS

176. Taper problems can be very readily solved by using the principles developed for similar figures.

177. Example of a taper problem.—When the length, small diameter and outside diameter are given, to find the taper per foot:

PROBLEM: What is the taper per foot of the given plug, or what length does A equal in Fig. 253?

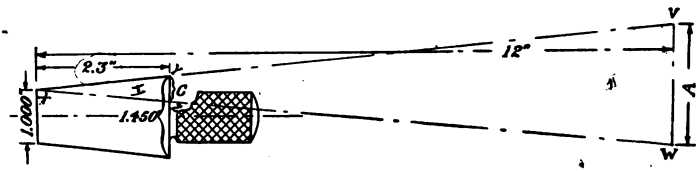


Fig. 253

SOLUTION: Triangles XYZ and XVW are similar.

$C = 1.450'' - 1'' = .450''$.

2.3″ must be multiplied by $\frac{12}{2.3}$ to obtain 12″; therefore, C must be multiplied by $\frac{12}{2.3}$ to obtain A. (See Article 174.)

Thus, $A = .450'' \times \frac{12}{2.3} = 2.348''$; therefore, the taper per foot of the given plug is 2.348″.

178. Example of a taper problem.—When the taper per foot, length, and one diameter are given, to find the other diameter:

PROBLEM 1: What diameter does A equal in Fig. 254?

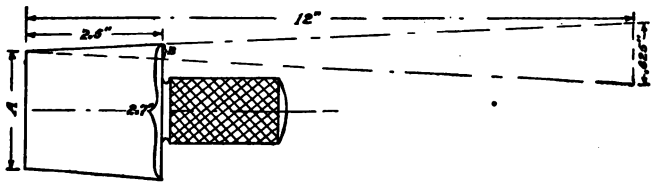


Fig. 254

SOLUTION: $B = .625'' \times \frac{2.5}{12} = .1302''$.
$A = 2.7'' - .1302'' = 2.5698''$.

PROBLEM 2: What diameter does X equal in Fig. 255?

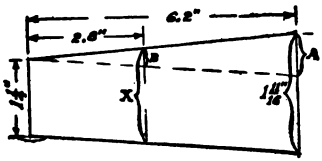


Fig. 255

SOLUTION:
$A = 1\frac{11}{16}'' - 1\frac{1}{4}'' = \frac{7}{16}''$.
$B = \frac{7}{16}'' \times \frac{2.8}{6.2} = .1976''$.
$X = 1\frac{1}{4}'' + .1976'' = 1.4476''$.

179. A **proportion rule** for tapers is as follows:

RULE: *Multiply that which is of the same denomination as the answer by the ratio of two like denominations.*

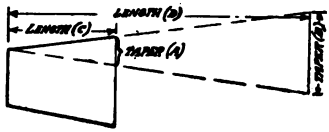


Fig. 256

In Fig. 256, $A = B \times \frac{C}{D}$; $B = A \times \frac{D}{C}$; $C = D \times \frac{A}{B}$; $D = C \times \frac{B}{A}$.

180. Dimensions for **standard tapers.**

Morse	
Number of taper	Taper per foot
0	.625″
1	.600″
2	.602″
3	.602″
4	.623″
5	.630″
6	.626″
7	.625″

Brown & Sharp	
Number of taper	Taper per foot
1 to 9	.500″
10	.5161″
11 to 18	.500″

Jarno
.600″ per foot

Jarno Taper

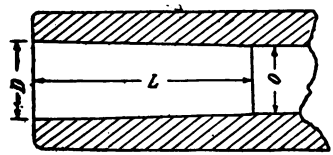


Fig. 257

L = Number of taper ÷ 2.
D = Number of taper ÷ 8.
O = Number of taper ÷ 10.

EXERCISE 33

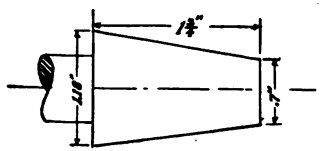


Fig. 258

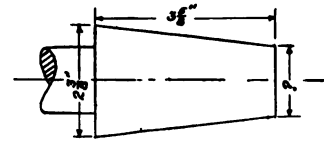


Fig. 259

1. In Fig. 258, from the dimensions given find the taper per foot.

2. In Fig. 259, if the taper per foot is .4375″, find the dimension indicated by ?.

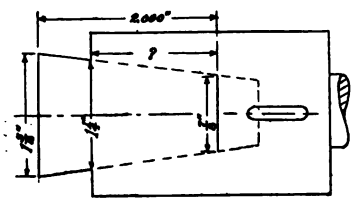


Fig. 260

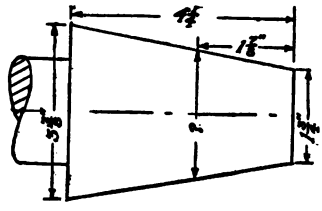


Fig. 261

3. In Fig. 260, from the dimensions given find the value of ?.

4. In Fig. 261, find the value of ?.

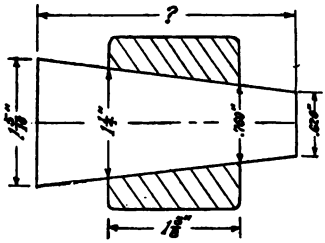


Fig. 262

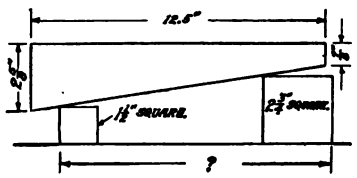


Fig. 263

5. In Fig. 262, solve for length ?.

6. In Fig. 263, find the value of ?.

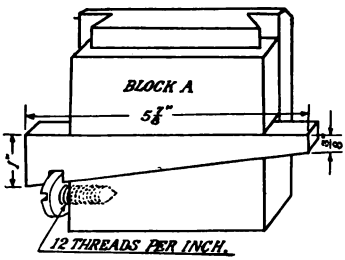


Fig. 264

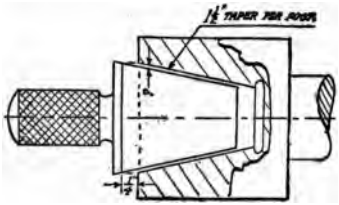


Fig. 265

7. In Fig. 264, how many revolutions must the screw make to raise A a distance of .009 inch?

8. In Fig. 265, find the value indicated by ?.

XXV

TRIGONOMETRY

181. The sides of a right triangle are termed the **hypotenuse,** the **side opposite** a given angle, and the **side adjoining** a given angle (Fig. 266).

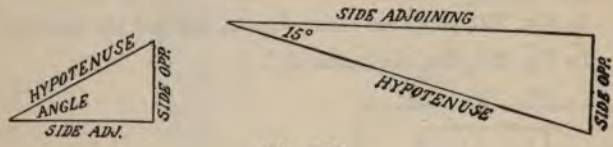


Fig. 266

The **hypotenuse** is the line opposite the right angle in any right-angled triangle and it is always longer than either side.

The **side opposite** means the side that is opposite a given angle.

The **side adjoining** means the side that adjoins a given angle.

The term **side,** strictly speaking, is used in a right triangle to designate only the side opposite and the side adjacent, the hypotenuse not being ordinarily designated as a side. In mathematical processes, however, the hypotenuse is treated as a "side."

182. One multiplied by a number equals the number. Thus, $1 \times 12 = 12$.

If the lengths of corresponding sides of two similar triangles are one and any other given number, it is only necessary to multiply the lengths of the other given sides of the first triangle by the given number in the second triangle to obtain the corresponding sides of the second triangle.

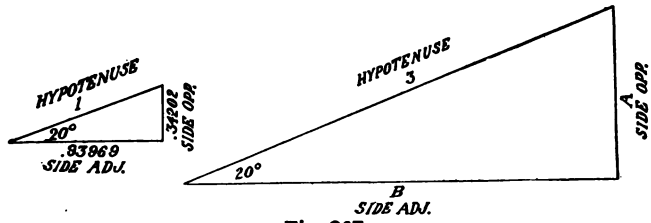


Fig. 267

Thus, in Fig. 267, since the right triangles given are similar figures, if the hypotenuse of one triangle is three times the length of the hypotenuse of the other, there must be a corresponding relationship between the *sides* of the two triangles. That is, the opposite side of the one will be three times the opposite side of the other and the adjoining side of the one will be three times the adjoining side of the other. With the lengths of the sides of the smaller triangle given we can find the lengths of the corresponding sides of the larger by using three as a multiplier.

Therefore, $A = 3 \times .34202 = 1.02606$, and $B = 3 \times .93969 = 2.81907$.

183. If the sides of a given triangle are divided by a number equal to the number of units in one of its sides, and if the quotients are used to construct another triangle, the triangle formed will be similar to the original triangle.

Thus, similar triangles to the triangle in Fig. 268 can be constructed by reducing each side in turn to unity by dividing by the number of units in that side into each of the other sides to get the respective proportional lengths for the sides of the new similar triangle. The three similar triangles and the processes are shown in Figs. 268A, 268B, and 268C.

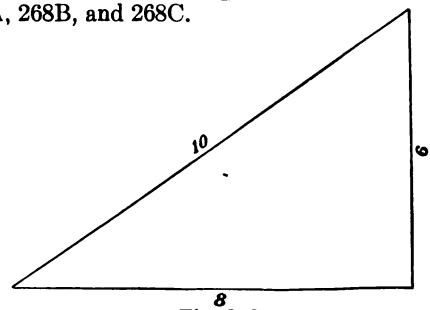


Fig. 268

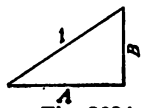


Fig. 268A

$10 \div 10 = 1$

$A = 8 \div 10 = .8$

$B = 6 \div 10 = .6$

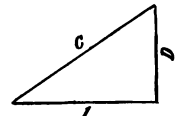


Fig. 268B

$8 \div 8 = 1$

$C = 10 \div 8 = 1.25$

$D = 6 \div 8 = .75$

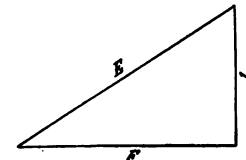


Fig. 268C

$6 \div 6 = 1$

$E = 10 \div 6 = 1.666$

$F = 8 \div 6 = 1.333$

EXERCISE 34

Fig. 269

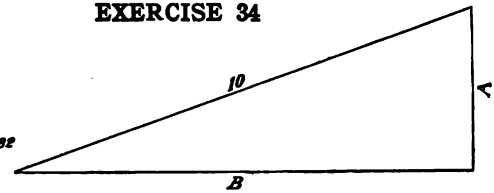


Fig. 269A

1. From Figs. 269 and 269A, find the altitude A.
2. From the same figures, find the length of the base B.

Fig. 270

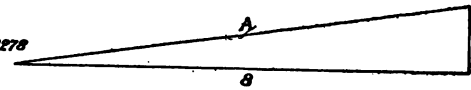


Fig. 270A

3. From Figs. 270 and 270A, find the length of the hypotenuse A.
4. From the same figures, find the altitude B.

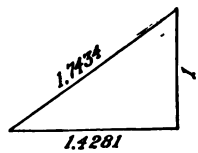


Fig. 271

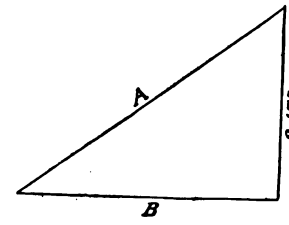


Fig. 271A

5. From Figs. 271 and 271A, find the length of the hypotenuse A.
6. From the same figures, find the length of the base B.

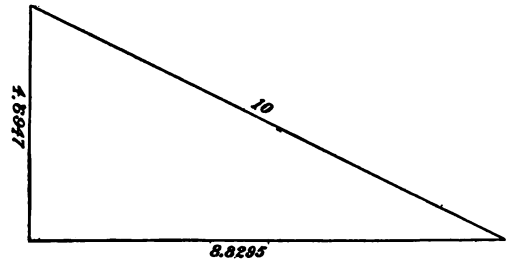


Fig. 272

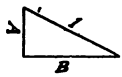


Fig. 272A

7. From Figs. 272 and 272A, find the altitude A.
8. From the same figures, find the length of the base B.

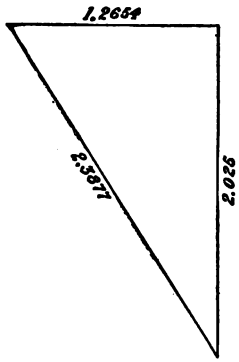


Fig. 273

Fig. 273A

9. From Figs. 273 and 273A, find the length A.
10. From the same figures, find the length B.

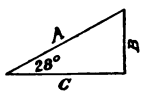


Fig. 274

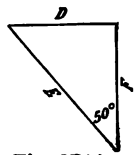


Fig. 274A

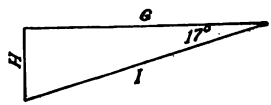


Fig. 274B

In Figs. 274, 274A, and 274B:

11. Which side is the hypotenuse?
12. Which side is the side opposite the given angle?
13. Which side is the side adjoining the given angle?

Functions of Angles

184. The **trigonometric functions** of an angle are called the **sine, cosine, tangent, cotangent, secant,** and **cosecant.** When the hypotenuse is 1, the length of the side opposite the angle is called the **sine** of the angle (Fig. 275).

For example, .5000 = sine of 30° angle (see Trigonometric Tables).

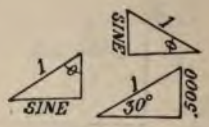


Fig. 275

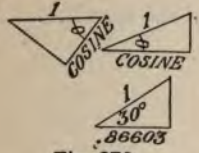


Fig. 276

185. When the hypotenuse is 1, the length of the side adjoining the angle is called the **cosine** of the angle (Fig. 276).

For example, .86603 = cosine of 30° angle (see Trigonometric Tables).

186. When the side adjoining the angle is 1, the length of the side opposite the angle is called the **tangent** of the angle (Fig. 277).

For example, .57735 = tangent of 30° angle (see Trigonometric Tables).

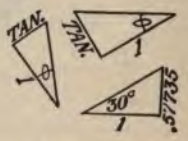


Fig. 277

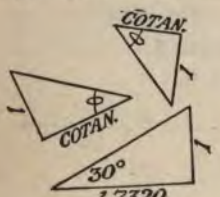


Fig. 278

187. When the side opposite the angle is 1, the length of the side adjoining the angle is called the **cotangent** of the angle (Fig. 278).

For example, 1.7320 = cotangent of 30° angle (see Trigonometric Tables).

188. When the side adjoining the angle is 1, the length of the hypotenuse is called the **secant** of the angle (Fig. 279).

For example, 1.1547 = secant of 30° angle (see Trigonometric Tables).

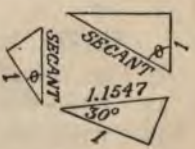


Fig. 279

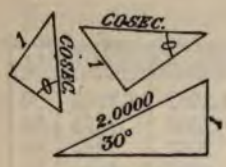


Fig. 280

189. When the side opposite the angle is 1, the length of the hypotenuse is called the **cosecant** of the angle (Fig. 280).

For example, 2.0000 = cosecant of 30° angle (see Trigonometric Tables).

Use of a Trigonometric Table

Minutes 34°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.55919	.82904	.67451	1.4826	1.2062	1.7883	60
1	.55943	.82887	.67493	.4816	.2064	.7875	59
2	.55967	.82871	.67535	.4807	.2067	.7867	58
3	.55992	.82855	.67578	.4798	.2069	.7860	57
4	.56016	.82839	.67620	.4788	.2072	.7852	56
5	[illegible]	.82822	.67663	1.4779	[illegible]	1.7844	[illegible]
[illegible]		[illegible]	.67705		.218		8
53	.57191	.82032	[illegible]	.4343	.2190	.7485	7
54	.57214	.82015	.69761	.4335	.2193	.7478	6
55	.57238	.81998	.69804	1.4326	1.2195	1.7471	5
56	.57262	.81982	.69847	.4317	.2198	.7463	4
57	.57286	.81965	.69891	.4308	.2200	.7456	3
58	.57310	.81948	.69934	.4299	.2203	.7449	2
59	.57334	.81932	.69977	.4290	.2205	.7442	1
60	.57358	.81915	.70021	1.4281	1.2208	1.7434	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

55°

Fig. 281

For angles less than 45° read table from the top down.

Thus, sine 34° 54′ = .57214.
Tangent 34° 1′ = .67493.

Note: For angles less than 45° pay no attention to the lower and right-hand columns.

34°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.55919	.82904	.67451	1.4826	1.2062	1.7883	60
1	.55943	.82887	.67493	.4816	.2064	.7875	59
2	.55967	.82871	.67535	.4807	.2067	.7867	58
3	.55992	.82855	.67578	.4798	.2069	.7860	57
4	.56016	.82839	.67620	.4788	.2072	.7852	56
5	.56040	.82822	.67663	1.4779	1.2074	1.7844	55
6	.56064		.67705	1.4770	.2076	.7837	54
7		.8205		.4761		.7829	
	.57214	.82015	.69761		.2195		6
52	.57238	.81998	.69804	1.4526	1.2195	1.7471	5
53	.57262	.81982	.69847	.4317	.2198	.7463	4
57	.57286	.81965	.69891	.4308	.2200	.7456	3
58	.57310	.81948	.69934	.4299	.2203	.7449	2
59	.57334	.81932	.69977	.4290	.2205	.7442	1
60	.57358	.81915	.70021	1.4281	1.2208	1.7434	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

55°

Minutes

Fig. 282

For angles greater than 45° read table from the bottom up.

Thus, tangent 55° 54′ = 1.4770.

Sine 55° 3′ = .81965.

NOTE: For angles greater than 45° pay no attention to the upper and left-hand columns.

EXERCISE 35

1. When the hypotenuse is 1, what is the side opposite the angle called?

2. When the hypotenuse is 1, what is the side adjoining the angle called?

3. When the side adjoining the angle is 1, what is the side opposite the angle called?

4. When the side adjoining the angle is 1, what is the hypotenuse called?

5. When the side opposite the angle is 1, what is the side adjoining the angle called?

6. When the side opposite the angle is 1, what is the hypotenuse called?

Use the Trigonometric Tables in the back of the book in solving the following problems.

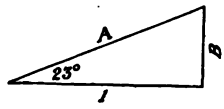


Fig. 283

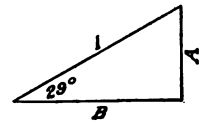


Fig. 284

7. In Fig. 283, what is A called and what is the value of this function?

8. In Fig. 283, what is B called and what is the value of this function?

9. In Fig. 284, what is A called and what is the value of this function?

10. In Fig. 284, what is B called and what is the value of this function?

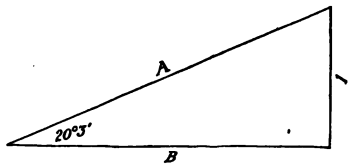


Fig. 285

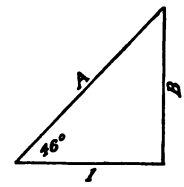


Fig. 286

11. In Fig. 285, what is A called and what is the value of this function?

12. In Fig. 285, what is B called and what is the value of this function?

13. In Fig. 286, what is A called and what is the value of this function?

14. In Fig. 286, what is B called and what is the value of this function?

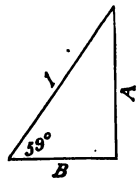


Fig. 287

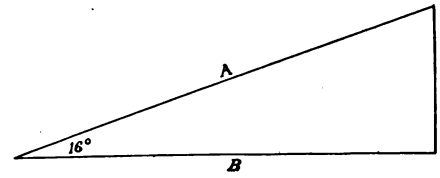


Fig. 288

15. In Fig. 287, what is A called and what is the value of this function?

16. In Fig. 287, what is B called and what is the value of this function?

17. In Fig. 288, what is A called and what is the value of this function?

18. In Fig. 288, what is B called and what is the value of this function?

Solution of Right Triangles When the Angle and One Side Are Given

190. By the use of the values of the **trigonometric functions,** the solutions of problems involving right triangles are evolved.

Example of the use of the **sine:**

Problem 1: In Fig. 289, what is the length of side marked X?

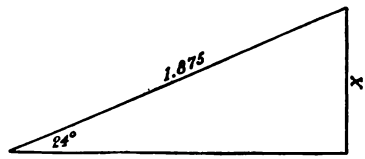


Fig. 289

Fig. 289A

Solution: The hypotenuse (1.875) is given; therefore, construct a similar figure whose hypotenuse is 1 (Fig. 289A). When the hypotenuse is 1, the side opposite the angle can be found by

looking up the sine of the given angle in the table. If the hypotenuse in the problem is 1.875 times as long as the other hypotenuse which is unity, the side opposite the angle must be 1.875 times the corresponding side in Fig. 289A, which is the sine of 24°.

Thus, A = sine 24° or .40674.

Then, X = 1.875 × A or 1.875 × sine 24° =
1.875 × .40674 = .7626375 or using 5 decimal places only, .76264.

191. The **cosine** may be used in a similar manner in the solution of problems involving right triangles.

Example of the use of the **cosine**:

PROBLEM 2: In Fig. 290, what is the length of side marked X?

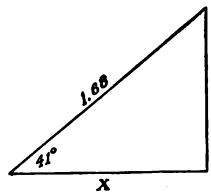


Fig. 290

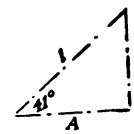


Fig. 290A

SOLUTION: The hypotenuse (1.68) is given; therefore, construct a similar figure whose hypotenuse is 1. When the hypotenuse is 1, the side adjoining the angle can be found by looking up the cosine of the given angle in the table. If the hypotenuse in the problem is 1.68 times as long as the other hypotenuse which is unity, the side adjoining must be 1.68 times the corresponding side in Fig. 290A, which is the cosine of 41°.

Thus, A = cosine 41° or .75471.

Then, X = 1.68 × A or 1.68 × cosine 41° =
1.68 × .75471 = 1.2679128 or, using 5 decimal places, 1.26791.

192. The **tangents, cotangents, secants,** and **cosecants** may be used in solving problems in a similar manner.

Example of the use of the **tangent:**

PROBLEM 3: In Fig. 291, what is the length of the side marked X?

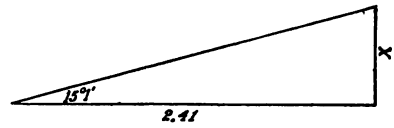


Fig. 291

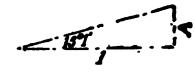


Fig. 291A

SOLUTION: Construct Fig. 291A as in the former problems. The triangles are similar; therefore, if the side adjoining the angle in Fig. 291 is 2.41 times the side adjoining the angle in Fig. 291A, the side opposite also must be 2.41 times the side opposite in the other triangle, but this is the tangent of 15° 1′.

Thus, A = tangent 15° 1′ or .26826.

Therefore, X = 2.41 × A or 2.41 × .26826 or .64650.

193. Example of the use of the **cotangent:**

PROBLEM 4: In Fig. 292, what is the length of the side marked X?

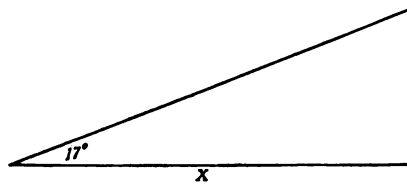


Fig. 292

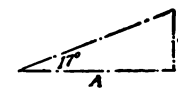


Fig. 292A

SOLUTION: Construct Fig. 292A. The triangles are similar; therefore, if the side opposite the angle in Fig. 292 is 4.1 times as long as the side opposite the angle in Fig. 292A, the side adjoining also must be 4.1 times the side adjoining in the other triangle, but this is the cotangent of 17°.

Thus, A = cotangent 17° or 3.2708.

Therefore, X = 4.1 × A or 4.1 × 3.2708 or 13.410.

194. Example of the use of the **secant:**

PROBLEM 5: In Fig. 293, what is the length of side marked X?

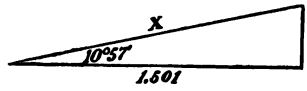


Fig. 293

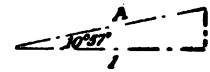


Fig. 293A

SOLUTION: Construct Fig. 293A. The triangles are similar; therefore, if the side adjoining the angle in Fig. 293 is 1.501 times the side adjoining the angle in Fig. 293A, the hypotenuse also must be 1.501 times the hypotenuse in the other triangle, but this is the secant of 10° 57′.

Thus, A = secant 10° 57′ or 1.0185.
Therefore, X = 1.501 × A or 1.501 × 1.0185 or 1.5287.

195. Example of the use of the **cosecant:**

PROBLEM 6: In Fig. 294, what is the length of side marked X?

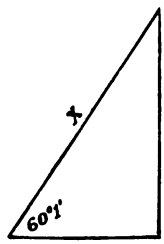


Fig. 294

Fig. 294A

SOLUTION: Construct Fig. 294A. The triangles are similar; therefore, if the side opposite the angle in Fig. 294 is 1.2 times as long as the side opposite the angle in Fig. 294A, the hypotenuse also must be 1.2 times the hypotenuse in the other triangle, but this is the cosecant of 60° 1′.

Thus, A = cosecant 60° 1′ or 1.1545.
Therefore, X = 1.2 × A or 1.2 × 1.1545 or 1.3854.

EXERCISE 36

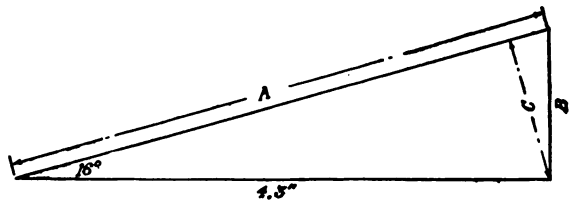


Fig. 295

1. In the right triangle (Fig. 295), in which an angle and an adjacent side are given, find A.

2. In the same figure, determine the length of the opposite side B.

3. What is the length of C, which is the perpendicular distance from the vertex of the right angle to the hypotenuse?

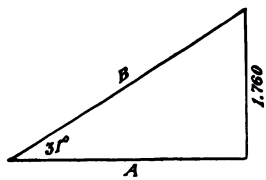


Fig. 296

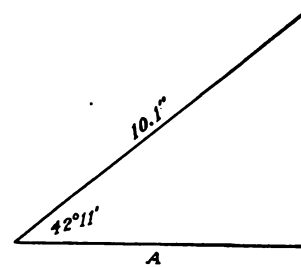


Fig. 297

4. In the right triangle (Fig. 296), one angle and the altitude being given, determine the adjacent side A.

5. In the same figure, find the length of the hypotenuse B.

6. In the right triangle (Fig. 297), one angle and the hypotenuse being given, find the adjacent side A.

Solution of Right Triangles When Any Side Is One and Another Side Is Given

196. When the hypotenuse is one, the length of the side opposite is the **sine** of the angle, and the length of the side adjoining is the **cosine** of the angle. By referring to a trigonometric table the corresponding angles may be determined.

Thus,

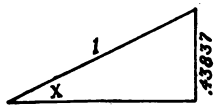


Fig. 298

In Fig. 298, .43837 = sine of angle X, and in the table, .43837 = sine 26°.

Therefore, angle X = 26°.

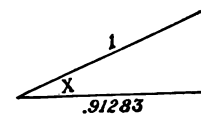


Fig. 298A

In Fig. 298A, .91283 = cosine of angle X, and in the table, .91283 = cosine 24° 6′.

Therefore, angle X = 24° 6′.

197. When the side adjoining the angle is one, the length of the side opposite is the **tangent** of the angle, and the length of the hypotenuse is the **secant** of the angle. By referring to a trigonometric table the corresponding angles may be determined.

Thus,

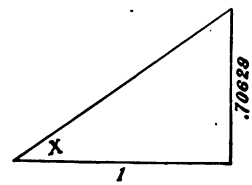


Fig. 299

In Fig. 299, .70629 = tangent of angle X, and in the table, .70629 = tangent 35° 14′.

Therefore, angle X = 35° 14′.

In Fig. 299A, 1.0326 = secant of angle X, and in the table, 1.0326 = secant of 14° 26′.

Therefore, angle X = 14° 26′.

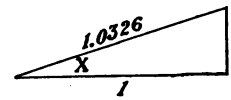


Fig. 299A

198. When the side opposite the angle is one, the length of the side adjoining is the **cotangent** of the angle, and the length of the hypotenuse is the **cosecant** of the angle. By referring to a trigonometric table the corresponding angles may be determined.

Thus,

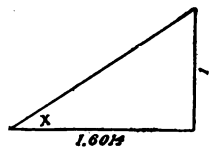


Fig. 300

In Fig. 300, 1.6014 = cotangent of angle X, and in the table, 1.6014 = cotangent 31° 59′.

Therefore, angle X = 31° 59′.

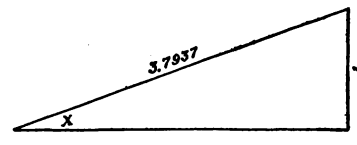


Fig. 300A

In Fig. 300A, 3.7937 = cosecant of angle X, and in the table, 3.7937 = cosecant 15° 17′.

Therefore, angle X = 15° 17′.

EXERCISE 37

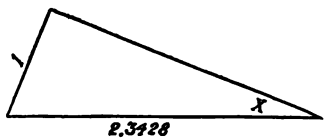


Fig. 301

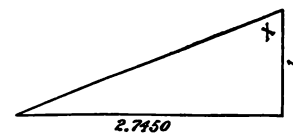


Fig. 302

1. In Fig. 301, find angle X, the opposite side being 1 and the hypotenuse 2.3428.

2. In Fig. 302, find angle X, the adjacent side being 1 and the opposite side 2.7450.

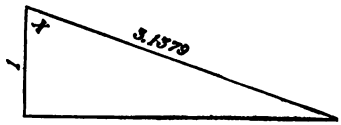


Fig. 303

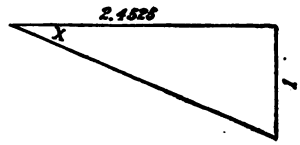


Fig. 304

3. In Fig. 303, find angle X, the adjacent side being 1 and the hypotenuse 3.1379.

4. In Fig. 304, find angle X, the opposite side being 1 and the adjacent side 2.4525.

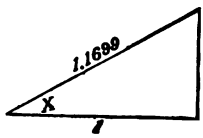


Fig. 305

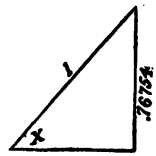


Fig. 306

5. In Fig. 305, find angle X, the adjacent side being 1 and the hypotenuse 1.1699.

6. In Fig. 306, find angle X, the hypotenuse being 1 and the opposite side .76754.

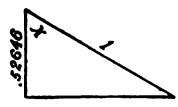


Fig. 307

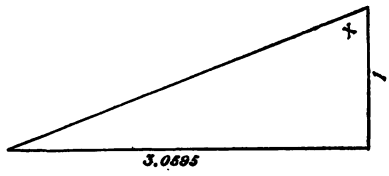


Fig. 308

7. In Fig. 307, find angle X, the hypotenuse being 1 and the adjacent side .52646.

8. In Fig. 308, find angle X, the adjacent side being 1 and the opposite side 3.0595.

Solution of Right Triangles When Two Sides Are Given

199. To determine the angle, when the hypotenuse and the opposite side are known:

If the sides of a triangle be divided by the same number and these quotients used to construct another triangle, the triangles will be similar.

Problem: In Fig. 309, find angle X.

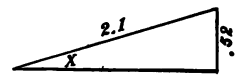


Fig. 309

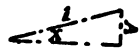


Fig. 309A

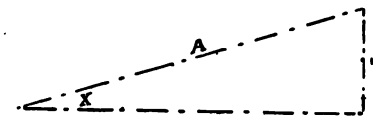


Fig. 309B

Solution 1: The hypotenuse (Fig. 309) must be divided by 2.1 to make it equal the hypotenuse in Fig. 309A. Therefore, the side opposite must be divided by 2.1 to make it equal the corresponding opposite side.

Thus,

$A = .52 \div 2.1 = .24761.$

.24761 = sine of angle X.

Therefore, angle X = 14° 20′.

Solution 2: The side opposite the angle (Fig. 309) must be divided by .52 to make it equal the side opposite the angle in Fig. 309B. Therefore, the hypotenuse must be divided by .52 to make it equal the hypotenuse of Fig. 309B.

Thus,

$A = 2.1 \div .52 = 4.0384.$

4.0384 = cosecant of angle X.

Therefore, angle X = 14° 20′.

In looking up an angle in the table use the angle that is closest to the number.

200. To determine the angle when the hypotenuse and the adjacent side are known:

PROBLEM: In Fig. 310, find angle X.

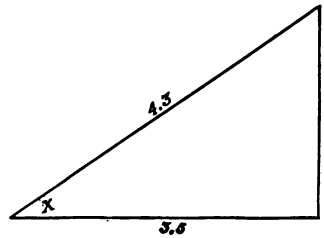


Fig. 310

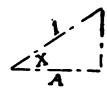


Fig. 310A

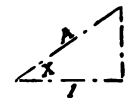


Fig. 310B

SOLUTION 1: The hypotenuse (Fig. 310) must be divided by 4.3 to make it equal the hypotenuse in Fig. 310A. Therefore, the side adjoining must be divided by 4.3 to make it equal the corresponding adjoining side.

Thus,

$A = 3.5 \div 4.3 = .813956.$

.813956 = cosine of angle X.

Therefore, angle X = 35° 31′.

SOLUTION 2: The side adjoining the angle (Fig. 310) must be divided by 3.5 to make it equal the side adjoining the angle in Fig. 310B. Therefore, the hypotenuse must be divided by 3.5 to make it equal the hypotenuse of Fig. 310B.

Thus,

$A = 4.3 \div 3.5 = 1.22857.$

1.22857 = secant of angle X.

Therefore, angle X = 35° 31′.

201. To determine the angle when the side adjacent and the side opposite are known:

PROBLEM: In Fig. 311, find angle X.

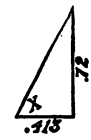


Fig. 311

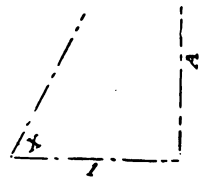


Fig. 311A

Fig. 311B

SOLUTION 1: The side adjoining the angle (Fig. 311) must be divided by .413 to make it equal the side adjoining in Fig. 311A. Therefore, the side opposite must be divided by .413 to make it equal the corresponding opposite side.

Thus,

$A = .72 \div .413 = 1.7433.$

1.7433 = tangent of angle X.

Therefore, angle X = 60° 10′.

SOLUTION 2: The side opposite the angle (Fig. 311) must be divided by .72 to make it equal the side opposite in Fig. 311B. Therefore, the side adjoining must be divided by .72 to make it equal the corresponding adjoining side of Fig. 311B.

Thus,

$A = .413 \div .72 = .57361.$

.57361 = cotangent of angle X.

Therefore, angle X = 60° 10′.

EXERCISE 38

Fig. 312

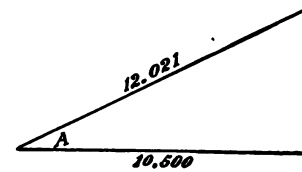


Fig. 313

1. In Fig. 312, determine angle A, two sides of the right triangle being given.

2. In Fig. 313, determine angle A, the hypotenuse and the adjacent side being given.

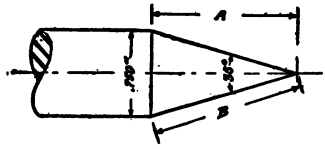


Fig. 314

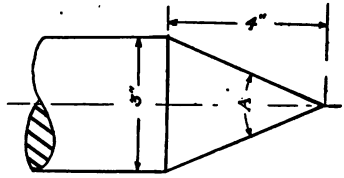


Fig. 315

3. In Fig. 314, determine the length of A from the data given.
4. In the same figure, find the distance indicated by B.
5. In Fig. 315, determine the value of angle A.

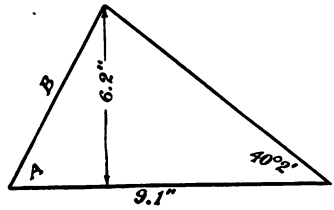


Fig. 316

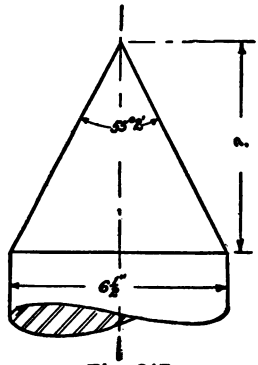


Fig. 317

6. In Fig. 316, find angle A.
7. In the same figure, find length of side B.
8. In Fig. 317, find the length indicated by ?.

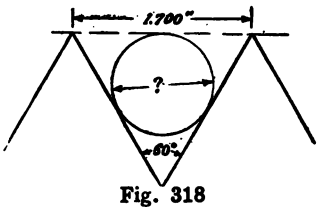


Fig. 318

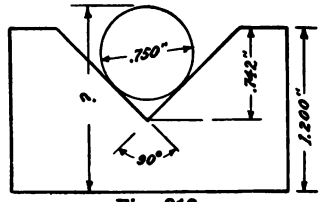


Fig. 319

9. In Fig. 318, find the diameter indicated by ?.
10. In Fig. 319, find the length indicated by ?.

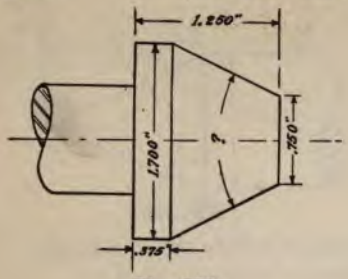


Fig. 320

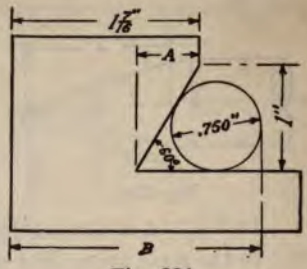


Fig. 321

11. In Fig. 320, find the angle indicated by ?.

12. In Fig. 321, find the length indicated by A.

13. In the same figure, determine the length B.

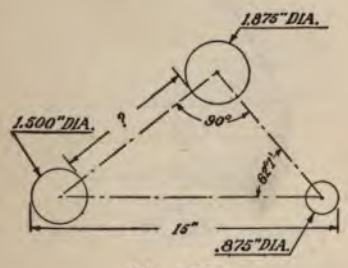


Fig. 322

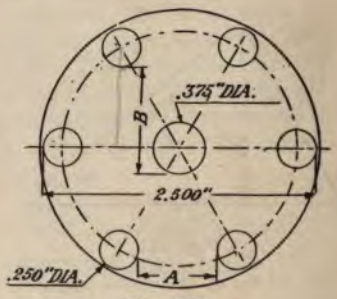


Fig. 323

14. Determine in Fig. 322 the length indicated by ?.

15. In Fig. 323, if the 6 holes in the disk are equally spaced, find the distance A.

16. In the same figure, determine the distance B

202. To determine the unknown side when two sides are given: When two sides of a right triangle are given the unknown side can be determined by first finding one of the angles.

To find the hypotenuse, when the two sides are given:

PROBLEM: In Fig. 324, find length of side marked X.

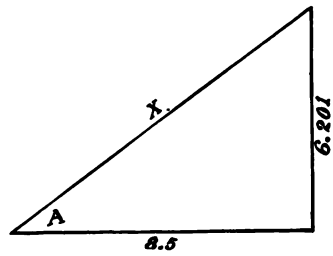


Fig. 324

SOLUTION: Tangent of $A = \frac{6.201}{8.5} = .72953$.

Angle $A = 36° 7'$.

Therefore, $X = 8.5 \times \text{secant } 36° 7' =$
$8.5 \times 1.2379 = 10.522$.

203. To find the other side, when the hypotenuse and one side are given:

PROBLEM: In Fig. 325, find length of side marked X.

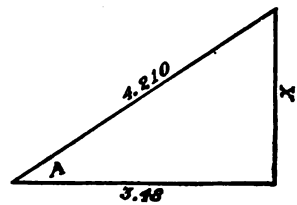


Fig. 325

SOLUTION: Cosine of $A = \frac{3.48}{4.21} = .82660$.

Angle $A = 34° 15'$.

Therefore, $X = 3.48 \times \text{tangent } 34° 15' =$
$3.48 \times .68087 = 2.3694$.

EXERCISE 39

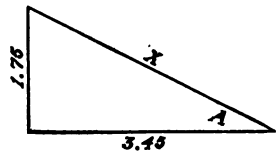


Fig. 326

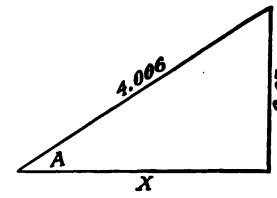


Fig. 327

1. In Fig. 326, determine the angle A.
2. In the same figure, find the length of the hypotenuse.
3. In Fig. 327, determine the angle A.
4. In the same figure, find the length of the adjacent side X.

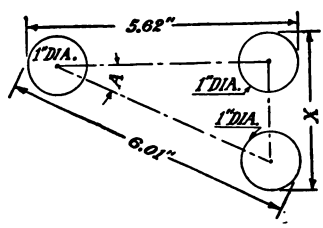


Fig. 328

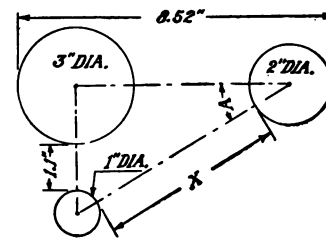


Fig. 329

5. In Fig. 328, determine the angle A.
6. In the same figure, find the length X.
7. In Fig. 329, determine the angle A.
8. In the same figure, find the length X.

Perpendiculars of a Triangle

204. The point where the sides of a triangle meet is called a **vertex.**

205. An **oblique triangle** is a triangle that has no right angle.

206. The **perpendiculars** of a triangle are lines drawn from the vertices, perpendicular to the sides or extended sides opposite.

Thus, in Figs. 330, 330A, and 330B, the perpendiculars are marked A.

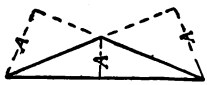


Fig. 330

Fig. 330A

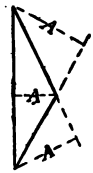


Fig. 330B

These perpendiculars divide any triangle into right triangles and are extremely useful in solving problems involving oblique triangles.

A good method for drawing the perpendiculars is shown in Figs. 331, 331A, and 331B. Place a ruler on one side of the triangle; then, take a square and draw the perpendicular from the vertex to the side the ruler is on, extending the side if necessary.

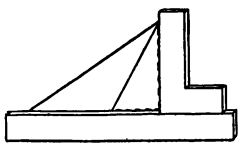


Fig. 331

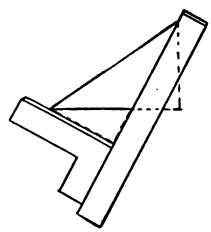


Fig. 331A

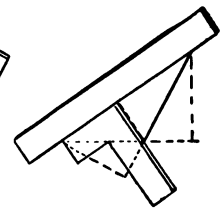


Fig. 331B

EXERCISE 40

Draw the three perpendiculars of each of the following triangles. If necessary, extend the sides.

Place an A on each perpendicular.

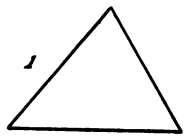


Fig. 332

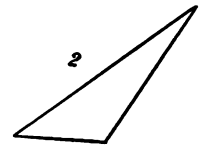


Fig. 332A

Fig. 332B

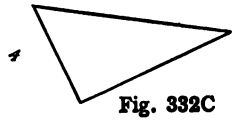


Fig. 332C

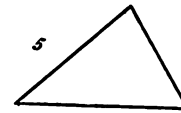


Fig. 332D

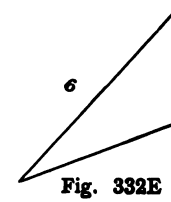


Fig. 332E

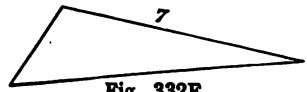


Fig. 332F

Solution of Oblique Triangles When Three Sides Are Given

207. In an oblique triangle, the distance between the midpoint of a side and the foot of the perpendicular to this side can always be determined by using the formula, $Y = \frac{(A \times A) - (C \times C)}{2 \times B}$, in which B = the side being considered, A = the longer of the two remaining sides, and C = the other side. (See Article 149 for the development of this formula.)

208. This formula is extremely useful in determining the angles when three sides of an oblique triangle are given.

To find the angles of a triangle, when three sides are given, and when the perpendicular from the vertex falls within the base of the triangle:

PROBLEM 1: In Fig. 333, find angles X and Z.

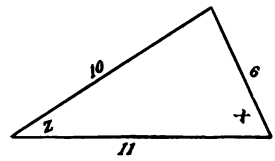


Fig. 333

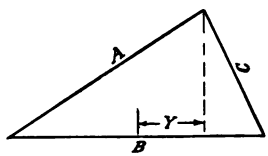


Fig. 333A

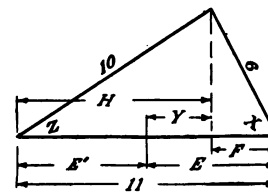


Fig. 333B

SOLUTION: In Fig. 333A, if A = 10, B = 11, and C = 6;

$$\text{then, } Y = \frac{(10\times10)-(6\times6)}{2\times11} = \frac{100-36}{22} = \frac{64}{22} = \frac{32}{11} = 2\tfrac{10}{11} = 2.9091.$$

$$\text{Likewise in Fig. 333B, } Y = \frac{(10\times10)-(6\times6)}{2\times11} = 2.9091,$$

$E = \frac{11}{2} = 5.5,$
$F = E - Y = 5.5 - 2.9091 = 2.5909.$
Cosine of $X = \frac{2.5909}{6} = .43181.$ Angle $X = 64°\ 25'.$

$H = E' + Y = 5.5 + 2.9091 = 8.4091.$
Cosine of $Z = \frac{8.4091}{10} = .84091.$ Angle $Z = 32°\ 46'.$

209. To find the angles of a triangle, when three sides are given and when the perpendicular from the vertex falls without the base of the triangle:

PROBLEM 2: In Fig. 334, find angles Z and X.

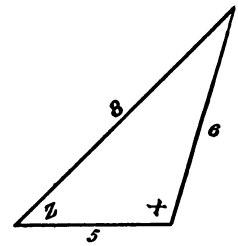


Fig. 334

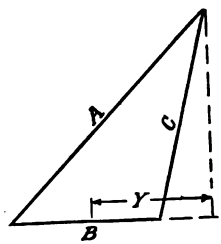


Fig. 334A

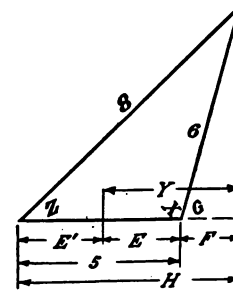


Fig. 334B

SOLUTION: In Fig. 334A, if A=8, B=5, and C=6; then, $Y=\frac{(8\times8)-(6\times6)}{2\times5}=\frac{64-36}{10}=\frac{28}{10}=\frac{14}{5}=2\frac{4}{5}=2.8$.

Likewise in Fig. 334B, $Y=\frac{(8\times8)-(6\times6)}{2\times5}=2.8$,

$E=\frac{5}{2}=2.5$,

$F=Y-E=2.8-2.5=.3$.

Cosine of $X=\frac{3}{6}=.050$. Angle $G=87°\ 8'$.

Angle $X=180°-87°\ 8'=92°\ 52'$.

$H=Y+E'=2.8+2.5=5.3$.

Cosine of angle $Z=\frac{5.3}{8}=.6625$. Angle $Z=48°\ 31'$.

210. Additional illustration of this method of solution:

PROBLEM 3: Find angles X and Z.

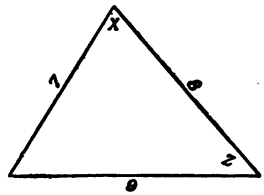


Fig. 335

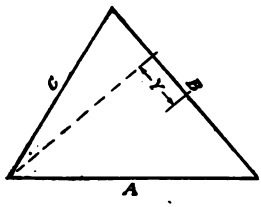


Fig. 335A

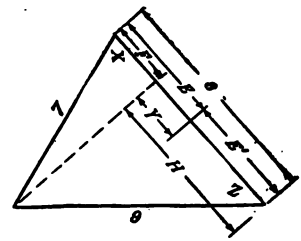


Fig. 335B

SOLUTION: In Fig. 335A, if A=9, B=8, and C=7;

$$\text{then, } Y=\frac{(9\times 9)-(7\times 7)}{2\times 8}=\frac{81-49}{16}=\frac{32}{16}=2.000.$$

$$\text{Likewise in Fig. 335B, } Y=\frac{(9\times 9)-(7\times 7)}{2\times 8}=2.000,$$

$E=\frac{8}{2}=4.000,$
$F=E-Y=4.00-2.00=2.00.$
Cosine of $X=\frac{2}{7}=.28571.$ Angle $X=73°\ 24'.$

$H=E'+Y=4.00+2.00=6.00.$
Cosine of $Z=\frac{6.00}{9}=.66667.$ Angle $Z=48°\ 11'.$

EXERCISE 41

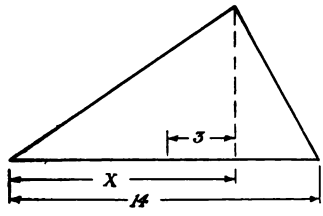


Fig. 336

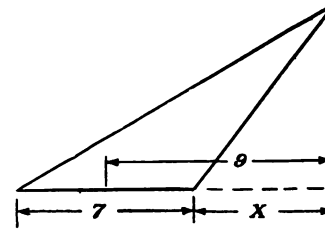


Fig. 337

1. In Fig. 336, find X, which measures the distance from the vertex of the acute angle to the foot of the perpendicular.

2. In Fig. 337, find X, which is the extended base to the foot of the perpendicular.

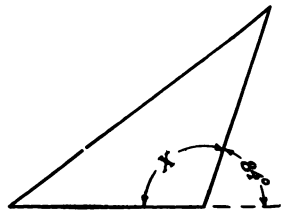


Fig. 338

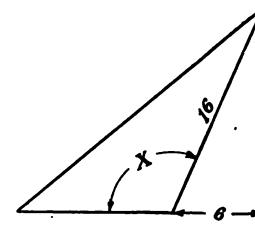


Fig. 339

3. In Fig. 338, find the value of the angle X.

4. In Fig. 339, find the value of the angle X.

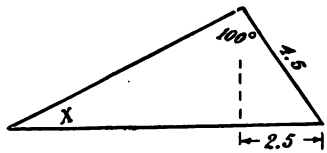


Fig. 340

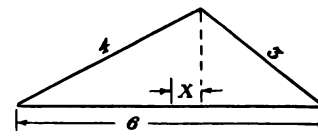


Fig. 341

5. In Fig. 340, find the value of the angle X.

6. In Fig. 341, find the distance X, which measures from the midpoint of the base to the foot of the perpendicular.

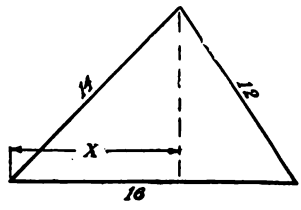


Fig. 342

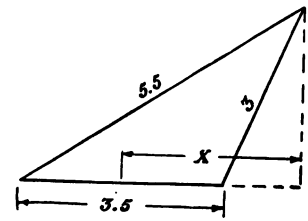


Fig. 343

7. In Fig. 342, find the distance X from the vertex to the foot of the perpendicular.

8. In Fig. 343, find the distance X from the midpoint of the base to the foot of the perpendicular.

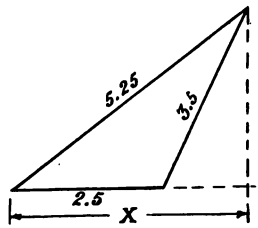


Fig. 344

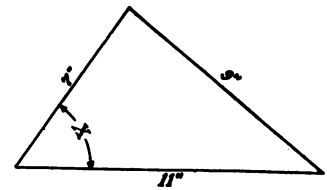


Fig. 345

9. In Fig. 344, find the distance X from the vertex of the acute angle of the base to the foot of the perpendicular.

10. In the oblique triangle (Fig. 345), three sides being given, find angle X.

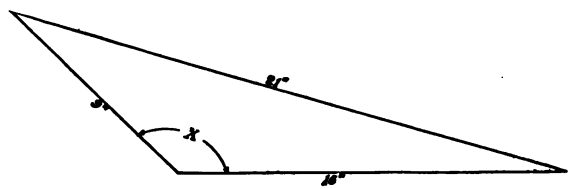


Fig. 346

11. In the oblique triangle (Fig. 346), find the angle X opposite the longest side, the lengths of the three sides being given.

Solution of Oblique Triangles When Two Sides and One Angle Are Given

211. When solving oblique triangles, first draw the three perpendiculars; then, place an A on those that can be figured in one operation. The perpendiculars that assist most are the ones to be used in finding the unknown sides or angles.

To solve an oblique triangle, when the unknown side is adjacent to both of the unknown angles:

Problem 1: In Fig. 347, find angles B and C and side X.

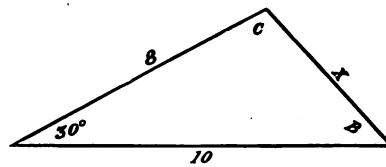


Fig. 347

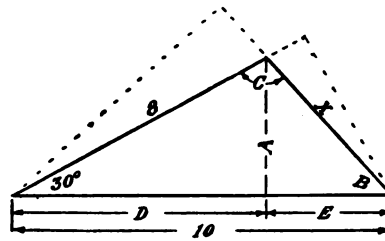


Fig. 348

Solution 1:

In Fig. 348, $A = 8 \times \text{sine } 30° = 8 \times .50000 = 4.0000.$

$D = 8 \times \text{cosine } 30° = 8 \times .86603 = 6.92824.$

$E = 10 - D = 10 - 6.92824 = 3.07176.$

Cotangent of angle $B = \frac{E}{A} = \frac{3.07176}{4.0000} = .76799.$

Angle $B = 52° 29'.$

$X = A \times \text{cosecant } 52° 29' = 4 \times 1.2607 = 5.0428.$

$C = 180° - (52° 29' + 30°) = 97° 31'.$

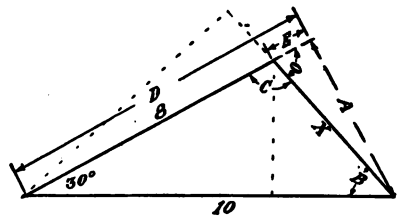


Fig. 349

SOLUTION 2: In Fig. 349, A = 10 × sine 30° = 10 × .50000 = 5.0000.
D = 10 × cosine 30° = 10 × .86603 = 8.6603.
E = D − 8 = 8.6603 − 8.0000 = .6603.
Cotangent of angle O = $\frac{E}{A}$ = $\frac{.6603}{5.0000}$ = .13206.
Angle O = 82° 29′.
C = 180° − 82° 29′ = 97° 31′.
X = A × cosecant 82° 29′ = 5 × 1.0087 = 5.0435.
B = 180° − (30° + 97° 31′) = 52° 29′.

212. To solve an oblique triangle, when the unknown side is adjacent to the given angle, and when the perpendicular to this side falls without the side; that is, on the side extended:

PROBLEM 2: In Fig. 350, find angles B and C and side marked X.

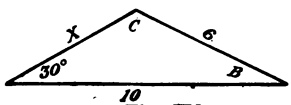


Fig. 350

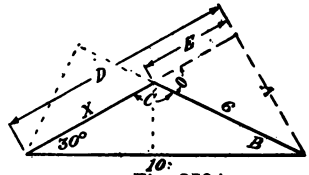


Fig. 350A

SOLUTION: In Fig. 350A, A = 10 × sine 30° = 10 × .50000 = 5.0000.
D = 10 × cosine 30° = 10 × .86603 = 8.6603.
Sine of angle O = $\frac{A}{6}$ = $\frac{5.0000}{6}$ = .83333.
Angle O = 56° 27′.
C = 180° − 56° 27′ = 123° 33′.
E = 6 × cosine 56° 27′ = 6 × .55266 = 3.3160.
X = D − E = 8.6603 − 3.3160 = 5.2443.
Angle B = 180° − (30° + 123° 33′) = 26° 27′.

213. To solve an oblique triangle, when the unknown side is adjacent to the given angle, and when the perpendicular to this side falls within this side:

PROBLEM 3: In Fig. 351, find angles B and C and side marked X.

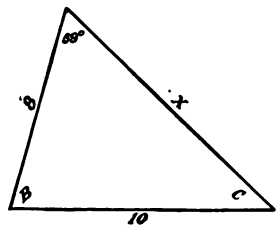


Fig. 351

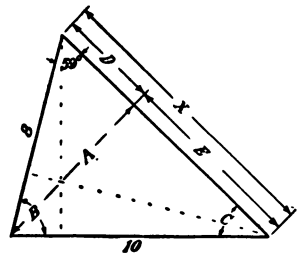


Fig. 351A

SOLUTION: In Fig. 351A, $A = 8 \times \text{sine } 59° = 8 \times .85717 = 6.85736$.
$D = 8 \times \text{cosine } 59° = 8 \times .51504 = 4.12032$.
Sine of angle $C = \frac{A}{10} = \frac{6.85736}{10} = .685763$.
Angle C × 43° 18′.
$E = 10 \times \text{cosine } 43° 18' = 10 \times .72777 = 7.2777$.
$X = E + D = 7.2777 + 4.1203 = 11.3980$.
Angle $B = 180° - (59° + 43° 18') = 77° 42'$.

EXERCISE 42

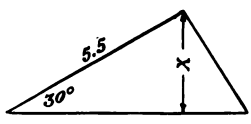


Fig. 352

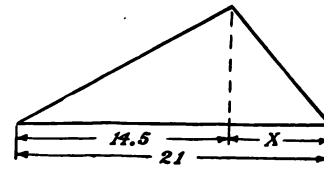


Fig. 353

1. In Fig. 352, find the length of X.
2. In Fig. 353, find the length of X.

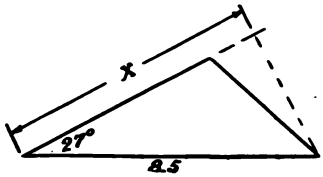


Fig. 354

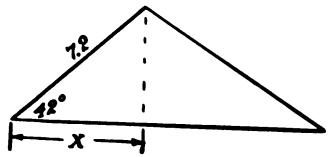


Fig. 355

3. In Fig. 354, find the length of X.
4. In Fig. 355, find the length of X.

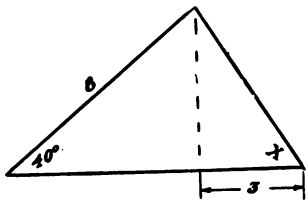


Fig. 356

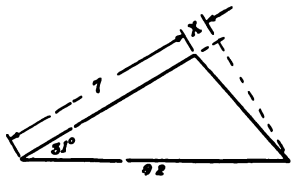


Fig. 357

5. In Fig. 356, find the angle X.
6. In Fig. 357, find the length of X.

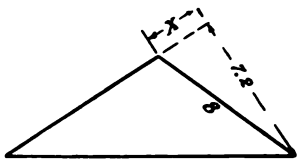


Fig. 358

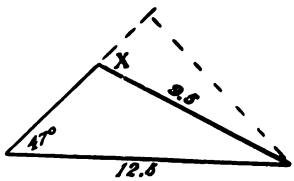


Fig. 359

7. In Fig. 358, find the length of X.
8. In Fig. 359, find the angle X.

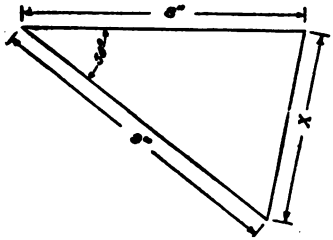


Fig. 360

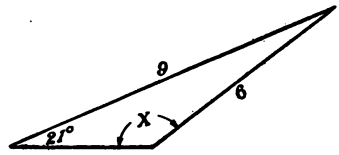


Fig. 361

9. In Fig. 360, find the length of X.

10. In Fig. 361, find the angle X.

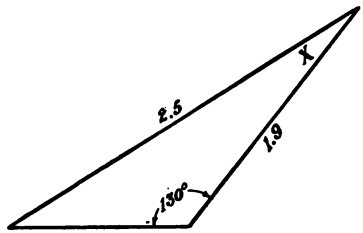


Fig. 362

11. In Fig. 362, find the angle X.

Solution of Oblique Triangles When Two Angles and One Side Are Given

214. To find the other two sides, when the given side is adjacent to the two given angles, and the other angle is *greater* than 90°:

Problem 1: In Fig. 363, find sides B and C and angle X.

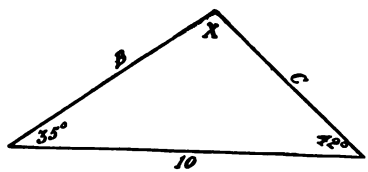


Fig. 363

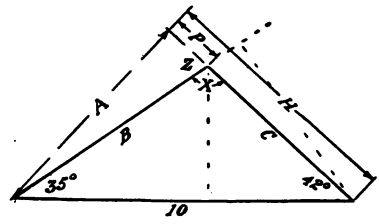


Fig. 364

SOLUTION 1: In Fig. 364, $X = 180° - (35° + 42°) = 103°$.

$A = 10 \times \text{sine } 42° = 10 \times .66913 = 6.6913.$
Angle $Z = 180° - X = 180° - 103° = 77°.$
$B = A \times \text{cosecant } 77° = 6.6913 \times 1.0263 = 6.8673.$
$H = 10 \times \text{cosine } 42° = 10 \times .74314 = 7.4314.$
$P = A \times \text{cotangent } 77° = 6.6913 \times .23087 = 1.5448.$
$C = H - P = 7.4314 - 1.5448 = 5.8866.$

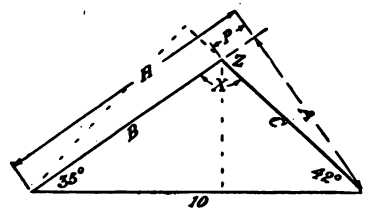


Fig. 365

SOLUTION 2: In Fig. 365, $X = 180° - (35° + 42°) = 103°$.

$A = 10 \times \text{sine } 35° = 10 \times .57358 = 5.7358.$
Angle $Z = 180° - X = 180° - 103° = 77°.$
$C = A \times \text{cosecant } 77° = 5.7358 \times 1.0263 = 5.8866.$
$H = 10 \times \text{cosine } 35° = 10 \times .81915 = 8.1915.$
$P = A \times \text{cotangent } 77° = 5.7358 \times .23087 = 1.3242.$
$B = H - P = 8.1915 - 1.3242 = 6.8673.$

215. To find the other two sides, when the given side is adjacent to the two given angles, and the other angle is *less* than 90°:

PROBLEM 2: In Fig. 366, find sides B and C and angle X.

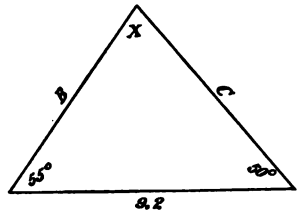


Fig. 366

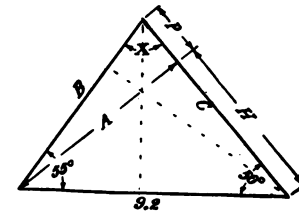


Fig. 367

SOLUTION 1: In Fig. 367, $X = 180° - (55° + 50°) = 75°$.

$A = 9.2 \times \text{sine } 50° = 9.2 \times .76604 = 7.0476.$
$B = A \times \text{cosecant } 75° = 7.0476 \times 1.0353 = 7.2964.$
$H = 9.2 \times \text{cosine } 50° = 9.2 \times .64279 = 5.9137.$
$P = A \times \text{cotangent } 75° = 7.0476 \times .26795 = 1.8884.$
$C = H + P = 5.9137 + 1.8884 = 7.8021.$

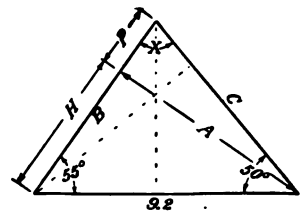


Fig. 368

SOLUTION 2: In Fig. 368, $X = 180° - (55° + 50°) = 75°$.

$A = 9.2 \times \text{sine } 55° = 9.2 \times .81915 = 7.5362.$
$C = A \times \text{cosecant } 75° = 7.5362 \times 1.0353 = 7.8022.$
$H = 9.2 \times \text{cosine } 55° = 9.2 \times .57358 = 5.2769.$
$P = A \times \text{cotangent } 75° = 7.5362 \times .26795 = 2.0193.$
$B = H + P = 5.2769 + 2.0193 = 7.2962.$

216. To find the other two sides, when they include one of the given angles:

PROBLEM 3: In Fig. 369, find sides B and C and angle X.

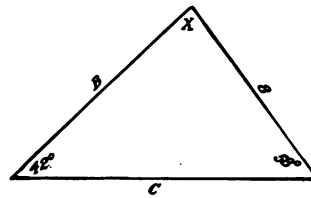


Fig. 369

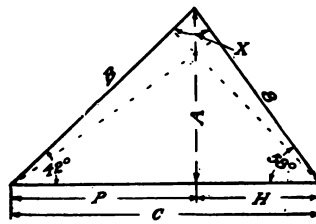


Fig. 370

SOLUTION 1: In Fig. 370, $X = 180° - (42° + 58°) = 80°$.

$A = 8 \times \text{sine } 58° = 8 \times .84805 = 6.7844.$
$B = A \times \text{cosecant } 42° = 6.7844 \times 1.4945 = 10.1393.$
$H = 8 \times \text{cosine } 58° = 8 \times .52992 = 4.23936.$
$P = A \times \text{cotangent } 42° = 6.7844 \times 1.1106 = 7.5348.$
$C = H + P = 4.2394 + 7.5348 = 11.7742.$

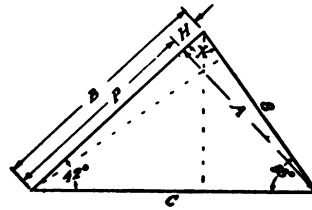


Fig. 371

SOLUTION 2: In Fig. 371, $X = 180° - (42° + 58°) = 80°$.

$A = 8 \times \text{sine } 80° = 8 \times .98481 = 7.8785.$

$C = A \times \text{cosecant } 42° = 7.8785 \times 1.4945 = 11.7744.$

$H = 8 \times \text{cosine } 80° = 8 \times .17365 = 1.3892.$

$P = A \times \text{cotangent } 42° = 7.8785 \times 1.1106 = 8.7498.$

$B = H + P = 1.3892 + 8.7498 = 10.139.$

EXERCISE 43

In the following figures, find the length indicated by X.

1.

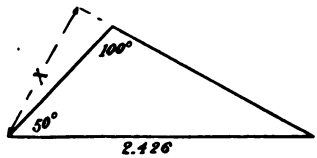


Fig. 372

2.

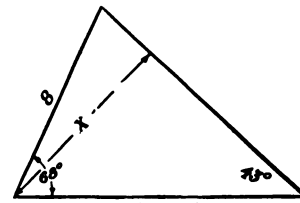


Fig. 373

3.

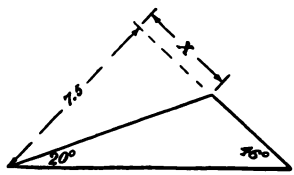


Fig. 374

4.

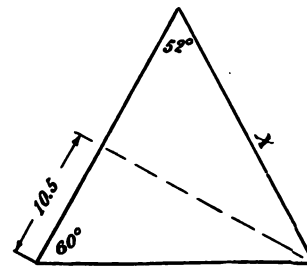


Fig. 375

5.

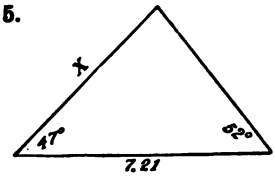


Fig. 376

6.

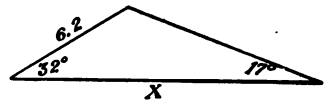


Fig. 377

7.

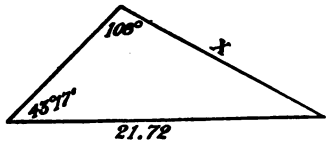


Fig. 378

8.

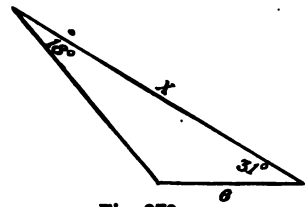


Fig. 379

To Find the Altitude of a Triangle

217. To find the altitude of an oblique triangle when two angles and the base are given and the altitude line falls *within* the base:

Construct a similar triangle whose altitude is one and use the following process:

Problem 1: In Fig. 380, find X.

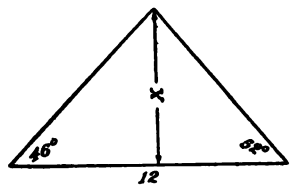


Fig. 380

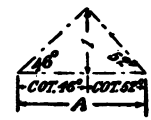


Fig. 380A

Solution: In Fig. 380A, A = cotangent 46° + cotangent 52° = .96569 + .78129 = 1.74698.

If the base of the one triangle is $\frac{12}{1.74698}$ times as long as the base of the other triangle, the altitude of the one triangle must be $\frac{12}{1.74698}$ times the altitude of the other.

Therefore, $X = 1 \times \frac{12}{1.74698} = 6.8690$.

218. To find the altitude of an oblique triangle when two angles and the base are given and the altitude line falls *without* the base:

Construct a similar triangle whose altitude is one and use the following process:

PROBLEM 2: In Fig. 381, find X.

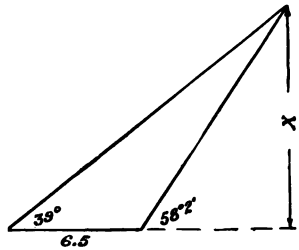


Fig. 381

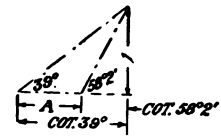


Fig. 381A

SOLUTION: In Fig. 381A, A = cotangent 39° − cotangent 58° 2′ = 1.2349 − .62406 = .61084.

If the base of the one is $\frac{6.5}{.61084}$ times as long as the base of the other triangle, the altitude of the one must be $\frac{6.5}{.61084}$ times the altitude of the other.

Therefore, $X = 1 \times \frac{6.5}{.61084} = 10.641$.

EXERCISE 44

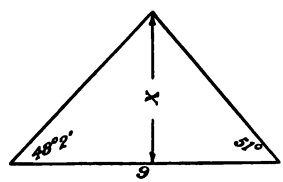


Fig. 382

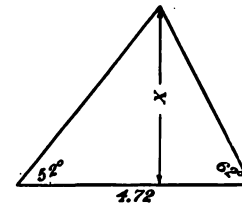


Fig. 383

1. In Fig. 382, find the altitude X.
2. In Fig. 383, find the altitude X.

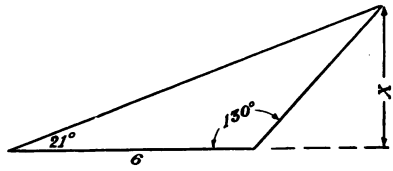


Fig. 384

3. In Fig. 384, find the altitude X.

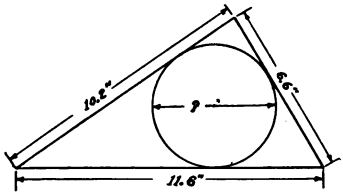


Fig. 385

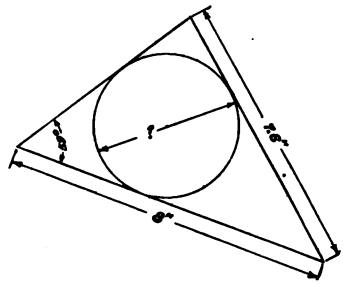


Fig. 386

4. In Fig. 385, find the diameter of the inscribed circle.

5. In Fig. 386, find the diameter of the inscribed circle.

Practical Problems

EXERCISE 45

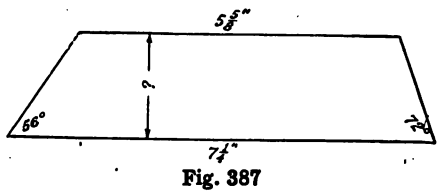
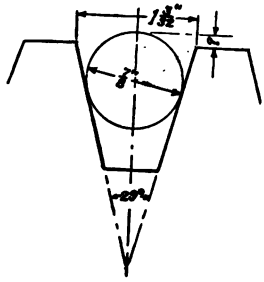


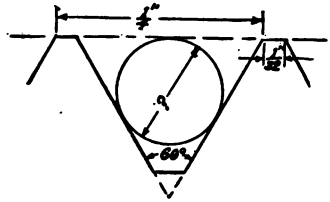
Fig. 387

1. In Fig. 387, solve for ?.



Method of Checking Acme and Worm Threads

Fig. 388



Method of Checking U. S. Form Threads

Fig. 389

2. In Fig. 388, solve for ?.

3. In Fig. 389, solve for ?.

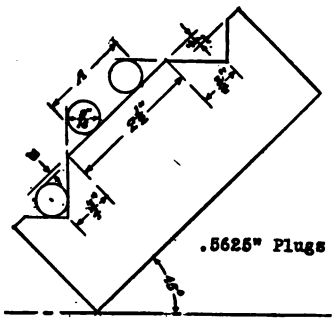
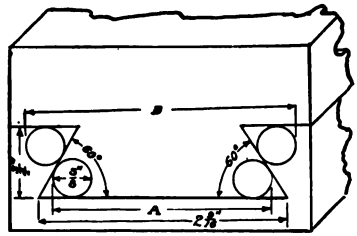


Fig. 390



Method of Checking Dove Tails

Fig. 391

4. In Fig. 390, find value of A.

5. In Fig. 390, find value of B.

6. In Fig. 391, what does A equal?

7. In Fig. 391, what does B equal?

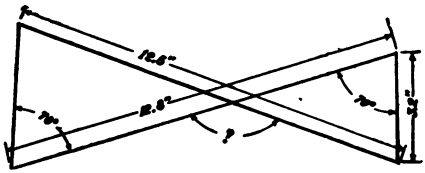


Fig. 392

8. In Fig. 392, find angle ?.

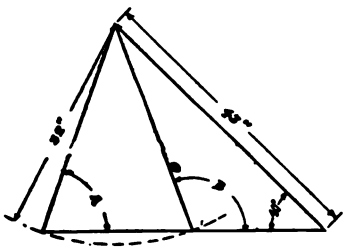


Fig. 393

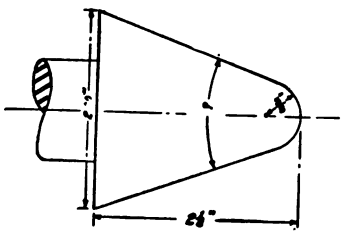


Fig. 394

9. In Fig. 393, find angle A.

10. In Fig. 393, find angle B.

11. In forming punch shown in Fig. 394, find angle ?.

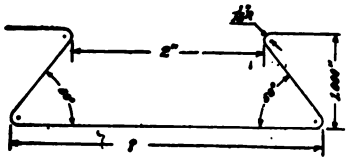


Fig. 395

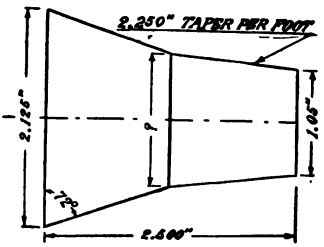
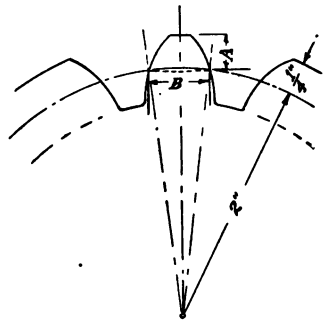


Fig. 396

12. In Fig. 395, find length ?.

13. In Fig. 396, find length ?.



Method of Checking Gear Teeth

Fig. 397

14. In Fig. 397, if the gear has 16 teeth and a diametral pitch of 4, find A.

15. In the same figure, find B.

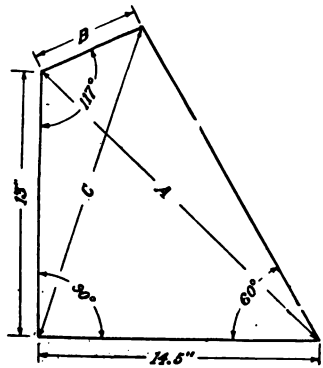


Fig. 398

16. In Fig. 398, find length A.

17. In the same figure, find length B.

18. In the same figure, find length C.

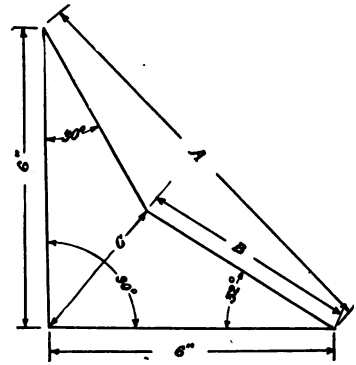


Fig. 399

19. In Fig. 399, find length A.

20. In the same figure, find length B.

21. In the same figure, find length C.

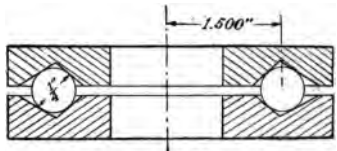
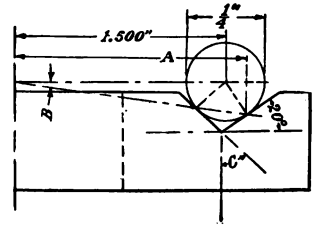


Fig. 400



Application of Trigonometry in Ball Race Designing

Fig. 400A

22. In Fig. 400A, find length A.

23. In the same figure, find angle B.

24. In the same figure, find angle C.

EXERCISE 46

NOTE: Do not use the trigonometric table in solving the following problems.

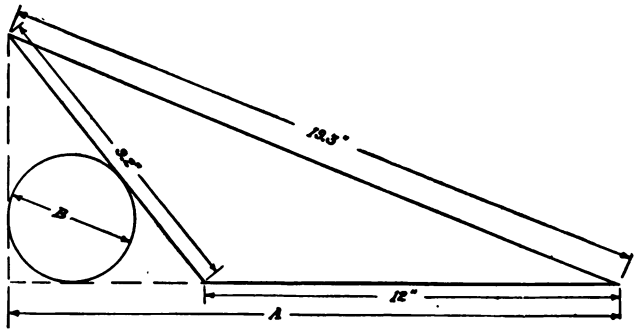


Fig. 401

1. In Fig. 401, find the dimension indicated by A.
2. In the same figure, find B.

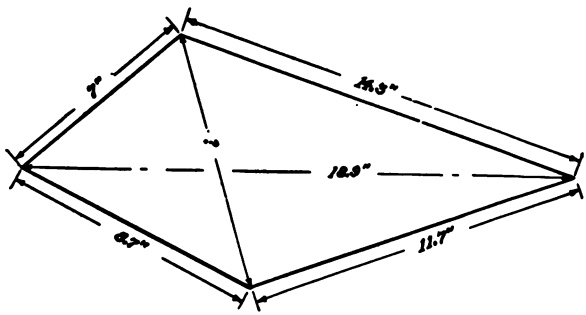


Fig. 402

3. In Fig. 402, find the dimension indicated by ?.

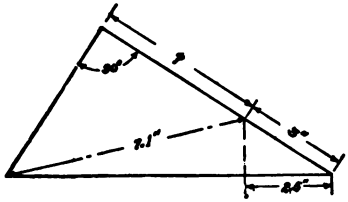


Fig. 403

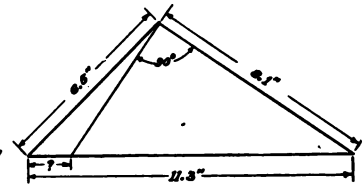


Fig. 404

4. In Fig. 403, find the dimension indicated by ?.
5. In Fig. 404, find the dimension indicated by ?.

Sine Bar Problems

EXERCISE 47

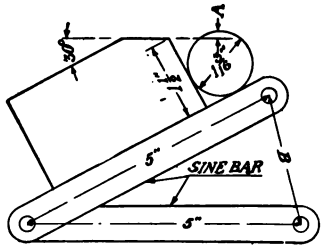


Fig. 405

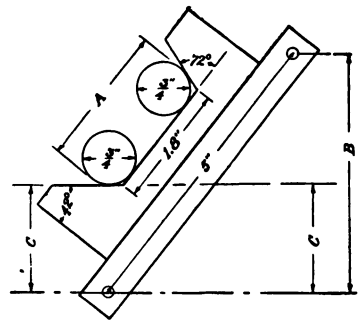


Fig. 406

1. In Fig. 405, what does A equal?
2. In Fig. 405, what does B equal?
3. In Fig. 406, what does A equal?
4. In Fig. 406, what does B equal?

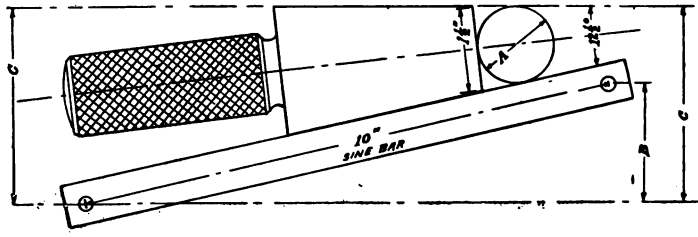


Fig. 407

5. In Fig. 407, what does A equal?

6. In Fig. 407, what does B equal?

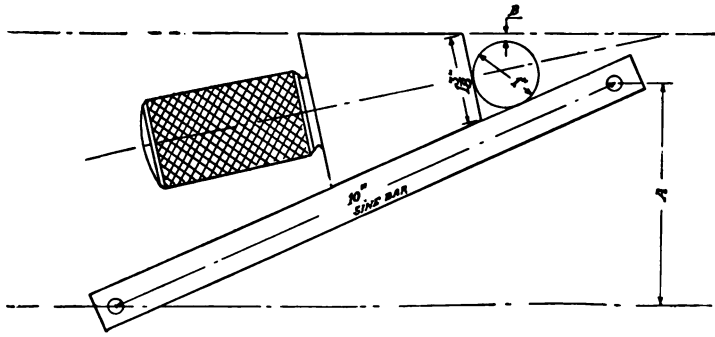


Fig. 408

7. In Fig. 408, if the taper per foot is 4.500″, what does A equal?

8. In the same figure, what does B equal?

Gage Problems

EXERCISE 48

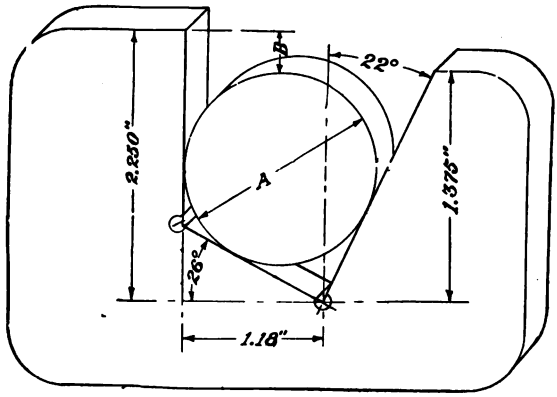


Fig. 409

1. In Fig. 409, find the dimension A.
2. In the same figure, find the dimension B.

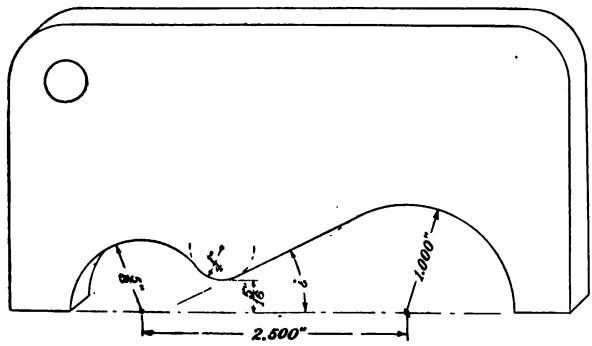


Fig. 410

3. In Fig. 410, find the angle indicated by ?.

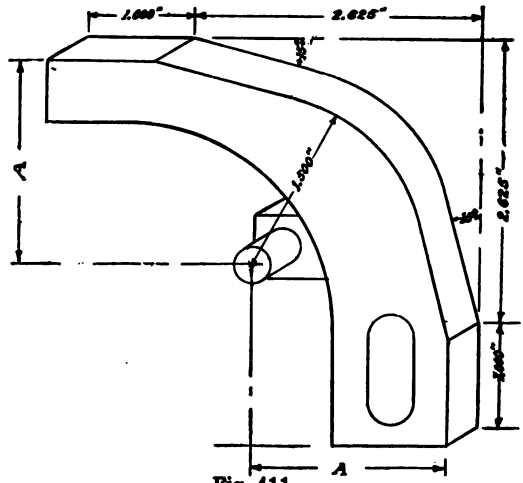


Fig. 411

4. In Fig. 411, find the length A.

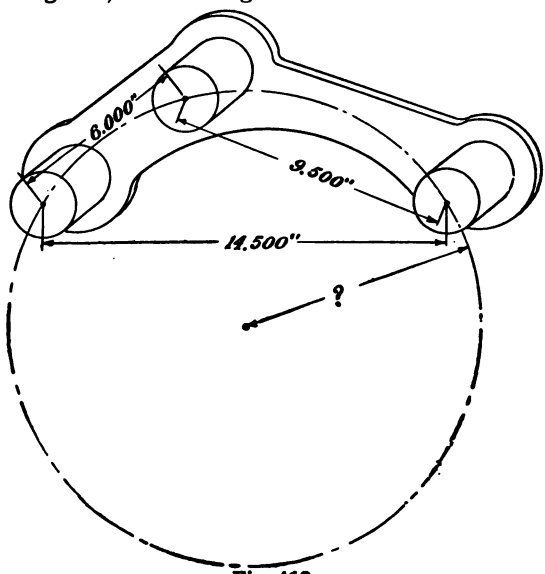


Fig. 412

5. In Fig. 412, find the length of the radius indicated by ?.

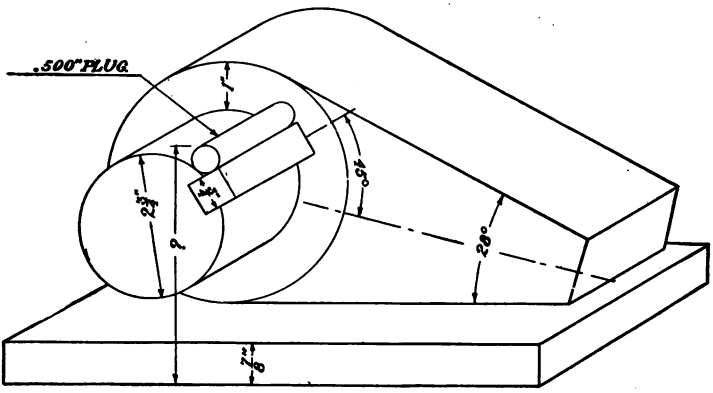


Fig. 413

6. In Fig. 413, find the diameter indicated by ?.

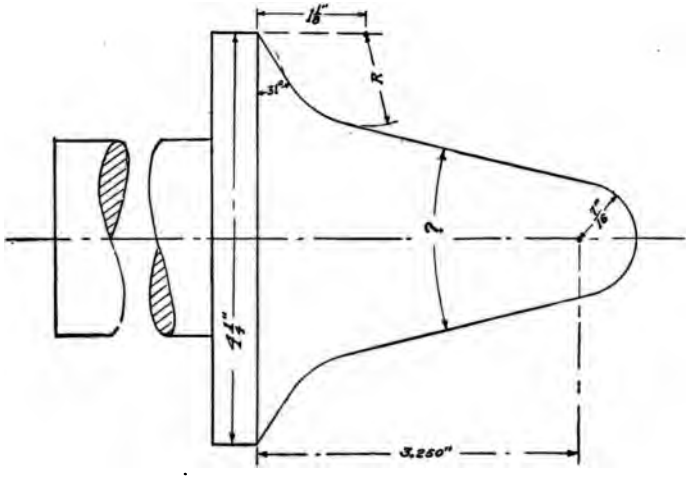


Fig. 414

7. In Fig. 414, find R.

8. In Fig. 414, find angle indicated by ?.

Checking Holes Bored at an Angle

EXERCISE 49

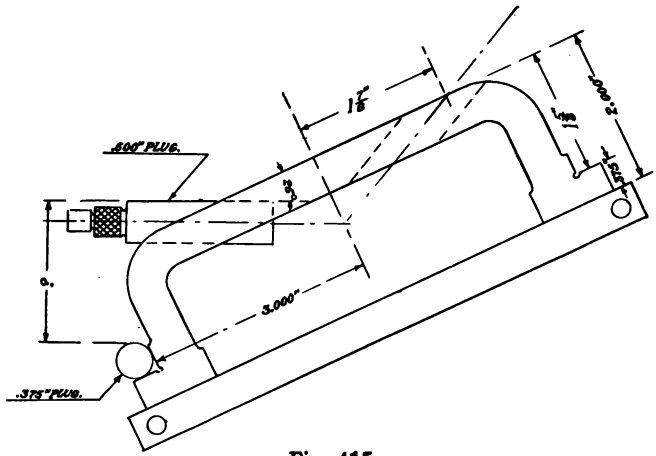


Fig. 415

1. In Fig. 415, find the dimension indicated by ?.

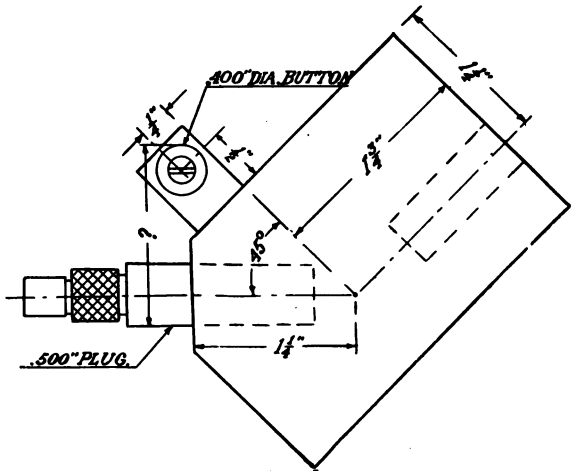


Fig. 416

2. In Fig. 416, find the dimension indicated by ?.

XXVI

SPUR GEAR AND RACK

219. The **standard $14\frac{1}{2}°$ involute gear** has a $14\frac{1}{2}°$ **pressure angle.**

The **rack teeth** have straight sides $14\frac{1}{2}°$ from the vertical line.

Diametral pitch (D. P.) is the number of teeth to each inch of the pitch diameter, or the number of teeth in 3.1416 inches measured along the pitch line. Diametral pitch is sometimes simply referred to as the **pitch.**

Circular pitch (C. P.) is the distance from the edge of one tooth to the corresponding edge of the next tooth measured along the pitch line.

$$\text{D.P.} = \frac{3.1416}{\text{C.P.}} \text{ or C.P.} = \frac{3.1416}{\text{D.P.}}.$$

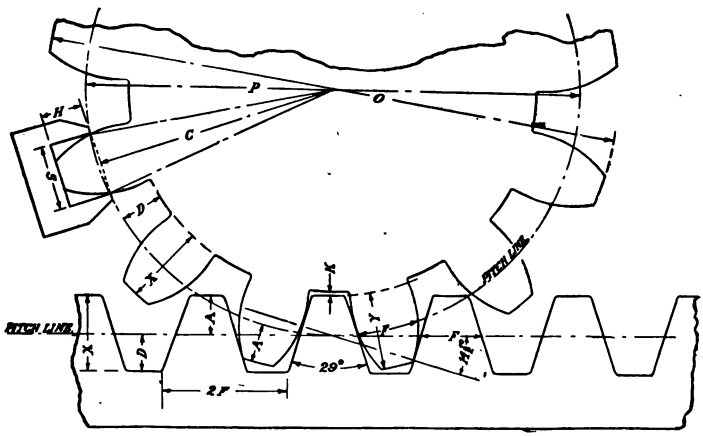


Fig. 417

N = number of teeth in gear.

P = pitch diameter = $\frac{N}{D.P.}$.

O = outside diameter = $\frac{N+2}{D.P.}$.

F = thickness of tooth measured along the pitch line = $\frac{1}{2}$(C.P.).

A = addendum = (.3183)(C.P) or $\frac{1}{D.P.}$.

K = clearance = (.05)(C.P.) or $\frac{.157}{D.P.}$.

D = dedendum = (.3683)(C.P) or $\frac{1.157}{D.P.}$.

Y = working depth = (.6366)(C.P.) or $\frac{2}{D.P.}$.

X = whole depth = (.6866)(C.P.) or $\frac{2.157}{D.P.}$.

Checking Gear and Rack with Gear Tooth Vernier Calipers

220. Dimensions for gear tooth vernier calipers for checking on the pitch line when the pressure angle is $14\frac{1}{2}°$.

N	R	U	N	R	U
8	1.5607	1.0769	17	1.5686	1.0362
9	1.5628	1.0684	21	1.5694	1.0294
10	1.5643	1.0616	26	1.5698	1.0237
11	1.5654	1.0559	35	1.5702	1.0176
12	1.5663	1.0514	55	1.5706	1.0112
14	1.5675	1.0440	135	1.5707	1.0047

In Fig. 417, C = $\frac{1}{2}$P $\left(\text{cosine } \frac{90°}{N}\right)$.

S = chordal thickness = P $\left(\text{sine } \frac{90°}{N}\right)$ or $\frac{R}{D.P.}$.

H = depth to set calipers = ($\frac{1}{2}$O − C) or $\frac{U}{D.P.}$.

Checking Gear and Rack with Plugs

221. Dimensions for checking gear teeth with plugs tangent at the pitch line with any pressure angle, external and internal.

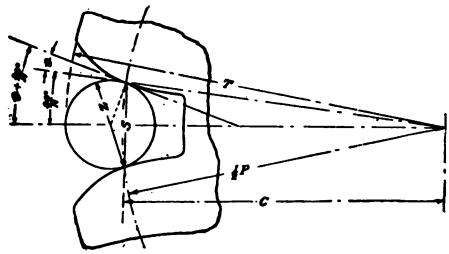


Fig. 418

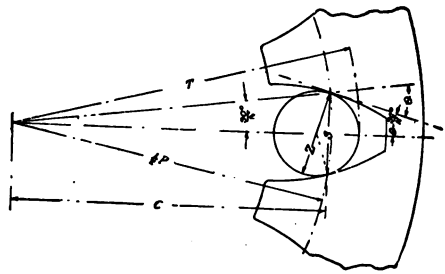


Fig. 419

θ = pressure angle.
S = chordal thickness.
$C = \frac{1}{2}P\left(\text{cosine } \frac{90^\circ}{N}\right)$.
Z = diameter of plug to be used.
T = dimension from outside of plug to center of gear.

External gears, any pressure angle.

$$Z = S\left[\text{secant } \left(\theta + \frac{90^\circ}{N}\right)\right].$$

$$T = C + \frac{Z}{2} + \frac{S}{2}\left[\text{tangent } \left(\theta + \frac{90^\circ}{N}\right)\right].$$

Internal gears, any pressure angle.

$$Z = S\left[\text{secant}\left(\theta - \frac{90^\circ}{N}\right)\right].$$

$$T = C + \frac{Z}{2} - \frac{S}{2}\left[\text{tangent}\left(\theta - \frac{90^\circ}{N}\right)\right].$$

222. Dimensions for checking rack teeth with a plug tangent at the pitch line when the pressure angle is 14½°.

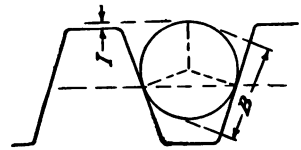


Fig. 420

$$B = \frac{1.6225}{D.P.} \text{ and } I = \frac{.0143}{D.P.}.$$

CUTTERS FOR GEAR TEETH

223. Table of cutters for gear teeth when the pressure angle is 14½°.

NUMBER OF CUTTER	NUMBER OF TEETH	NUMBER OF CUTTER	NUMBER OF TEETH
1	135 to rack	5	21 to 22
1½	80 " 134	5½	19 " 20
2	55 " 79	6	17 " 18
2½	42 " 54	6½	15 " 16
3	35 " 41	7	14
3½	30 " 34	7½	13
4	26 " 29	8	12
4½	23 " 25		

224. Checking spur gears with plugs for 14½° and 20° pressure angles. External and internal gears.—In this system the plugs are not tangent at the pitch line. It is a very simple and practical means of checking, and the system is theoretically perfect. The pitch diameter passes through the center of the plug being used.

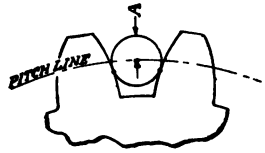


Fig. 421

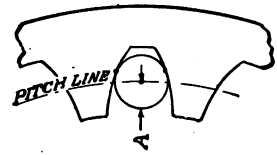


Fig. 422

$14\frac{1}{2}°$ Pressure Angle		
Diametral pitch	Diameter of plug	A
1	1.5207	.7603
2	.7603	.3802
3	.5069	.2534
4	.3802	.1901
5	.3041	.1520
6	.2534	.1267
7	.2172	.1086
8	.1901	.0950
9	.1689	.0845
10	.1521	.0760
12	.1267	.0633
14	.1086	.0543
16	.0950	.0475

20° Pressure Angle		
Diametral pitch	Diameter of plug	A
1	1.4760	.7380
2	.7380	.3690
3	.4920	.2460
4	.3690	.1845
5	.2952	.1476
6	.2460	.1230
7	.2109	.1054
8	.1845	.0923
9	.1640	.0820
10	.1476	.0738
12	.1222	.0611
14	.1054	.0527
16	.0922	.0461

Diameter of plug for a $14\frac{1}{2}°$ pressure angle $= \frac{1.5207}{\text{D.P.}}$. $A = \frac{.7603}{\text{D.P.}}$.

Diameter of plug for a 20° pressure angle $= \frac{1.4760}{\text{D.P.}}$. $A = \frac{.7380}{\text{D.P.}}$.

To check an even number of teeth:

Example: If two plugs are used in checking a 20-tooth, 10 D.P. gear, the dimension over the plugs will be the pitch diameter plus 2A. Thus, the dimension over the plugs $= 2 + .1520 = 2.1520''$ for a $14\frac{1}{2}°$ pressure angle, and $2 + .1476 = 2.1476''$ for a 20° pressure angle.

If the gear were an internal gear the dimension over the plugs would be the same as in the above example.

To check an uneven number of teeth:

EXAMPLE: If one plug is used in checking a 19-tooth, 10 D.P. gear the dimension over the plug to a point opposite on the outside diameter of the gear will be the pitch diameter plus the addendum plus A. Thus, the dimension over the plug to a point opposite on the outside diameter = 1.9 + .1 + .0760 = 2.076″ for a 14½° pressure angle.

225. Checking spur gears with a square gauge for 14½° and 20° pressure angles. External and internal gears.—In this system the dimensions are not at the pitch line. It is a very simple and practical means of checking, and the system is theoretically perfect.

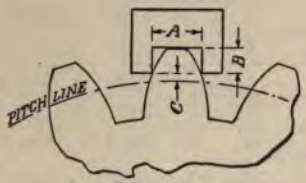


Fig. 423

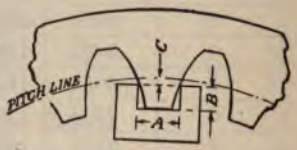


Fig. 424

14½° PRESSURE ANGLE		
Diametral pitch	A	B
1	1.4723	.8096
2	.7361	.4048
3	.4908	.2699
4	.3681	.2024
5	.2945	.1619
6	.2454	.1349
7	.2103	.1157
8	.1840	.1012
9	.1636	.0900
10	.1472	.0810
12	.1227	.0675
14	.1052	.0578
16	.0920	.0506

20° PRESSURE ANGLE		
Diametral pitch	A	C
1	1.3870	.2524
2	.6935	.1262
3	.4623	.0841
4	.3468	.0631
5	.2774	.0505
6	.2312	.0421
7	.1981	.0361
8	.1734	.0316
9	.1541	.0280
10	.1387	.0252
12	.1156	.0210
14	.0990	.0181
16	.0867	.0158

For a $14\frac{1}{2}°$ pressure angle, $A = \frac{1.4723}{D.P.}$ and $B = \frac{.8096}{D.P.}$.

For a 20° pressure angle, $A = \frac{1.3870}{D.P.}$ and $C = \frac{.2524}{D.P.}$.

When a 20° pressure angle is used the gears usually have a shorter addendum and dedendum than the $14\frac{1}{2}°$ pressure angle gears; therefore, B equals the addendum minus C.

EXERCISE 50

1. What is the outside diameter of a 22-tooth, 8-diametral pitch gear?
2. What dimension does the addendum in this gear equal?
3. What is the pitch diameter of a 20-tooth, 7-diametral pitch gear?
4. What number cutter should be used to cut this gear?
5. What is the whole depth of the tooth space in this gear?

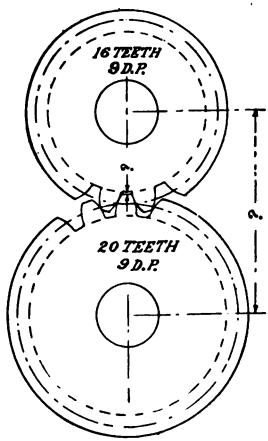


Fig. 425

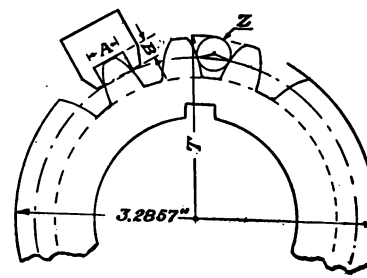


Fig. 426

Use Fig. 425:

6. Find center distance indicated by ?.
7. Find clearance indicated by ?.

In Fig. 426, where the pressure angle is $14\frac{1}{2}°$ and the number of teeth in the gear is 21:

8. Find the diametral pitch.

9. Find the chordal thickness A.

10. Find the depth to set calipers, indicated by B.

11. What will be the diameter of a plug Z that will be tangent at the pitch line?

12. What will the distance T from the outside of this plug to the center of the gear be?

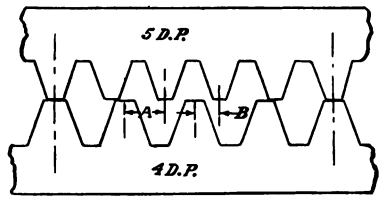


Fig. 427

13. In Fig. 427, find A.

14. In Fig. 427, find B.

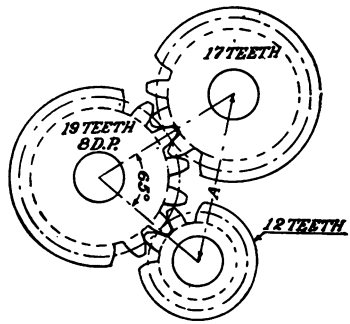


Fig. 428

15. In Fig. 428, find A.

16. How should the vernier calipers be set so that any 8-diametral pitch gear could be checked without resetting the calipers?

XXVII

BEVEL GEARS

226. Illustration of bevel gears with shafts at right angles.

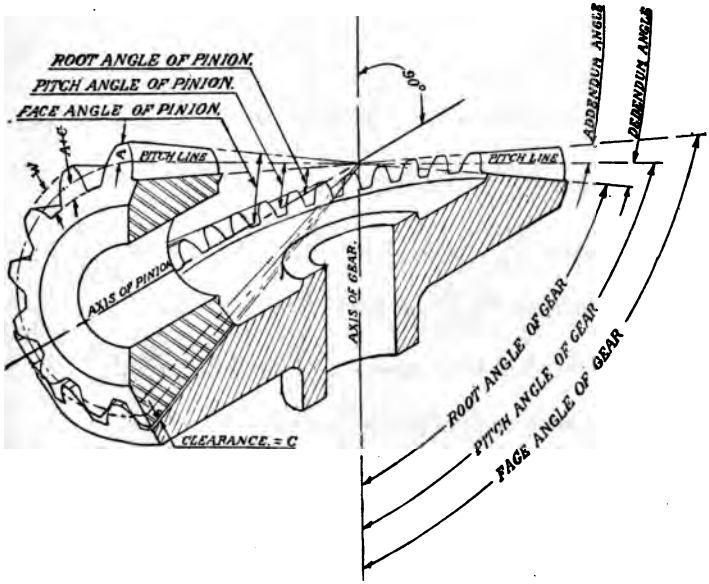


Fig. 429

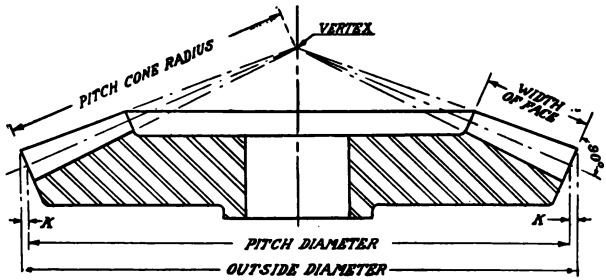


Fig. 430

D.P. = diametral pitch.

C.P. = circular pitch.

N = number of teeth in gear.

n = number of teeth in pinion.

$$\text{Pitch diameter of gear} = \frac{N}{D.P.} \text{ or } \frac{N(C.P.)}{3.1416}.$$

$$\text{Pitch diameter of pinion} = \frac{n}{D.P.} \text{ or } \frac{n(C.P.)}{3.1416}.$$

(The pitch diameters of bevel gears are the same as those of spur gears.)

$$A = \text{addendum} = \frac{1}{D.P.} \text{ or } .3183(C.P.).$$

$$C = \text{clearance} = \frac{.157}{D.P.} \text{or } .05(C.P.).$$

$$A+C = \text{dedendum} = \frac{1.157}{D.P.} \text{ or } .3683(C.P.).$$

$$W = \text{whole depth of tooth space} = \frac{2.157}{D.P.} \text{ or } .6866(C.P.).$$

$$\text{Tangent of pitch angle of gear} = \frac{N}{n}.$$

$$\text{Tangent of pitch angle of pinion} = \frac{n}{N}.$$

K = angular addendum = A(cosine of pitch angle).

Outside diameter = pitch diameter + 2K.

$$\text{Pitch cone radius} = \frac{(\text{pitch diameter})(\text{cosecant of pitch angle})}{2}.$$

$$\text{Cotangent of addendum angle} = \frac{\text{pitch cone radius}}{A}.$$

$$\text{Cotangent of dedendum angle} = \frac{\text{pitch cone radius}}{A+C}.$$

Face angle = pitch angle + addendum angle.

Root angle = pitch angle − dedendum angle.

Checking Bevel Gear Teeth with Gear Tooth Vernier Calipers

227. Dimensions for checking bevel gear teeth.—These dimensions are the same as those required for spur gears

(see Article 225). In this system the dimensions are not at the pitch line.

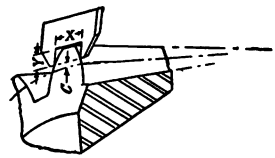


Fig. 431

For $14\frac{1}{2}°$ pressure angle, $X=\frac{1.4723}{D.P.}$ and $Y=\frac{.8096}{D.P.}$

For 20° pressure angle, $X=\frac{1.3870}{D.P.}$ and $C=\frac{.2524}{D.P.}$

When a 20° pressure angle is used, the gears usually have a shorter addendum than the $14\frac{1}{2}°$ pressure angle; therefore, Y equals the addendum minus C.

EXERCISE 51

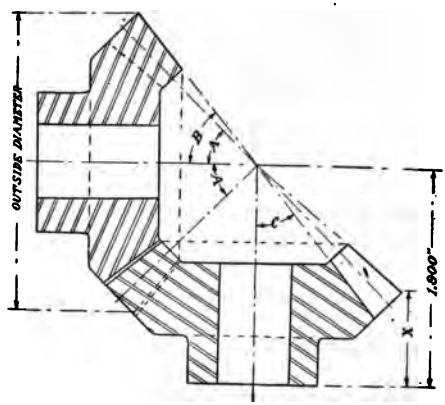


Fig. 432

In Fig. 432, where the diametral pitch is 6 and the number of teeth in the miter gears is 15:

EXERCISE 52

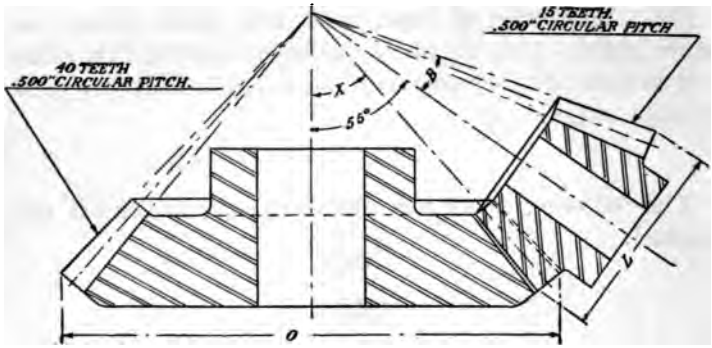


Fig. 435

Use Fig. 435:

1. Find pitch cone angle X of gear.
2. Find face angle B of pinion.
3. Find outside diameter O of gear.
4. Find outside diameter L of pinion.

XXVIII

WORM GEARING

A worm gear consists of a worm and a worm wheel.

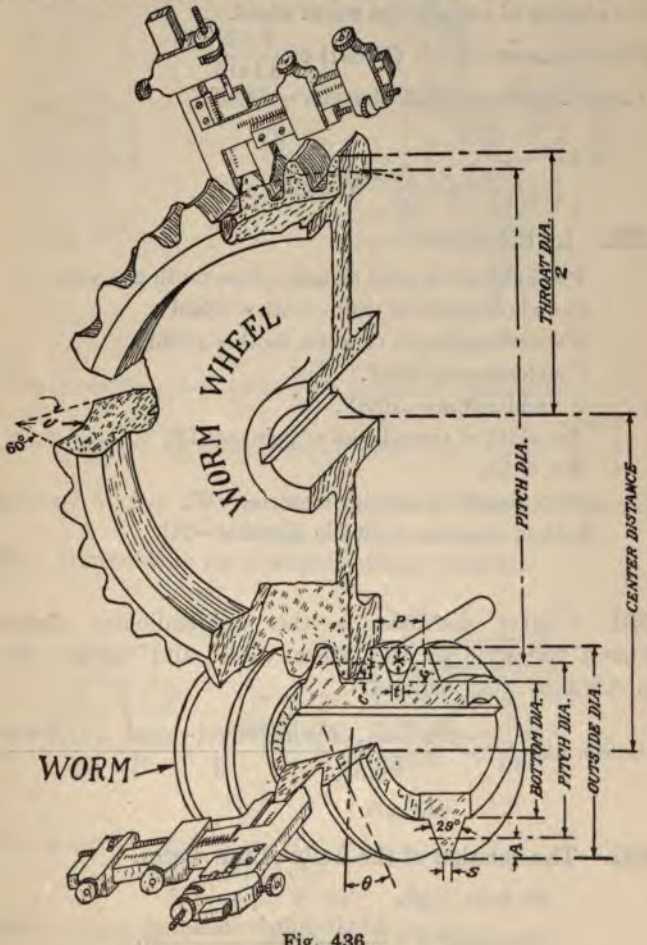


Fig. 436

Worm Wheel

229. In the worm wheel:

P = linear pitch = distance from tooth to tooth measured along the pitch line.

N = number of teeth in the worm wheel.

Pitch diameter = $P \times N$ (.31831) or $\frac{P \times N}{3.1416}$.

Throat diameter = pitch diameter − .6366P.

Worm

230. In the worm:

P = linear pitch = the distance from tooth to tooth.

G = whole depth of worm tooth = .6866P.

W = working depth of worm tooth = .6366P.

C = clearance = .050P.

A = addendum = .3183P.

t = width of thread tool at point = .31P.

S = .335P.

Pitch diameter = outside diameter − W.

Bottom diameter = outside diameter − 2G.

231. Center distance is the perpendicular distance between the axes of the worm wheel and worm. It is expressed by the formula:

$$\text{Center distance} = \frac{\text{pitch dia. of worm wheel} + \text{pitch dia. of worm}}{2}.$$

232. The relation of the helix of the worm.

θ = helix angle.

$$\text{Cotangent } \theta = \frac{3.1416 \text{ (pitch diameter)}}{\text{lead}}.$$

Checking Worm Wheel Teeth with Gear Tooth Vernier Calipers

233. Dimensions for checking worm wheel teeth.

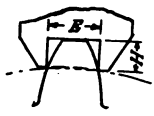


Fig. 437

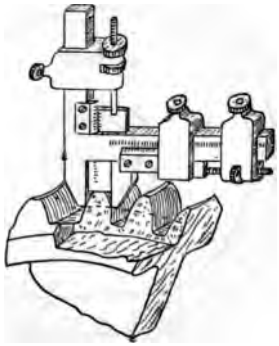


Fig. 438

$$E = (\text{pitch diameter}) \left(\text{sine}\frac{90°}{N}\right) (\text{cosine of helix angle}).$$

$$H = \tfrac{1}{2}\left[(\text{throat diameter}) - (\text{pitch diameter}) \left(\text{cosine } \frac{90°}{N}\right)\right].$$

Checking Worm Threads with Gear Tooth Vernier Calipers or Plug.

234. Dimensions for checking worm threads.

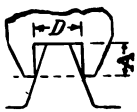


Fig. 439

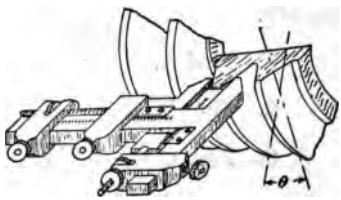


Fig. 440

When a vernier caliper is used:

$$A = .3183P \text{ and}$$
$$D = (\text{cosine } \theta)(.5P).$$

When a plug is used for checking worm threads:

$X = .5149P$ (cosine θ). (See X in Fig. 436.)

EXERCISE 53

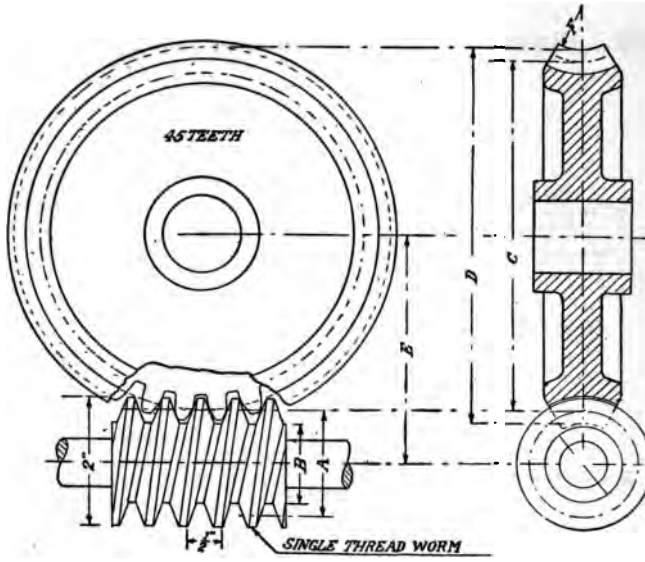


Fig. 441

Use Fig. 441:

1. Find A.
2. Find B.
3. Find C.
4. Find D.
5. Find E.
6. Find F.

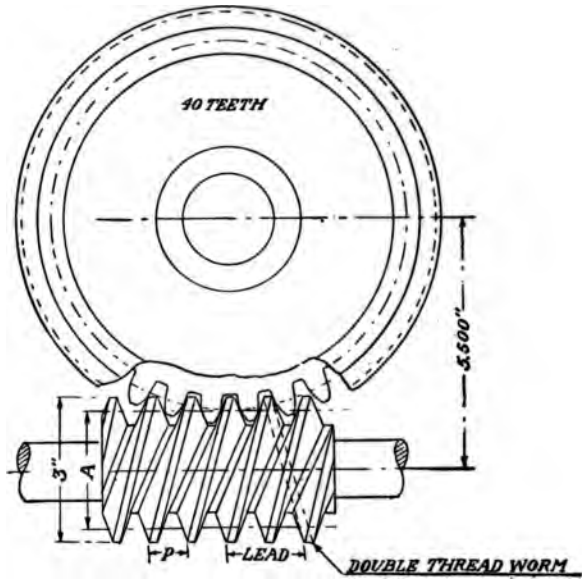


Fig. 442

Use Fig. 442:

7. Find pitch. **8.** Find lead. **9.** Find A.

XXIX

DIVIDING HEAD INDEXING

Simple Indexing

235. The **dividing head** is chiefly used on a milling machine to divide work into different parts, such as drills, reamers, gears, etc.

The principal parts of a dividing head are the **index plate, spindle, worm,** and **worm wheel.** The worm wheel generally has 40 teeth and a single thread worm is used; therefore, the ratio is 40 to 1.

1 revolution of the spindle will take place while the crank A makes 40 revolutions.

10 revolutions of the spindle will take place while the crank A makes 10×40 or 400 revolutions.

$\frac{1}{7}$ of a revolution of the spindle will take place while the crank A makes $\frac{1}{7}$ of 40 or $5\frac{5}{7}$ revolution.

$\frac{5}{6}$ of a revolution of the spindle will take place while the crank A makes $\frac{5}{6}$ of 40 or $33\frac{1}{3}$ revolutions.

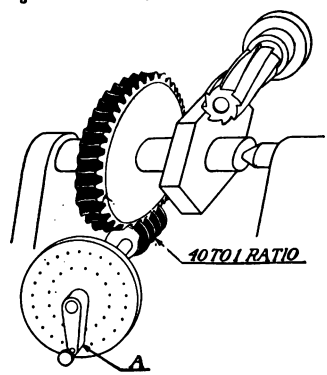


Fig. 443

Fig. 443 shows the working parts of a dividing head, while Fig. 445 shows how the dividing head actually appears.

Assume that a hexagon is to be milled on a piece of metal. Then while the work rotates from one side to the next the spindle makes $\frac{1}{6}$ of a revolution. To do this the crank A must make $\frac{1}{6}$ of 40 or

$6\frac{2}{3}$ revolutions. Index plates furnished with the dividing head take care of fractional parts of a revolution. Thus, $\frac{2}{3}$ of a revolution equals $\frac{18}{27}$. The denominator of the fraction expresses the number of holes in the circle to be used and the numerator the number of holes the crank is to be moved in that circle.

After revolving the crank 6 complete revolutions, the $\frac{2}{3}$ of a revolution is taken care of by moving the crank 18 holes in the 27 circle.

Angular Indexing

236. One revolution of the crank revolves the spindle $\frac{1}{40}$ of a revolution, or 9°; then, $\frac{1}{9}$ of a revolution will revolve the spindle 1°.

Problem: What are the requirements for indexing $17\frac{1}{2}$°?

Solution: If $\frac{1}{9}$ of a revolution of the crank revolves the spindle 1°, to revolve it $17\frac{1}{2}$° will require $17\frac{1}{2} \times \frac{1}{9}$ or $1\frac{7}{18}$ revolutions. The fractional part of a revolution is taken care of by revolving the crank 17 holes in the 18 circle.

Differential Indexing

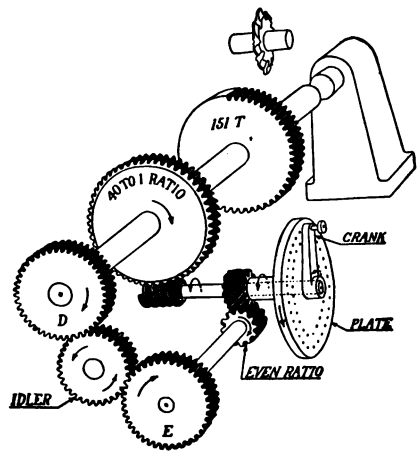


Fig. 444

Fig. 444 shows the working parts of a dividing head when used for differential indexing, while Fig. 446 shows how the dividing head actually appears.

237. With the plates that are furnished with a dividing head only a limited number of divisions can be obtained, and in many cases it is necessary to use **differential indexing.** That is, gears are used to revolve the index plate a small amount in either direction, while the crank is being revolved a number of holes in a certain circle. The amount the gears revolve the plate and the amount the crank is revolved on the plate equal the total movement of the crank.

For example, if the crank revolves 3 holes in the 15 circle, while the plate revolves $\frac{1}{50}$ of a revolution in the same direction, the total movement of the crank will be $\frac{3}{15}+\frac{1}{50}$ or $\frac{11}{50}$ of a revolution.

238. While the crank revolves $\frac{11}{50}$ of a revolution, the spindle will revolve $\frac{11}{50}\times\frac{1}{40}$ or $\frac{11}{2000}$ of a revolution. With suitable gears it is possible to revolve the plate, or gear E, $\frac{1}{50}$ of a revolution, while the spindle or gear D revolves $\frac{11}{2000}$ of a revolution. The number of teeth in the gears is obtained by inverting the numbers of revolutions of the gears. (See Article 71.)

Thus, if E makes $\frac{1}{50}$ of a revolution while D makes $\frac{11}{2000}$ of a revolution, E must have $\frac{11}{2000}$ of a tooth and D must have $\frac{1}{50}$ of a tooth. By multiplying these numbers by 2000 to clear of fractions, $E=\frac{11}{2000}\times 2000$ or 11 teeth and $D=\frac{1}{50}\times 2000$ or 40 teeth.

When these gears are used the spindle will make $\frac{11}{2000}$ of a revolution while the crank is revolved 3 holes in the 15 circle.

239. A simple method to obtain the gears D and E for differential indexing is as follows:

Assume that it is necessary to cut a gear with 151 teeth. After cutting the first tooth, the gear must be revolved $\frac{1}{151}$ part of a revolution. To do this the crank must make $\frac{1}{150}\times 40$ or $\frac{40}{151}$ of a revolution. In simple indexing, 40 holes in the 151 circle would be required; but since there is no index plate with 151 holes, it is necessary to use differential indexing.

In differential indexing the fraction $\frac{40}{151}$ is reduced to any fraction expressed in smaller numbers and having nearly the same

numerical value, as $\frac{1}{4}$, or 5 holes in the 20 circle. By revolving the crank $\frac{1}{4}$ of a revolution, the amount of variation would be the difference between $\frac{40}{151}$ and $\frac{1}{4}$ or $\frac{9}{4\times151}$.

Thus, $\frac{4\times40}{4\times151}-\frac{1\times151}{4\times151}=\frac{160}{4\times151}-\frac{151}{4\times151}=\frac{9}{4\times151}$. In other words, for the crank or worm to have a total movement of $\frac{40}{151}$ of a revolution, the plate must revolve $\frac{9}{4\times151}$ of a revolution while the crank moves an additional $\frac{1}{4}$ of a revolution on the same plate, because $\frac{1}{4}+\frac{9}{4\times151}=\frac{40}{151}$, which is the part of a revolution the crank must make to move the spindle $\frac{1}{151}$ of a revolution.

With suitable gears it is possible to make the plate, or gear E, revolve $\frac{9}{4\times151}$ of a revolution, while the spindle makes $\frac{1}{151}$ of a revolution. The number of teeth in the gears is obtained by inverting the number of the revolutions of the gears (see Article 71).

Thus, the gear on spindle D will require $\frac{9}{4\times151}$ of a tooth, and gear E will require $\frac{1}{151}$ of a tooth. By multiplying these by 4×151 to clear of fractions, the gear on the spindle will require 9 teeth and gear E will require 4 teeth.

Thus, $\frac{9}{4\times151}\times(4\times151)=9$ and $\frac{1}{151}\times(4\times151)=4$.

By multiplying these last numbers of teeth by any small number as 6, the gear on the spindle will have 9×6 or 54 teeth and gear E will have 4×6 or 24 teeth.

This arrangement will cause the spindle to make $\frac{1}{151}$ part of a revolution while the crank is revolved 5 holes in the 20 circle.

The same results can be obtained by applying the formula,

$$\frac{D}{E}=40-(X)\ (N) \text{ or } \frac{D}{E}=(X)\ (N)-40.$$

D = the number of teeth in the gear on the spindle.
E = the number of teeth in gear E.
N = the number of divisions.
X = the approximate value of $\frac{40}{N}$.

The formula, $\frac{D}{E}=40-(X)\ (N)$, requires one idler gear.

The formula, $\frac{D}{E}=(X)\ (N)-40$, requires two idler gears.

The solution of the above example is as follows:

$$\frac{D}{E}=40-\frac{1}{4}\times 151=40-\frac{151}{4}=\frac{160}{4}-\frac{151}{4}=\frac{9}{4}=\frac{54}{24}.$$

EXERCISE 54

The plates furnished with the dividing head have the following series of holes, 15, 16, 17, 18, 19, 20, 21, 23, 27, 29, 31, 33, 37, 39, 41, 43, and 47.

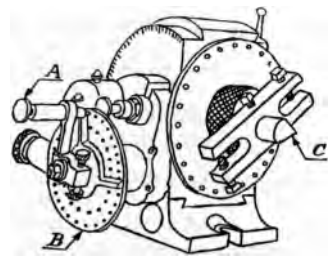


Fig. 445

Use Fig. 445:

1. How many revolutions must A make to cause C to make one complete revolution?

2. How many revolutions must A make to cause C to make $\frac{8}{7}$ of a revolution?

3. How many revolutions must A make to cause C to make $\frac{1}{9}$ of a revolution?

4. How many revolutions must A make to cause C to make $\frac{1}{17}$ of a revolution?

5. How many holes are required in the index plate used in indexing 13 divisions?

6. How many holes would the crank be moved in the above circle to take care of the fractional part of a revolution that remains, when indexing 13 divisions?

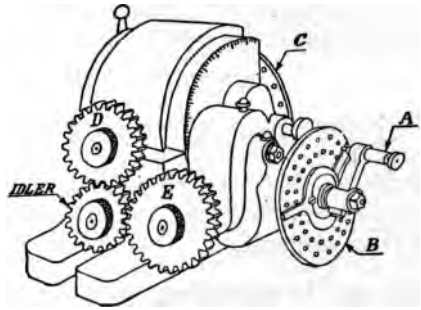


Fig. 446

Use Fig. 446, in which B = the number of holes in the circle:

7. How many revolutions must A make to cause C to make $\frac{1}{206}$ of a revolution?

8. How many revolutions must A make to cause C to move 22 degrees?

9. How many revolutions must A make to cause C to make $2\frac{1}{16}$ revolutions?

10. How many revolutions must A make to cause C to move 21° 27′ 21″?

In the following problems keep the number of teeth in the gears between the limits of 24 and 100.

	Number of Divisions	Index Circle B	Number of Holes to Move A	Gear on Spindle D	Gear E
11.	107	?	?	?	?
12.	123	?	?	?	?
13.	506	?	?	?	?
14.	1151	?	?	?	?

XXX

CONTINUED FRACTIONS

240. The principle of **continued fractions** is used to reduce a proper or improper fraction to a series of approximate fractions, called **convergents.**

This process is used in calculating change gears for milling machines, lathes, hobbing machines, etc.

241. The method of procedure used can be explained best by an example.

EXAMPLE: What are the convergents of $\frac{5263}{8647}$ or $\frac{8647}{5263}$?

SOLUTION: 1. Place the numerator and denominator on either side of the line C, as shown. (See Article 22.)

2. Divide the larger number by the smaller and place the quotient in column X. Thus, 8647 contains 5263 once with a remainder 3384.

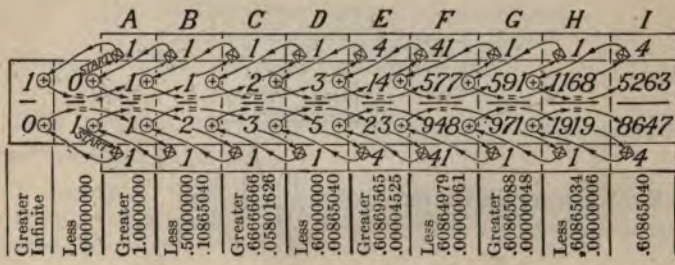
3. Divide the last divisor by this remainder and place the quotient in column X. Thus, 5263 contains the remainder 3384 once with a remainder 1879.

The process of dividing the last divisor by the last remainder is continued until the final remainder becomes 0.

	C		X
	8647	5263	1
	5263	3384	1
	3384	1879	1
	1879	1505	1
	1505	374	4
	1496	369	41
	9	5	1
	5	4	1
	4	1	4
	4		
	0		

The next step is to construct a block arrangement, as shown below and place $\frac{1}{0}$ and $\frac{0}{1}$ in the first two sets of blocks of the middle rows. The quotients obtained in column X are then placed in the top and bottom rows as shown.

Then, at the point marked *start*, proceed as shown by the arrows. The sign ⊠ indicates multiplication and the sign ⊕ indicates addition.



The following will help to simplify the above process:

A	B	C
$0\times1+1=1$	$1\times1+0=1$	$1\times1+1=2$
$1\times1+0=\overline{1}$	$1\times1+1=\overline{2}$	$2\times1+1=\overline{3}$
D	**E**	**F**
$2\times1+1=3$	$3\times4+2=14$	$14\times41+3=577$
$3\times1+2=\overline{5}$	$5\times4+3=\overline{23}$	$23\times41+5=\overline{948}$
G	**H**	**I**
$577\times1+14=591$	$591\times1+577=1168$	$1168\times4+591=5263$
$948\times1+23=\overline{971}$	$971\times1+948=\overline{1919}$	$1919\times4+971=\overline{8647}$

The convergents of $\frac{5263}{8647}$ are $\frac{1}{0}$, $\frac{0}{1}$, $\frac{1}{1}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{14}{23}$, $\frac{577}{948}$, etc.

The convergents of $\frac{8647}{5263}$ are $\frac{0}{1}$, $\frac{1}{0}$, $\frac{1}{1}$, $\frac{2}{1}$, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{23}{14}$, $\frac{948}{577}$, etc.

242. The values of the convergents obtained by the principle of continued fractions are alternately greater and less than the exact value of the fraction from which the convergents were obtained. The convergents approach the true value of the fraction as they advance to the right. The values of the convergents and the variations are shown at the bottom of the block arrangement.

EXERCISE 55

1. What convergent does the block marked X equal?

		3	5	1	12	1
$\frac{1}{0}$	$\frac{0}{1}$				X	
		3	5	1	12	1

2. What are the convergents of $\frac{46}{471}$?

Continued Fractions for Simple Gear Ratios

243. The number of teeth in a gear is usually between the limits of 20 and 100; therefore, if either gear in a simple ratio has more than 100 teeth, the ratio should be reduced by continued fractions to a smaller approximate form.

To find the fraction that is a close approximation of another common fraction:

Example 1: What fraction is a very close approximation of $\frac{526}{864}\frac{3}{7}$ if neither the numerator nor the denominator may exceed 100?

		1	1	1	1	4	~~41~~ 4
$\frac{1}{0}$	$\frac{0}{1}$	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{14}{23}$	$\frac{59}{97}$
		1	1	1	1	4	~~41~~ 4

Solution: From Article 241, the quotients are 1, 1, 1, 1, 4, 41, etc. The work is carried to a point where the next step would make the convergent greater than 100. That is, it can be seen that if 41 is used the denominator of the convergent will be $23\times41+5$, which is greater than 100. The largest number that could be used instead of 41 is 4. By using 4 the fraction $\frac{59}{97}$ is formed.

$\frac{1}{0}$, $\frac{0}{1}$, $\frac{1}{1}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{14}{23}$ are the convergents, and $\frac{59}{97}$ is an approximate fraction. In many cases this approximate fraction is closer than the convergent.

Thus, $\frac{52}{86}\frac{63}{47}=.608650$.
$\frac{14}{23}=.608695$. The variation equals .000045.
$\frac{59}{97}=.608247$. The variation equals .000403.
Then, $\frac{14}{23}$ is the closest approximation of $\frac{52}{86}\frac{63}{47}$.

244. To find the common fraction that is a close approximation of a decimal:

EXAMPLE 2: What fraction is a close approximation of .6689 if neither the numerator nor the denominator of the fraction may exceed 100.

		1	2	~~49~~ 33
$\frac{1}{0}$	$\frac{0}{1}$	$\frac{1}{1}$	$\frac{2}{3}$	$\frac{67}{100}$
		1	2	~~49~~ 33

SOLUTION: $.6689=\frac{6689}{10000}$. By using the rules given under Article 241 the quotients 1, 2, 49, 2, etc., and the convergents $\frac{1}{0}$, $\frac{0}{1}$, $\frac{1}{1}$, $\frac{2}{3}$, are obtained.

If 49 were to be used, the denominator of the convergent would be $3\times49+1$, which is greater than 100; therefore, the largest number that could be used instead of 49 is 33.

By using 33 the fraction $\frac{67}{100}$ is formed.
$\frac{6689}{10000}=.66890$.
$\frac{2}{3}=.66666$. The variation equals .00224.
$\frac{67}{100}=.67000$. The variation equals .00110.
Then, $\frac{67}{100}$ is the closest approximation of .6689.

EXERCISE 56

1. What improper fraction is a very close approximation of $\sqrt{2}$ if neither the numerator nor the denominator may exceed 100?

2. What improper fraction is a very close approximation of $\frac{9063}{7420}$ if neither the numerator nor the denominator may exceed 100?

3. What common fraction is a very close approximation of .8143 if neither the numerator nor the denominator may exceed 100?

Uniting Fractions

245. If the sum of the numerators of two fractions is used to form the numerator of another fraction, and the sum of the denominators is used to form the denominator, the value of the fraction formed will be between the values of the given fractions. This process is called **uniting fractions.**

Example 1: By adding the numerators and denominators of the fractions $\frac{2}{5}$ and $\frac{9}{22}$ the fraction $\frac{11}{27}$ is formed, and its value will be between the values of $\frac{2}{5}$ and $\frac{9}{22}$.

Solution: $\frac{2+9=11}{5+22=27}$ The given fraction $\frac{2}{5}=.40000$.
The given fraction $\frac{9}{22}=.40909$.
The formed fraction $\frac{11}{27}=.40740$.

The value of the formed fraction, .40740, is between the values .40000 and .40909.

By the use of this method other fractions between certain limits can be ascertained.

Example 2: What fractions whose numerators and denominators do not exceed 20 are between the values $\frac{1}{4}$ and $\frac{1}{5}$?

$$\frac{1}{4}\ \frac{4}{17}\ \frac{3}{13}\ \frac{2}{9}\ \frac{3}{14}\ \frac{4}{19}\ \frac{1}{5}$$

The fractions $\frac{4}{17}$, $\frac{3}{13}$, $\frac{2}{9}$, $\frac{3}{14}$, and $\frac{4}{19}$ are between the values of $\frac{1}{4}$ and $\frac{1}{5}$.

Example 3: Assume that the following convergents were obtained from a certain gear ratio.

		8	2	1	1	1	6
$\frac{1}{0}$	$\frac{0}{1}$	$\frac{1}{8}$	$\frac{2}{17}$	$\frac{3}{25}$	$\frac{5}{42}$	$\frac{8}{67}$	$\frac{53}{444}$
		8	2	1	1	1	6

If the convergents $\frac{8}{67}$ and $\frac{53}{444}$ are close enough for the gear ratio, other fractions between these values could be formed as shown,

$$\frac{8}{67}\ \frac{69}{578}\ \frac{61}{511}\ \frac{114}{955}\ \frac{53}{444}$$

The fractions $\frac{69}{578}$, $\frac{61}{511}$, and $\frac{114}{955}$ are between the values of the convergents $\frac{8}{67}$ and $\frac{53}{444}$.

Continued Fractions for Compound Gearing

246. Very often in gear ratios for compound gearing, all the factors obtained from the simple ratios will not be less than 100, which should be the case because the number of teeth in a gear is usually less than 100.

Example 1: If $\frac{173374}{100000}$ is the simple ratio, the factors would be $\frac{2\times23\times3769}{2\times250\times200}$.

Example 2: If $\frac{47}{149}$ is the simple ratio, the factors would be $\frac{47\times1}{149\times1}$.

Example 3: If $\frac{400750026}{1000000000}$ is the simple ratio, the factors would be $\frac{2\times3\times6679171}{2\times5000\times10000}$.

By the principle of continued fractions it is possible to obtain the closest set of factors so that the product of two of them divided by the product of the other two will equal approximately any given ratio and have no factor greater than 100.

The method used in obtaining the closest set of factors entails a rather lengthy and strenuous amount of calculating and should only be used where very accurate work is required, or where a machine is to be set for a large number of the same pieces.

247. The solution of Example 1 is as follows:

Solution 1: What product of two factors divided by the product of two other factors is the closest approximation of 1.73374, if the factors are not to be greater than 100?

$$1.73374 = \frac{173374}{100000}$$

100000	173374	1
73374	100000	1
26626	73374	2
20122	53252	1
6504	20122	3
6100	19512	10
404	610	1
206	404	1
198	206	1
192	198	24
	8	

											24 11	
		1	1	2	1	3	10	1	1	1	5735	Upper end fraction
1	0	1	1	3	4	15	154	169	323	492	9943	
0	1	1	2	5	7	26	267	293	560	853	5581	Lower end fraction
		1	1	2	1	3	10	1	1	↑1	9676	
										A.	24 12	

Neither the numerator nor the denominator of the fraction to be factored must exceed 10000, because 100×100 equals 10000.

This work is carried to a point where the next step would make the convergent greater than 10000. That is, it can be seen that if 24 is used the denominator will be $24 \times 492 + 323$, which is greater than 10000. The largest number that could be used instead of 24 is 11. By using 11 the fraction $\frac{5735}{9943}$ is obtained.

Thus, $11 \times 492 + 323 = 5735$, and $11 \times 853 + 560 = 9943$.

This fraction $\frac{5735}{9943}$ is called the **upper end fraction** and is not a convergent.

To obtain the denominator of the **lower end fraction** it is necessary, in the last step, to subtract 560 instead of adding. Thus, if 24 were used, the denominator of the fraction would be greater than 10000. Therefore, 12 is the largest number that could be used instead of 24. By using 12 the denominator 9676 is obtained. Thus, $12 \times 853 - 560 = 9676$. In this case 560 is subtracted instead of being added.

To obtain the numerator of the **lower end fraction** it is necessary, in the last step, to subtract 323 instead of adding. Thus, $12\times492-323=5581$. In this case, 323 is subtracted instead of being added.

This fraction $\frac{5581}{7676}$ is called the **lower end fraction** and is not a convergent.

The closest results are obtainable from the largest convergent and the two end fractions; therefore, try to factor these and if none of them can be factored, place the end fractions on each side of the lines C and D, as shown below.

Then continuously subtract the numerator and denominator of the largest convergent that can possibly be subtracted from the numerators and denominators of the end fractions, until some fraction whose numerator and denominator can be factored is obtained.

If two of the fractions can be united and still be less than 10000 unite them by adding the numerators for a new numerator and the denominators for a new denominator for the formed fraction. This is shown at *below.

	C			D		
1.73373947–>	9676	5581		9943	5735	<–1.7337401
	853	492	NOTE: Do not	853	492	
1.73373943–>	8823	5089	unite the num-	9090	5243	<–1.7337402
	853	492	bers that are	853	492	
1.73373939–>	7970	4597	being subtracted.	8237	4751	<–1.7337402
	853	492		853	492	
1.73373934–>	7117	4105		7384	4259	<–1.7337403
	853	492		853	492	
1.7337392–>	6264	3613		6531	3767	<–1.7337403
	853	492		853	492	
1.7337391–>	5411	3121		5678	3275	<–1.7337404
1.7337390–>	9969	853	492	*5750	853	492
1.7337390–>	4558	2629		4825	2783	<–1.7337405
1.7337389–>	8263	853	492	*4766 8797*	853	492	5074<–1.7337406
1.7337388–>	3705	2137		3972	2291	<–1.7337407
1.7337387–>	6557	853	492	*3782 7091*	853	492	4090<–1.7337408
1.7337386–>	2852	1645		3119	1799	<–1.7337409

NOTE: Numbers decrease in value.

NOTE: Numbers increase in value.

$\frac{2852}{1645}=\frac{46\times62}{35\times47}=1.7337386$. The variation equals .0000014. This is the first set that can be factored, therefore it is the closest set.

248. The solution of Example 2 is as follows:

SOLUTION: What product of two factors divided by the product of two other factors is the closest approximation of $\frac{47}{149}$, if the factors may not be greater than 100?

In this example the work is performed as in Example 1, except that the last possible division, $1 \div 0$, which equals infinity (∞), is involved.

If 1 is divided by a very small quantity a large quotient is obtained, and as the quantity being divided into 1 approaches 0, it can be reasoned that the quotient will approach infinity (∞), or a quantity greater than any assignable value.

Thus, $\frac{1}{.0000001} = 10000000.$

$\frac{1}{.00000000000000001} = 100000000000000000.$

$\frac{1}{0} = \infty$, or a quantity greater than any assignable value.

Any number divided by 0 equals ∞.

$$\frac{47}{149} = .315436$$

47	149	3
40	141	5
7	8	1
7	7	7
0	1	∞

					$\not\infty$ 66
	3	5	1	7	3108
1 0	1	5	6	47	9853
0 1	3	16	19	149	3143
	3	5	1	7	9964
					$\not\infty$ 67

For the upper end fraction, the largest number that can be used instead of ∞ is 66. By using 66 the upper end fraction $\frac{3}{9}\frac{1}{8}\frac{0}{5}\frac{8}{3}$ is obtained. Thus, $66 \times 47 + 6 = 3108$,

and $66 \times 149 + 19 = 9853$.

For the lower end fraction, the largest number that can be used instead of ∞ is 67. By using 67 the lower end fraction $\frac{3}{9}\frac{1}{9}\frac{4}{6}\frac{3}{4}$ is obtained. Thus, $67 \times 47 - 6 = 3143$,

and $67 \times 149 - 19 = 9964$.

In this case 6 and 19 were subtracted in forming the lower end fraction.

Try to factor the last or largest convergent and the two end fractions. If none of them can be factored place the end fractions on each side of the lines C and D, as shown. Then continuously subtract the numerator and denominator of the largest convergent that can possibly be subtracted from the numerators and denominators of the end fractions, until some fraction whose numerator and denominator can be factored is obtained.

C		D	
3108	9853	3143	9964
47	149	47	149
3061	9704	3096	9815
47	149	47	149
3014	9555	3049	9666
47	149	47	149
2967	9406	3002	9517
47	149	47	149
2920	9257	2955	9368
47	149	47	149
2873	9108	2908	9219
47	149	47	149
2826	8959	2861	9070
47	149	47	149
2779	8810	2814	8921
47	149	47	149
2732	8661	2767	8772
47	149	47	149
2685	8512	2720	8623
47	149	47	149
2638	8363	2673	8474
47	149	47	149
2591	8214	2626	8325
47	149	47	149
2544	8065	2579	8176
47	149	47	149
2497	7916	2532	8027
47	149	47	149
2450	7767	2485	7878
47	149	47	149
2403	7618	2438	7729
47	149	47	149
2356	7469	2391	7580

$\frac{2356}{7469}=\frac{38\times62}{77\times97}=.315437$. The variation equals .000001. This is the first set that can be factored; therefore, it is the closest set of factors.

CHECK: While A makes 1 revolution, D will make $1 \times \frac{4}{5}\frac{6}{5} \times \frac{6}{4}\frac{2}{7}$ or 1.733738 revolutions, which varies only .000002 from the true value.

EXERCISE 57

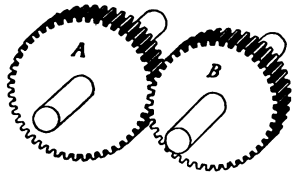


Fig. 448

1. In Fig. 448, while A makes 7.45 revolutions, how many teeth must A and B have so that B will make nearly 7.99 revolutions? (Gears to be between the limits of 24 and 100, and the variation on each revolution of B not to exceed .0001.)

2. Under the same conditions, while A makes 9 revolutions, how many teeth must A and B have so that B will make nearly 10.675 revolutions?

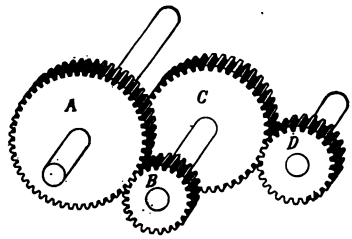


Fig. 449

3. In Fig. 449, while A makes 75 revolutions, how many teeth must A, B, C, and D have so that D will make 312.653 revolutions? (Gears to be between the limits of 20 and 100, and variation on each revolution of D not to exceed .00001.)

XXXI

GEARING OF THE LATHE

Simple Gearing of the Lathe

251. By using suitable change gears on a lathe, it is possible to cut almost any lead regardless of the number of threads per inch on the lead screw.

To cut a given lead, the carriage must move the distance of the lead to be obtained, while the spindle makes 1 revolution; therefore, it is necessary to determine the number of revolutions the lead screw must make to cause the carriage to move the distance of the lead to be cut.

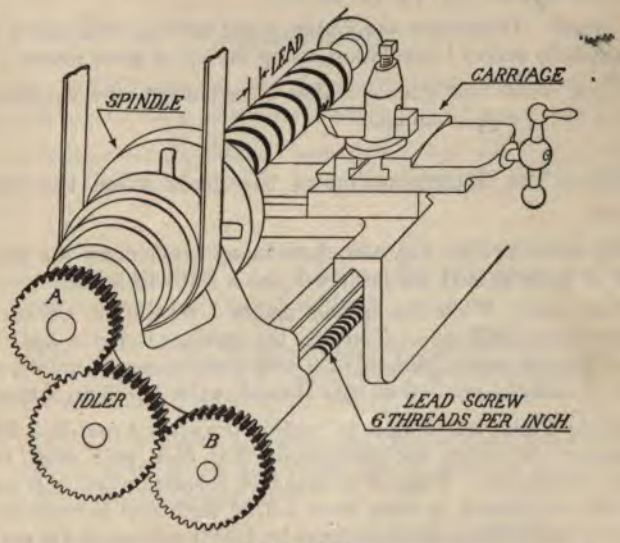


Fig. 450

252. The determination of the gears when threads per inch are given:

EXAMPLE 1: In Fig. 450, how many teeth must the gears A and B have so that the lathe will cut 11 threads per inch?

SOLUTION: While the spindle makes 1 revolution, the carriage must move $\frac{1}{11}$ of an inch. To cause the carriage to move $\frac{1}{11}$ of an inch, gear B must make $\frac{1}{11} \div \frac{1}{6}$ or $\frac{6}{11}$ of a revolution; therefore, while gear A makes 1 revolution, gear B must make $\frac{6}{11}$ of a revolution.

To determine the number of teeth in the gears A and B, it is only necessary to invert the number of revolutions that each gear must make. (See Article 71.) Thus, if A makes 1 revolution while B makes $\frac{6}{11}$ of a revolution, A must have $\frac{6}{11}$ of a tooth and B must have 1 tooth. Multiplying both of these numbers by 11 will cause the gears to have a whole number of teeth. Thus, $A = \frac{6}{11} \times 11$ or 6 teeth and $B = 1 \times 11$ or 11 teeth.

The ratio of these gears is equivalent to the ratio of any of following sets of gears, $\frac{12}{22}$, $\frac{18}{33}$, $\frac{24}{44}$, $\frac{30}{55}$, $\frac{36}{66}$.

CHECK: Determine the distance the carriage will move while the spindle makes 1 revolution, using the set of gears found.

Thus, while the spindle makes 1 revolution, the carriage will move $\frac{36}{66} \times \frac{1}{6} = \frac{1}{11}$ of an inch.

253. The determination of the gears when the lead is given:

EXAMPLE 2: In Fig. 450, how many teeth must the gears A and B have so that the lathe will cut a .3461-inch lead?

SOLUTION: While the spindle makes 1 revolution, the carriage must move .3461 inch. To cause the carriage to move .3461 inch, gear B must make $.3461 \div \frac{1}{6}$ or 2.0766 revolutions; therefore, while gear A makes 1 revolution, gear B must make 2.0766 revolutions.

To determine the number of teeth in the gears A and B, it is only necessary to invert the revolutions that each gear must make. (See Article 71.) Thus, if A makes 1 revolution while B makes 2.0766 revolutions, A must have 2.0766 teeth and B must have 1 tooth. Multiplying both of these by 10000 will cause the gears to have a whole number of teeth; thus, $A = 2.0766 \times 10000$ or 20766 teeth, and $B = 1 \times 10000$ or 10000 teeth.

Ordinarily it is not practicable to have a gear with more than 100 teeth, so the fraction $\frac{20766}{10000}$ is reduced by the principle of continued fractions to the approximate fraction $\frac{27}{13}$.

Thus,

20766	10000	2			2	13	18
20000	9958	13	1	0	1	13	235
766	42	18	0	1	2	27	488
756	40	8			2	13	18

$$\frac{27\times 2}{13\times 2}=\frac{54}{26}.$$

Then, A must have 54 teeth, and B must have 26 teeth.

CHECK: While the spindle makes one revolution, the carriage will move $\frac{54}{26}\times\frac{1}{6}$ or .34615 of an inch. The variation equals .00005 of an inch.

EXERCISE 58

1. If a .1763″ lead is required, how far must the tool move while the spindle of the lathe makes 1 revolution?

2. If there are 5 threads per inch on the lead screw of a lathe, how many revolutions must it make to cause the tool to move .4311″?

3. While the spindle of a lathe makes 1 revolution, how many revolutions must the lead screw make having 10 threads per inch, if the lathe is geared to cut a .1027″ lead?

4. What gears would be placed on the spindle and the lead-screw of a lathe, so that while the spindle makes 1 revolution the lead screw will make .5431 revolution?

5. What lead would be cut when the gear on the spindle has 45 teeth and the gear on the lead screw has 32 teeth, if there are 5 threads per inch on the lead screw?

6. What gears would be used on the spindle and the lead screw to cut a .3153″ lead, if there are 6 threads per inch on the lead screw?

$\frac{1}{1}\frac{3}{0}\frac{0}{0}\frac{9}{0}\frac{8}{0}$ = approximately $\frac{3}{2}\frac{3}{5}\frac{9}{9}\frac{5}{2}$. Then, D′ = 3395 teeth and G′ = 2592 teeth. After factoring (3395 = 35×97; 2592 = 81×32) and placing in proper compound order, D = 97 teeth, E = 81 teeth, F = 35 teeth, and G = 32 teeth. (See Article 73.)

CHECK: While the spindle makes 1 revolution, the carriage will move $\frac{9}{8}\frac{7}{1}\times\frac{3}{3}\frac{5}{2}\times\frac{1}{6}$ or .2182999″, which varies .0000001 from the true value.

EXERCISE 59

1. In a lathe (Fig. 451), when D = 50, E = 70, F = 37, and G = 90, what lead will the lathe cut, if there are 6 threads per inch on the lead screw?

2. What number of teeth must the gears D, E, F, and G have so that, while the spindle makes 1 revolution, the lead screw will make 4.011 revolutions? (Fig. 451.)

3. What number of teeth must the gears D, E, F, and G have so that the lathe will cut a 3.9248″ lead, if there are 4 threads per inch on the lead screw? (Fig. 451.)

4. If F = 96, and G = 24, what number of teeth must the gears D and E have, so that the lathe will cut a 3.1429″ lead, if there are 4 threads per inch on the lead screw? (Fig. 451.)

GEAR BOX OF A LATHE

255. The **gear box** is used on most of the modern lathes, and by placing the handles in different positions all the standard threads can be cut. The **standard threads** have leads of 1″, $\frac{1}{2}$″, $\frac{1}{3}$″, $\frac{1}{4}$″, $\frac{1}{5}$″, $\frac{1}{6}$″, $\frac{1}{7}$″, $\frac{1}{8}$″, etc.

In practical work many jobs require a special or **odd lead;** for instance, .1176″ or 2.6478″ lead.

To cut special or odd leads it is necessary to change the gears which connect the spindle with the gear box to other gears.

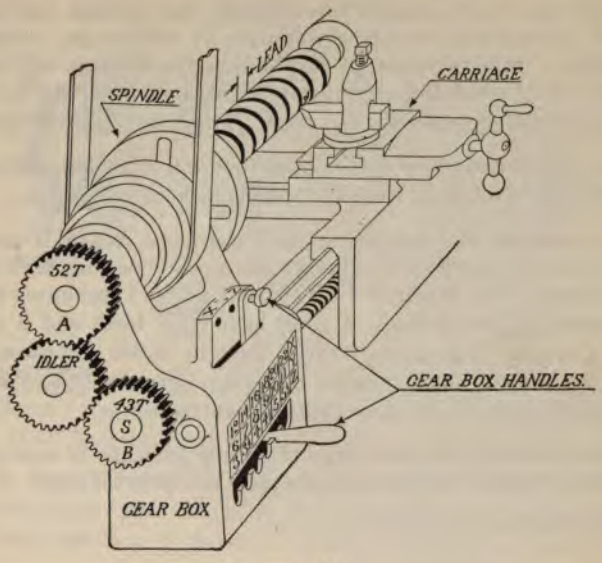


Fig. 452

The lathe shown in Fig. 452 cuts 5 threads per inch, when A = 52, B = 43, and the handles of the gear box are in the positions shown. Other standard leads could be cut by placing the handles of the gear box in different positions, but it would not be necessary to change the gears A and B.

256. For special or odd leads, A and B must be changed; consider the following example:

PROBLEM: In Fig. 452, what would A and B be changed to, if a .2633″ lead were required?

SOLUTION: By placing the handles in the position for 5 threads per inch a lead of .2000″ would be cut. There is no special reason for placing the handles for 5 threads per inch except that it is somewhat near the lead .2633″.

While the spindle makes 1 revolution, the carriage will move .2000″, and the shaft marked S will make $\frac{52}{43}$ revolutions; or stated otherwise: while S makes $\frac{52}{43}$ revolutions, the carriage will move .2000″. If $\frac{52}{43}$ revolutions of S move the carriage .2000″, to move it .2633″ the shaft S would have to make $\frac{52}{43}\times\frac{.2633}{.2000}$ or $\frac{13.6916}{8.6000}$ revolutions.

It is now necessary to obtain gears that will cause the shaft S to make $\frac{13.6916}{8.6000}$ revolutions while the spindle makes 1 revolution.

To determine the number of teeth the gears A and B should have, it is only necessary to invert the revolutions that each gear must make. (See Article 71.) Thus, if A makes 1 revolution while B makes $\frac{13.6916}{8.6000}$ revolutions, A must have $\frac{13.6916}{8.6000}$ teeth and B must have 1 tooth. Multiplying both of these by 86000 will cause the gears to have a whole number of teeth; thus, $A=\frac{13.6916}{8.6000}\times 86000$ or 136916 teeth, and $B=1\times 86000$ or 86000 teeth.

These numbers are then reduced by the principle of continued fractions to smaller numbers whose ratio is nearly the same.

This process is as follows:

136916	86000	1
86000	50916	1
50916	35084	1
35084	31664	2
15832	3420	4
13680	2152	1
2152	1268	1
1268		

		1	1	1	2	4	1	1
1	0	1	1	2	5	22	27	49
0	1	1	2	3	8	35	43	78
		1	1	1	2	4	1	1

$\frac{86000}{136916}$ = approximately $\frac{49}{78}$.

Then, A = 78 teeth, and B = 49 teeth. *Ans.*

CHECK: It is known from the preceding explanation that while the shaft S makes $\frac{52}{43}$ revolutions, the carriage will move .2000″. By using the set of gears just determined, while the spindle makes 1 revolution, the shaft S will make $\frac{78}{49}$ revolutions. If $\frac{52}{43}$ revolutions of S move the slide .2000″, $\frac{78}{49}$ revolutions will move it $.2000\times\frac{\frac{78}{49}}{\frac{52}{43}}$ or .26327″; therefore, the lead that these last gears will cut is .26327″. The variation equals .00003″.

EXERCISE 60

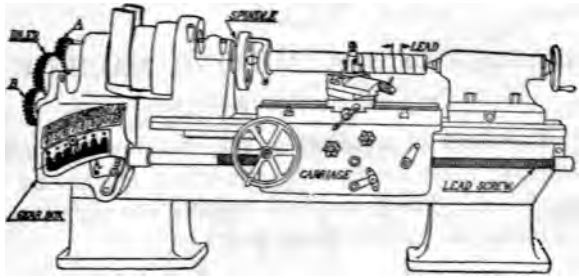


Fig. 453

1. When A=48 teeth and B=32 teeth, the lathe shown in Fig. 453 will cut 7 threads per inch. What would A and B be changed to if a .1537″ lead were required, the handle remaining as shown?

2. In Fig. 453, when A=48 teeth and B=32 teeth, the lathe will cut 7 threads per inch. What lead would be cut if A were changed to 47 teeth and B to 33 teeth, the handle remaining as shown?

XXXII

GEARING OF THE MILLING MACHINE

The Lead of the Spiral

257. The **lead of a spiral** is the distance the milling-machine table must move while the spiral makes one complete revolution. The gears that connect the dividing head and feed screw are called **change gears.**

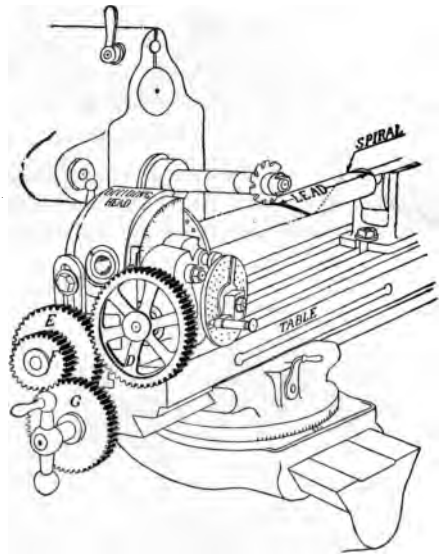


Fig. 454

The illustration in Fig. 454 shows how the milling machine is geared for cutting spirals. The illustration in Fig. 455 shows the working parts.

Process of determining the lead of the spiral when the change gears are given:

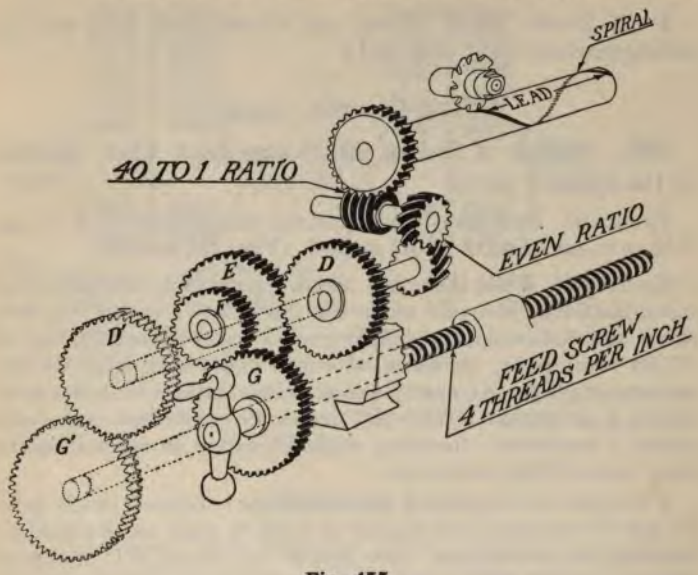


Fig. 455

PROBLEM: In Fig. 454, what lead would be cut, if D = 64 teeth, E = 70 teeth, F = 20 teeth, and G = 60 teeth?

SOLUTION: While the spiral makes 1 revolution, D will make 40 revolutions. While D makes 40 revolutions, G will make $40 \times \frac{64}{70} \times \frac{20}{60}$ or $\frac{256}{21}$ revolutions; therefore, while the spiral makes 1 revolution, G will make $\frac{256}{21}$ revolutions. If 1 revolution of the feed screw, or G, moves the table $\frac{1}{4}$ of an inch, $\frac{256}{21}$ revolutions will move it $\frac{256}{21} \times \frac{1}{4}$ or 3.047619 inches; therefore, while the spiral makes 1 revolution, the table will move 3.047619 inches, which is the lead the milling machine will cut when the above change gears are used.

EXERCISE 61

1. In Fig. 454, if D = 42, E = 28, F = 40, and G = 50, how many revolutions will G make while the spiral makes 1 revolution?

2. How far will the table move while the feed screw, or G, makes 11.05 revolutions? (Fig. 454.)

3. If D=64, E=57, F=62, and G=36, what lead will the milling machine cut? (Fig. 454.)

The Change Gears

258. Process of finding the change gears when the lead of the spiral is given:

Problem: How many teeth must the change gears D, E, F, and G have to cut a lead of 23.071 inches? (Figs. 454 and 455.)

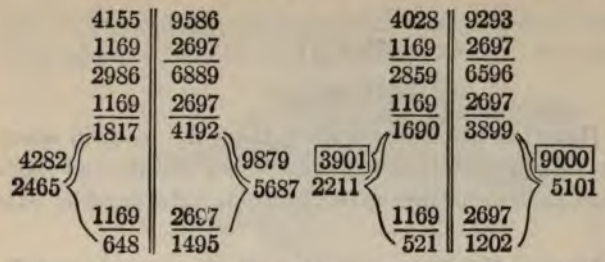
Solution: While the spiral makes 1 revolution, the table must move 23.071 inches. To move the table 23.071 inches, the feed screw and the imaginary simple gear G′ must make $23.071 \div \frac{1}{4}$ or 92.284 revolutions; therefore, to cut a 23.071 inch lead, the imaginary simple gear G′ must make 92.284 revolutions while the spiral makes 1 revolution. While D′ makes 40 revolutions, the spiral makes 1 revolution; therefore, while D′ makes 40 revolutions, G′ must make 92.284 revolutions.

If we know the number of revolutions the imaginary simple gears D′ and G′ will make, the number of teeth in them are obtained by inverting the revolutions. (See Article 76.) Thus, if D′ makes 40 revolutions while G′ makes 92.284 revolutions, D′ will have 92.284 teeth and G′ 40 teeth. By multiplying these by 1000 to clear of fractions, D′=99284 teeth, and G′=40000 teeth. The factors of these numbers are too large for compound gears; therefore, the numbers are reduced by the principle of continued fractions to other numbers from which suitable factors may be obtained. (See Articles 240 to 250.)

Thus,

40000	92284	2
36852	80000	3
3148	12284	3
2840	9444	1
308	2840	9
272	2772	4
36	68	1
32	36	1
4	32	8
	32	
	0	

										8 3
		2	3	3	1	9	4	1	1	4155
1	0	1	3	10	13	127	521	648	1169	9586
0	1	2	7	23	30	293	1202	1495	2697	4028
		2	3	3	1	9	4	1	1	9293
										8 4



$\frac{92284}{40000}$ = approximately $\frac{9000}{3901}$. Then, D′ = 9000 teeth, and G′ = 3901 teeth. After factoring (9000 = 90 × 100; 3901 = 47 × 83) and placing in proper compound order, D = 100 teeth, E = 83 teeth, F = 90 teeth, and G = 47 teeth. (See Article 73.)

CHECK: While the spiral makes 1 revolution, the table will move $40 \times \frac{100}{83} \times \frac{90}{47} \times \frac{1}{4}$ or 23.071007 inches, which varies only .000007 inch from the true value.

EXERCISE 62

1. How many teeth must the milling-machine change gears D, E, F, and G have to cut a 7.5103″ lead if there are four threads per inch on the feed screw? (Fig. 454.)

2. How many teeth must the milling-machine change gears D, E, F, and G have to cut a 50.0522″ lead if there are four threads per inch on the feed screw? (Fig. 454.)

3. How many teeth must the milling-machine change gears D, E, F, and G have to cut a 5.7291″ lead if there are four threads per inch on the feed screw? (Fig. 454.)

XXXIII

HOBS

259. Hobs are chiefly used for cutting the teeth in worm wheels, spur gears, and spiral gears. They are termed *right-hand* or *left-hand* according as the thread is right-hand or left-hand.

260. Linear pitch hobs.—Hobs for worm wheels usually have a **linear pitch** of 1″, $\frac{1}{2}$″, $\frac{1}{3}$″, $\frac{1}{4}$″, etc., or some lead that can be readily cut on any lathe without having to calculate the change gears. The outside diameter of hobs for worm wheels equals the diameter of the worm plus $\frac{\text{linear pitch}}{10}$. (See Article 229 for dimensions of worms.)

261. Diametral pitch hobs.—Hobs for spur or spiral gears have odd leads and are termed 5-, 6-, 7-, 8-, 9-, etc. diametral pitch. For example. a hob that is used for cutting 5-diametral pitch gears is called a 5-diametral pitch hob, or a 5-D. P. hob. (See Article 263 for determining the lead of diametral pitch hobs.) The outside diameter of diametral pitch hobs equals the pitch diameter plus $\frac{2\times1.157}{\text{D.P.}}$, or the pitch diameter equals the outside diameter minus $\frac{2\times1.157}{\text{D.P.}}$.

262. Spiral-fluted hobs.—For better cutting action, hobs are made with **spiral flutes.** These flutes are at right angles to the threads of the hob on the pitch diameter. They are also called **gashes.**

The Lead of Diametral Pitch Hobs

263. If a diametral pitch hob is constructed properly, it will fill all the space of a rack having the same diametral pitch, as is shown in Fig. 456. To do this the hob must be

placed at an angle on the rack. This angle is called the **helix angle** of the hob and is indicated by θ. It can be seen

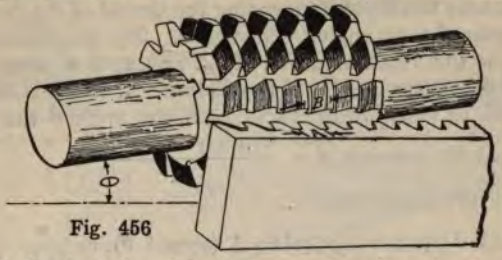


Fig. 456

that B, the lead of the thread, is greater than A, the distance from tooth to tooth on the rack. The lead of the thread is equal to the product of the distance from tooth to tooth of the rack and the secant of angle θ, or $B = A \text{ (secant } \theta) = \frac{\pi}{\text{D.P.}} \text{ (secant } \theta)$.

The following shows how angle θ is determined.

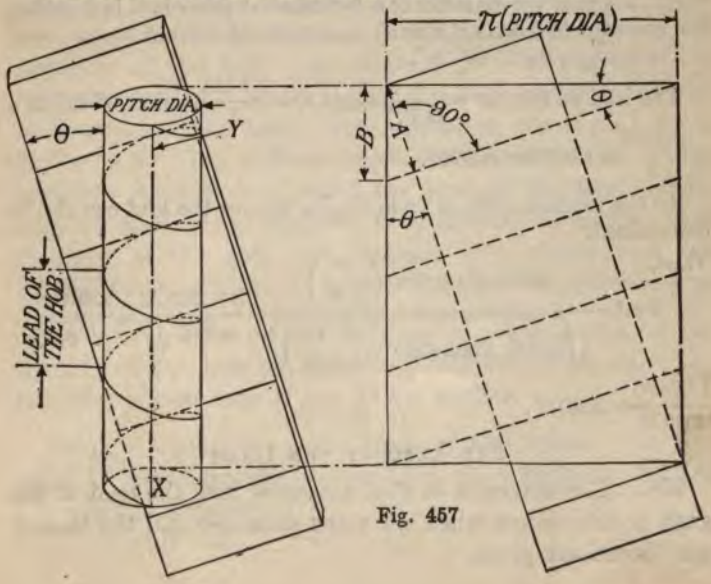


Fig. 457

A representation of the pitch diameter of a hob and a rack is shown in Fig. 457. For the lines representing the rack teeth to come directly under the lines representing the thread of the hob, the hob must be placed at an angle. See angle θ. The diagram at the right shows how this representation would appear if it were cut along XY and straightened out. If the distance A, which is the distance from tooth to tooth on the rack, is known, angle θ can readily be determined by cosecant $\theta = \dfrac{\pi(\text{pitch diameter})}{A}$,

or (D.P.) (pitch diameter).

The lead of the hob is equal to A (secant θ), or $\dfrac{\pi}{\text{D.P.}}$ (secant θ).

By the similar triangles in Fig. 457 it can be proved that

the lead of the hob also $= \dfrac{(\text{pitch diameter})\left(\dfrac{\pi}{\text{D.P.}}\right)}{\sqrt{(\text{pitch diameter})^2 - \left(\dfrac{1}{\text{D.P.}}\right)^2}}$, which will

produce the same results as the preceding formula.

Example: If the diameter of a 5-diametral pitch hob is 3 inches, the cosecant of $\theta = (\text{D.P.})(\text{pitch diameter}) = 5 \times 3 = 15$.

Then, angle $\theta = 3^\circ\, 49'\, 21''$.

The lead of the hob $= \dfrac{\pi}{\text{D.P.}}(\text{secant}\,\theta) = \dfrac{3.1416}{5}(\text{secant}\ 3^\circ\, 49'\, 21'')$

$= \dfrac{3.1416}{5} \times 1.0022 = .62970''$.

By the formula derived from similar figures the lead can also be determined.

Thus,

$$\text{lead} = \frac{(\text{pitch diameter})\left(\dfrac{\pi}{\text{D.P.}}\right)}{\sqrt{(\text{pitch diameter})^2 - \left(\dfrac{1}{\text{D.P.}}\right)^2}} = \frac{3 \times .62832}{\sqrt{3^2 - \frac{1}{2^5}}} = \frac{1.88496}{\sqrt{8.96}} =$$

$$\frac{1.88496}{2.9933} = .62970''.$$

The Lead of the Gash

264. The diagrams in Fig. 458 show how the **lead of the gash** is determined when the pitch diameter and the lead of the thread are given.

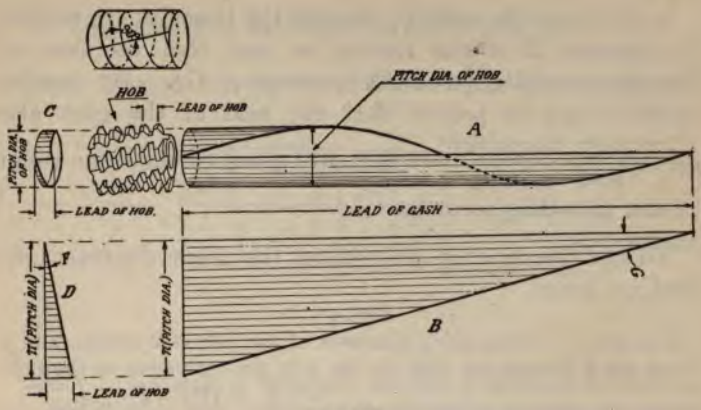


Fig 458

In diagram A, the heavy line represents one spiral flute of the hob. The distance that this spiral travels in one complete revolution is called the **lead of the gash.** Note that its diameter in diagram A is equivalent to the pitch diameter of the hob. In diagram B is shown how the representation of A would appear if it were straightened out. In diagram C, the heavy line represents one thread spiral of the hob. The distance that this spiral travels in one complete revolution is called the **lead of the hob.** Note that its diameter in diagram C is equivalent to the pitch diameter of the hob. In diagram D is shown how the representation of C would appear if it were straightened out.

The spiral flutes of the hob are at right angles to the threads of the hob on the pitch diameter; therefore, the triangles in diagrams B and D are similar.

Angle F equals the helix angle of the thread. In diagram D it can readily be seen that the cotangent of angle $F = \frac{\pi(\text{pitch diameter})}{\text{lead of hob}}$.

Angle G equals angle F, because the triangles are similar. In diagram B, it can readily be seen that the lead of the gash $=\pi$(pitch diameter)(cotangent G). By similar figures it can be proved that the lead of the gash also $=\frac{[\pi(\text{pitch diameter})]^2}{\text{lead of hob}}$, which will produce the same result as the preceding formula.

To find the lead of gash when the pitch diameter and lead are given:

EXAMPLE: If the pitch diameter of the hob is 2 inches, and if there are 3 threads per inch on the hob, the cotangent of the helix angle of the thread of the hob equals $\frac{\pi\times 2}{\frac{1}{3}}=\frac{3.1416\times 2}{\frac{1}{3}}=18.850$.

Then, angles G and F equal $3°\,2'\,12''$.

The lead of the gash $=\pi$ (pitch diameter) (cotangent G) $=3.1416\times 2\times 18.850=118.438''$.

By similar figures the lead of the gash can also be determined by $\frac{[\pi(\text{pitch diameter})]^2}{\text{lead of hob}}$. Thus, $\frac{[(3.1416\times 2)]^2}{\frac{1}{3}}=118.438''$.

To find the lead of gash when the pitch diameter and D. P. are given:

EXAMPLE: If the pitch diameter of a 5-diametral pitch hob is 3 inches, the cosecant $\theta=$(D.P.)(pitch diameter)$=5\times 3=15$. (See Article 263.) Then, the helix angle$=3°\,49'\,21''$. The lead of the gash $=\pi$(pitch diameter)(cotangent θ)$=3.1416\times 3\times 14.967=141.06''$.

EXERCISE 63

1. If the pitch diameter of a hob used for cutting 10-diametral pitch gears is 3 inches, what should be the outside diameter?

2. What should be the lead of this hob?

3. What should be the lead of the gash of this hob?

GEAR RATIOS FOR BACKING OFF SPIRAL-FLUTED HOBS

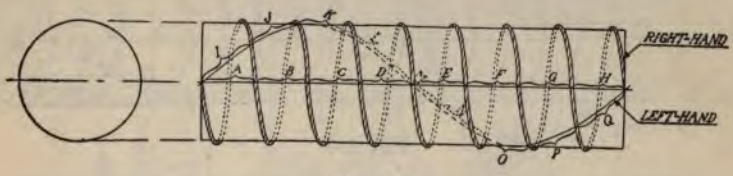


Fig. 459

265. If a string is wrapped around a cylinder to form a spiral as in Fig. 459, a straight line will cross as many spaces between successive turns of the string as there are turns in the spiral. In this case a straight line crosses A, B, C, D, E, F, G, and H, or 8 spaces, because there are 8 complete turns in the spiral.

If a line is drawn to form a spiral around the cylinder so that the direction will be opposite to the direction of the string, it will cross one more space between the successive turns of string than if it were straight. In this case the spiral line crosses I, J, K, L, M, N, O, P, and Q; that is 8+1 or 9 spaces.

266. Knowledge of the preceding demonstration is very essential in figuring the amount of rotation a spiral-fluted hob must make to cause the adjacent tooth in the next thread spiral to come on the center line.

EXAMPLE: If the lead of the gash is 10 inches and the lead of the hob is .500 of an inch, what part of a revolution must the hob make in order that the adjacent tooth in the next thread will come on the center line? In other words, in Fig. 460, when point A is on the center line, what part of a revolution must the cylinder representing the hob make to cause point B to come on the center line?

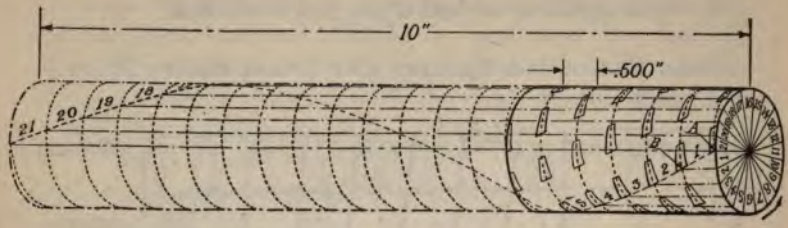


Fig. 460

SOLUTION: There are $10 \div .500$ or 20 turns of the thread within the distance of one turn of the gash. A straight line will cross 20 spaces between the threads of the hob. The lead of the gash makes one turn in the direction opposite to the turn of the thread; therefore, the lead of the gash crosses $20+1$ or 21 spaces between the threads.

If parallel lines are drawn from the intersecting points of the lead of the gash and the threads of the hob so that they project to the end of the hob as shown, the part of the whole circle between each of these lines will be $\frac{1}{21}$. The part of the whole circle between each of these lines can also be determined by the formula,—

$$?=\frac{1}{\frac{\text{lead of the gash}}{\text{lead of the hob}}+1}; \quad \text{thus, } \frac{1}{\frac{10}{.5}+1}=\frac{1}{21}.$$

If the cylinder representing the hob revolves in the direction as shown by the arrow, it will have to revolve $1-\frac{1}{21}$ or $\frac{20}{21}$ of a revolution to cause the adjacent tooth in the next thread to come on the center line, because tooth A is $\frac{1}{21}$ part of a division out of line with tooth B.

267. The principles explained in the preceding articles greatly simplify the calculation for the change gears controlling the backing-off mechanism.

EXAMPLE: How many teeth should the gears A, B, C, and D, in Fig. 461A, have, to back off a spiral-fluted hob with 5 flutes, if the lead of the gash is 88.5 inches, and the lead of the hob is $\frac{1}{3}$ of an inch? (Fig. 461.)

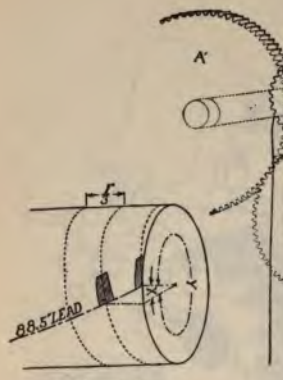


Fig. 461

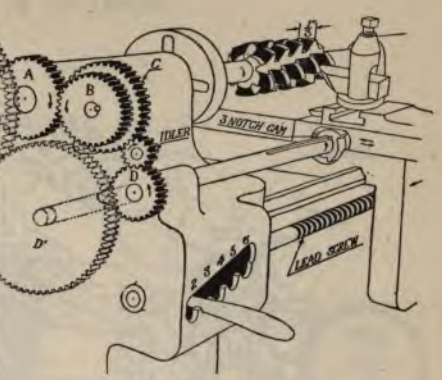


Fig. 461A

SOLUTION: Within the distance of the lead of the gash there are $88.5 \div \frac{1}{3}$ or 265.5 threads.

A straight line will cross 265.5 spaces within the lead of the gash; but the spiral line would cross $265.5+1$ or 266.5 spaces between the threads of the hob. In Fig. 461, $X=\frac{1}{266.5}$. Then $\frac{1}{266.5}$ is the part of a division the adjacent tooth in the next thread is out of line with the tooth being considered; therefore, when one tooth is on the center line, in order that the adjacent tooth in the next thread may come on the center line, the hob will have to revolve $1-\frac{1}{266.5}=\frac{265.5}{266.5}$ or $\frac{5\,3\,1}{5\,3\,3}$ of a revolution. In other words, Y in Fig. 461 equals $\frac{5\,3\,1}{5\,3\,3}$.

While the hob makes $\frac{5\,3\,1}{5\,3\,3}$ of a revolution, the tool must reciprocate 5 times, because there are 5 flutes in the hob. While the cam makes 1 revolution, the tool reciprocates 3 times; therefore, to cause the tool to reciprocate 5 times the cam or the imaginary simple gear D′ must make $5 \div 3$ or $\frac{5}{3}$ of a revolution.

If we know the number of revolutions the imaginary simple gears A′and D′ must make, the number of teeth in them will be obtained by inverting the revolutions. (See Article 76.) Thus, if A′ makes $\frac{5\,3\,1}{5\,3\,3}$ of a revolution while D′ makes $\frac{5}{3}$ of a revolution, A′ must have $\frac{5}{5}$ of a tooth and D′, $\frac{5\,3\,1}{5\,3\,3}$ of a tooth. By multiplying these by 533×3 to clear of fractions, $A'=\frac{5}{5}\times533\times3=2665$ teeth, and $D'=\frac{5\,3\,1}{5\,3\,3}\times533\times3=1593$ teeth. After factoring ($2665=41\times65$; $1593=27\times59$) and placing in proper compound order, A = 41 teeth, B = 27 teeth, C = 65 teeth, and D = 59 teeth.

EXERCISE 64

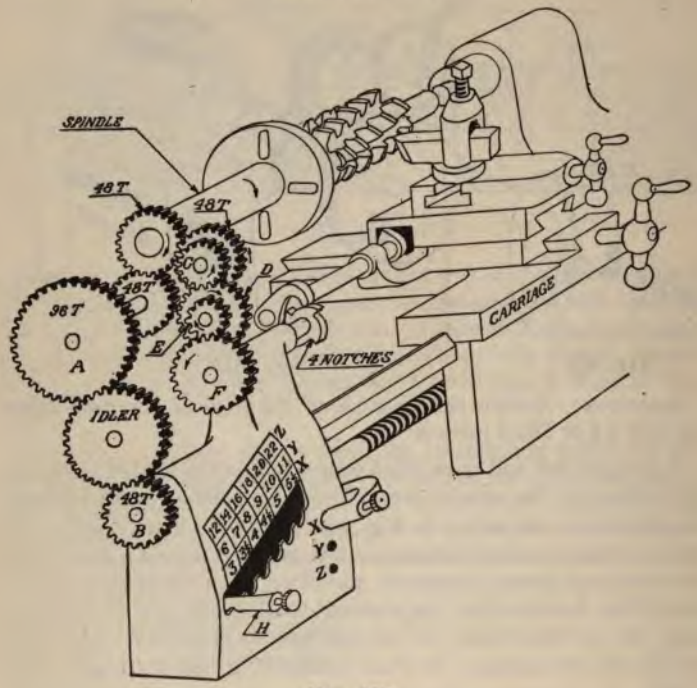


Fig. 462

1. The handle H being set for 3 threads per inch in Fig. 462, to what will the gears A and B be changed so as to cut a lead of .31432 inch?

2. How many teeth should the gears C, D, E, and F have in order to back off a spiral-fluted hob with 6 flutes, if the lead of the gash is 282.59 inches, and the lead of the hob is .31428 inch.

XXXIV

SPIRAL GEARS

Fig. 463

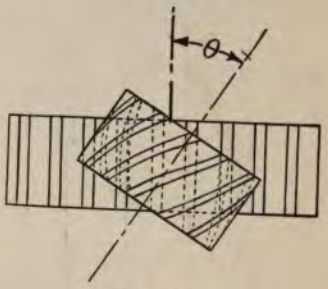


Fig. 464

268. If a spiral gear is constructed properly, it will mesh with a rack having the same pitch. To do this the spiral gear must be placed at an angle on the rack. See angle θ in Fig. 464. This angle is called the **angle of the spiral gear.** For example, if this angle were 20°, the gear would be called a 20° spiral gear. Fig. 463 shows a right-hand spiral gear.

Pitch Diameter and Lead of Spiral Gears

The diametral pitch D.P., the number of teeth N, and the spiral angle θ determine the dimensions for spiral gears.

269. Method of calculation:

θ = angle of the spiral gear.
N = number of teeth in the spiral gear.
D.P. = diametral pitch.

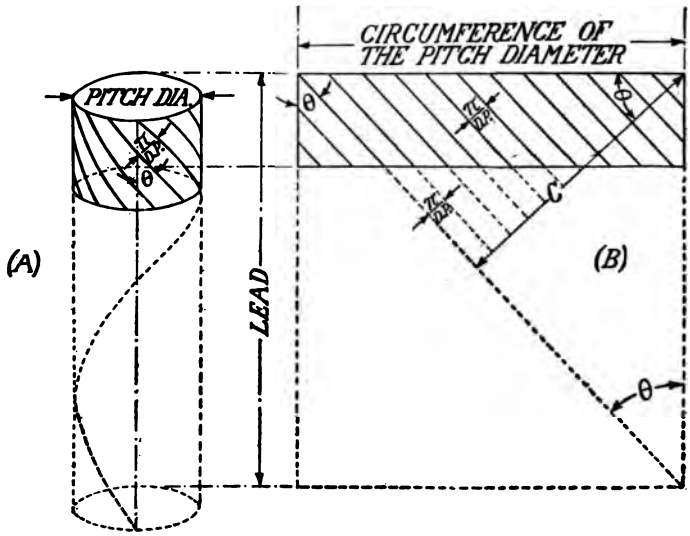


Fig. 465

$\frac{\pi}{\text{D.P.}}$ = distance from tooth to tooth on a rack that would mesh with the spiral gear.

In Fig. 465, diagram A is a representation of the pitch diameter of a spiral gear and an imaginary continuation of one tooth. The dimension $\frac{\pi}{\text{D.P.}}$ is the same as the distance from tooth to tooth on a rack with the same pitch. Diagram B shows how the spiral gear would appear if it were straightened out. Line C is perpendicular to the lines representing the teeth. Since $\frac{\pi}{\text{D.P.}}$ is the distance from tooth to tooth in diagram B, and since there are N teeth, it can readily be seen that $C = N\left(\frac{\pi}{\text{D.P.}}\right)$.

By trigonometry the circumference of the pitch diameter = C (secant θ) or $N\left(\frac{\pi}{\text{D.P.}}\right)$(secant. θ); therefore, the

pitch diameter $= N\dfrac{\left(\dfrac{\pi}{D.P.}\right)(\text{secant } \theta)}{\pi}$ or $\dfrac{N(\text{secant } \theta)}{D.P.}$, and the outside diameter $= \dfrac{N(\text{secant } \theta)+2}{D.P.}$.

PROBLEM: What is the pitch diameter and outside diameter of a 4-diametral pitch, 20-tooth, 22° spiral gear?

SOLUTION: Pitch diameter $= \dfrac{N(\text{secant } \theta)}{D.P.} = \dfrac{20(\text{secant } 22°)}{4} = \dfrac{20 \times 1.0785}{4} = 5.3925$ inches.

Outside diameter $= \dfrac{20 \times 1.0785 + 2}{4} = 5.8925$ inches.

By trigonometry it can readily be seen that:

Lead of the spiral gear $= C(\text{cosecant } \theta)$ or $N\left(\dfrac{\pi}{D.P.}\right)(\text{cosecant } \theta)$.

PROBLEM: What is the lead of a 4-diametral pitch, 20-tooth, 22° spiral gear?

SOLUTION: Lead $= N\left(\dfrac{\pi}{D.P.}\right)(\text{cosecant } \theta) =$

$20\left(\dfrac{\pi}{D.P.}\right)(\text{cosecant } \theta) = 20 \times \frac{3.1416}{4} \times 2.6695 = 41.8325$ inches.

EXERCISE 65

1. What is the pitch diameter of a 5-diametral pitch, 30-tooth, 25° spiral gear?

2. What is the pitch diameter of a 10-diametral pitch, 50-tooth, 40° spiral gear?

3. What is the lead of a 5-diametral pitch, 30-tooth, 25° spiral gear?

4. What is the lead of a 10-diametral pitch, 50-tooth, 40° spiral gear?

XXXV

HOBBING SPIRAL GEARS

Change-gear Ratios in Automatic Gear-hobbing Machines

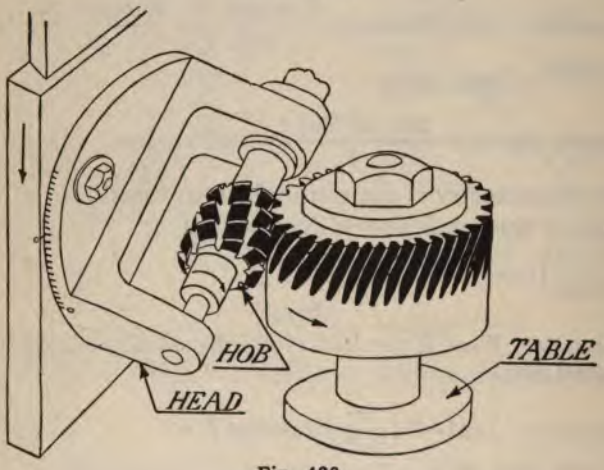


Fig. 466

270. When hobbing spiral gears in an automatic gear-hobbing machine, the hob and the table are mechanically connected in such a way that they revolve together. The motion of the hob with respect to the table depends on the type of the hob and the gear that are being considered. All the teeth are cut simultaneously as the hob feeds through the work.

271. A hob must be placed at an angle to mesh with a rack. (See Article 263.) So must a spiral gear be placed at an angle to mesh with a rack. (See Article 268.) There-

fore, the sum or the difference of these angles is the angle at which the head carrying the hob shown in Fig. 466, must be set when hobbing a spiral gear.

Feed of the Hob

272. As the hob and work revolve, the hob feeds through the face of the gear. The amount the hob feeds when the table makes 1 revolution is called the **feed of the hob.** For example, if the hob feeds .060 of an inch while the table makes 1 revolution, .060 of an inch is the feed of the hob, or the hob takes a .060 of an inch cut.

Hand of the Gear and Hob

273. It is not necessary for the **hand** of the gear and the **hand** of the hob to be the same. For example, a left-hand hob can be used for cutting either right- or left-hand spiral gears, and a right-hand hob can also be used for cutting either right- or left-hand gears.

Feed Spiral

274. If a spiral line is drawn around the gear blank which is being hobbed, so that the distance between each turn of this spiral line is the same as the feed being used, the hob will follow this line. The path that the center of the hob follows is similar to this line, and it is called the **feed spiral** of the gear. By using a very coarse feed with a hob taking a very light cut, these feed spirals can be traced, and if a right-hand hob is used, the feed spiral will be a right-hand spiral, and if a left-hand hob is used the feed spiral will be a left-hand spiral. To prove these statements study Figs. 467, 468, 469, and 470. The **feed spiral** is *not* the **tooth spiral** of the gear.

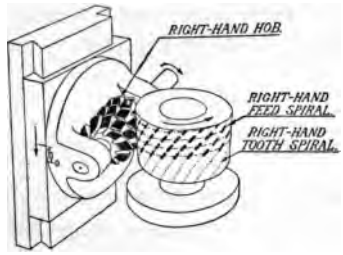


Fig. 467

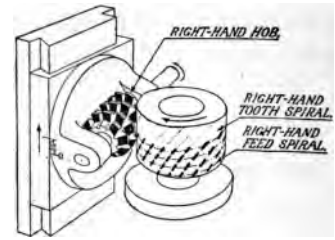


Fig. 468

From Fig. 467, it can be seen that, if a hob feeds down through the face of the work, it must revolve as shown by the small arrow. A right-hand hob revolving as shown by the small arrow will cause the work to revolve as shown by the large arrow. From this it can be seen that a right-hand feed spiral is obtained.

From Fig. 468, it can be seen that, if a hob feeds up through the face of the work, it must revolve as shown by the small arrow. A right-hand hob revolving as shown by the small arrow will cause the work to revolve as shown by the large arrow. From this it can be seen that a right-hand feed spiral is obtained.

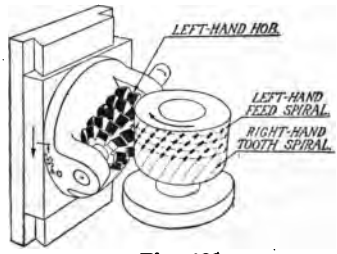


Fig. 469

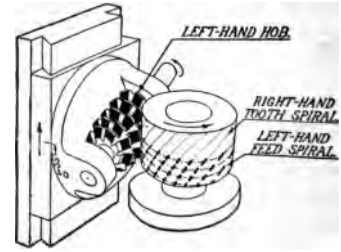


Fig. 470

From Fig. 469, it can be seen that, if a hob feeds down through the face of the work, it must revolve as shown by the small arrow. A left-hand hob revolving as shown by the small arrow will cause the work to revolve as shown by the large arrow. From this it can be seen that a left-hand feed spiral is obtained.

From Fig. 470, it can be seen that, if a hob feeds up through the face of the work, it must revolve as shown by the small arrow. A

left-hand hob revolving as shown by the small arrow will cause the work to revolve as shown by the large arrow. From this it can be seen that a left-hand feed spiral is obtained.

Right-hand hobs always produce right-hand feed spirals, and left-hand hobs always produce left-hand feed spirals.

THEORY OF THE INTERSECTING POINTS OF SPIRALS

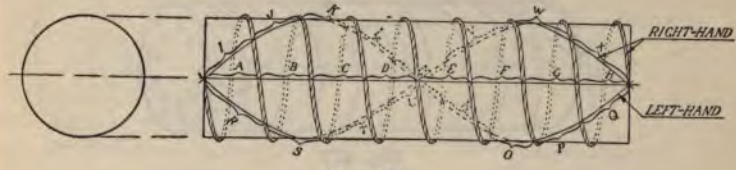


Fig. 471

275. If a string is wrapped around a cylinder to form a spiral as in Fig. 471, a straight line will cross as many spaces between the successive turns of the string as there are turns in the spiral. In this case a straight line crosses A, B, C, D, E, F, G, and H, or 8 spaces.

If a line is drawn to form a spiral around the cylinder so that the direction or "hand" will be opposite to the "hand" of the string, it will cross one more space between the successive turns of the string than if it were straight. In this case the spiral line, making one turn in the opposite hand from the string, crosses I, J, K, L, M, N, O, P, and Q; that is, 8+1 or 9 spaces.

If a line is drawn to form a spiral around the cylinder so that the "hand" will be the same as the "hand" of the string, it will cross the spaces made by the string one time less than if it were straight. In this case the spiral line, making one turn in the same hand as the string, crosses R, S, T, U, V, W, and X; that is, 8−1 or 7 spaces.

276. The preceding demonstration is used in problems dealing with the relation of the intersecting points of spirals,

and it is used as the basis for the computations of change gears for automatic gear-hobbing machines.

PROBLEM 1: In Fig. 472, if the lead of the left-hand spiral A is 5 inches, and if the lead of the left-hand spiral B is .500 of an inch, what part of the whole circle does X equal?

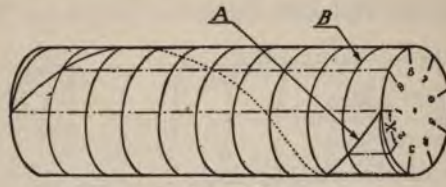


Fig. 472

SOLUTION: There are $5 \div .500$ or 10 turns of the spiral B within the distance of one turn of the spiral A. A straight line will cross 10 spaces between the turns of the spiral B; but the spiral A, making one turn in the same hand as the spiral B, crosses $10-1$ or 9 spaces. If parallel lines are drawn from the intersecting points of these spirals to the end of the cylinder, they will divide the circle into 9 parts; therefore, $\frac{1}{9}$ of a complete circle is between the successive intersecting points of the spirals.

277. Another case involving intersecting points of spirals.

PROBLEM 2: In Fig. 473, if the lead of the left-hand spiral A is 11 inches, what would be the lead of the right-hand spiral B, when the part of a complete circle between the intersecting points of the spirals is $\frac{1}{22.6}$?

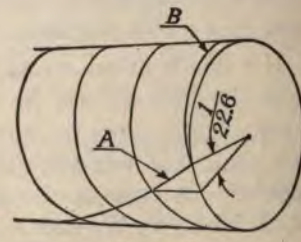


Fig. 473

SOLUTION: If the part of a complete circle between the intersecting points of the spirals is $\frac{1}{22.6}$, the spiral line A must cross 22.6 spaces between the turns of the spiral B. When the hands of the spirals are opposite, one spiral crosses the spaces between the turns of the other spiral one time more than a straight line would cross; therefore, a straight line would cross $22.6-1$ or 21.6 spaces

between the turns of the spiral B. If a straight line crosses 21.6 spaces within the distance of 11 inches, the lead of the spiral B must be $11 \div 21.6$ or $\frac{11}{21.6}$ of an inch.

EXERCISE 66

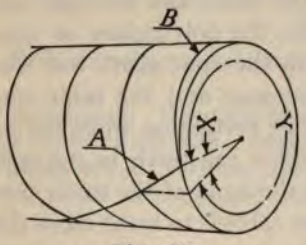


Fig. 474

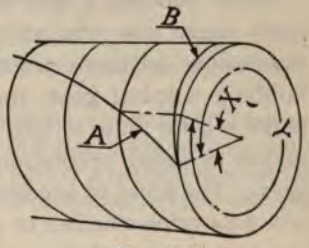


Fig. 475

1. In Fig. 474, if the lead of the spiral line A is 20 inches, and the lead of the spiral line B is .900 of an inch, what part of the complete circle is X?

2. In the same figure, what part of the complete circle is Y?

3. In Fig. 475, if the lead of the spiral line A is 20 inches, and the lead of the spiral line B is .900 of an inch, what part of the complete circle is X?

4. In the same figure, what part of the complete circle is Y?

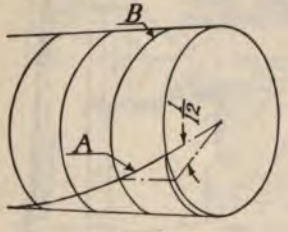


Fig. 476

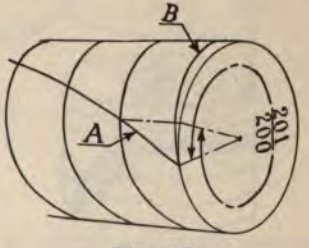


Fig. 477

5. In Fig. 476, if the lead of the spiral line A is 10 inches, what will be the lead of the spiral B, when the part of a complete circle between the intersecting points of the spirals is $\frac{1}{12}$?

6. In Fig. 477, if the lead of the spiral line A is 102 inches, what will be the lead of the spiral B, when $\frac{201}{200}$ of a complete circle is the relation of the intersecting points?

278. The principles in the preceding articles (275, 276, 277) are essential in calculating the amount of rotation the spiral gear must make, while the hob makes as many revolutions as there are teeth in the gear being considered.

When hobbing a spur gear, the table and the spur gear must make one revolution while the hob makes as many revolutions as there are teeth in the spur gear; but when hobbing a spiral gear, the spiral gear and the table must make a little more or less than one revolution while the hob makes as many revolutions as there are teeth in the spiral gear. The hand of the hob and the hand of the tooth spiral will determine whether or not the table is to make more than one complete revolution while the hob makes as many revolutions as there are teeth in the spiral gear.

PROBLEM 1: If 8 left-hand grooves have a lead of 10 inches and are being cut with a single tooth of a right-hand hob, by using a coarse feed of .500 of an inch, what part of a revolution must the table make while the hob makes 8 revolutions? (Figs. 478 and 479.)

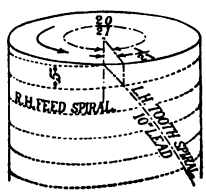


Fig. 478

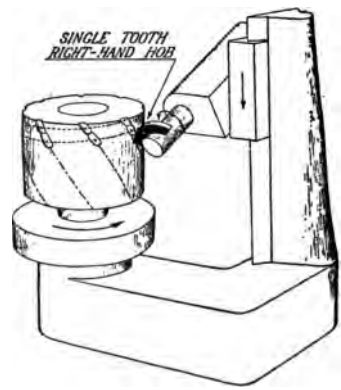


Fig. 479

SOLUTION: The feed spiral must be a right-hand spiral because a right-hand hob is being used. (See Article 274.) The hand of the

grooves and the hand of the feed spiral are opposite; therefore, $\frac{1}{21}$ is the part of a complete circle between the intersecting points of the spirals. (See Articles 275, 276.) If a right-hand hob feeds down through the work, the work must revolve as shown by the large arrow; therefore, while the hob makes 8 revolutions, the work will make $1-\frac{1}{21}$ or $\frac{20}{21}$ of a revolution.

279. Another case involving the rotation of the spiral gear:

PROBLEM 2. If 8 right-hand grooves have a lead of 10 inches, and are being cut with a single tooth of a right-hand hob, by using a coarse feed of .500 of an inch, what part of a revolution must the table make while the hob makes 8 revolutions? (Figs. 480 and 481.)

Fig. 480

Fig. 481

SOLUTION: The feed spiral must be a right-hand spiral because a right-hand hob is being used. (See Article 274.) The hand of the grooves and the hand of the feed spirals are the same; therefore, $\frac{1}{19}$ is the part of a complete circle between the intersecting points of the spirals. (See Articles 275, 276.) If a right-hand hob feeds down through the work, the work must revolve as shown by the large arrow; therefore, while the hob makes 8 revolutions, the work will make $1+\frac{1}{19}$ or $\frac{20}{19}$ revolutions.

Calculations of Change Gears

280. Method when using exactly the feed assumed:

Problem 1: In the automatic gear-hobbing machine in Fig. 482, what feed and index gears should be used in cutting a 20-tooth gear with a 44.62-inch right-hand lead, if a left-hand hob and a .040-inch feed are used?

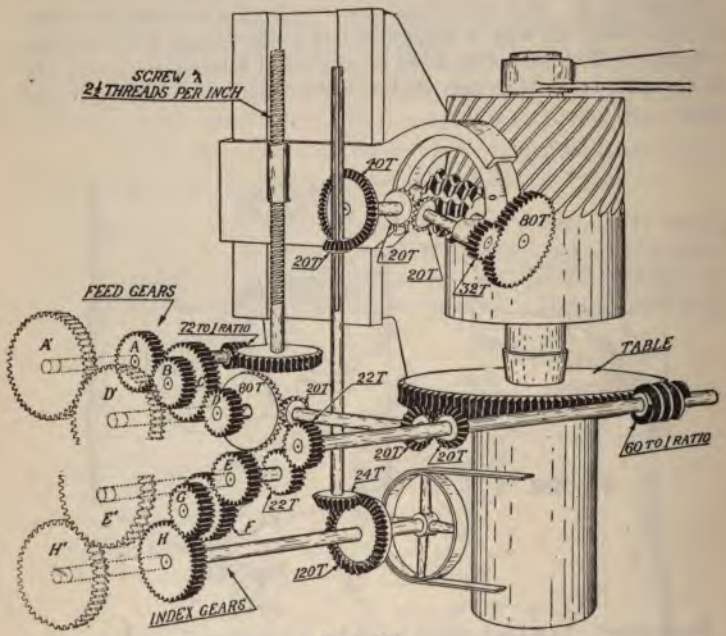


Fig. 482

281. Solution of the index gears.

Solution: In Fig. 483, the feed spiral will be left-hand because a left-hand hob is being used. (See Article 274.) The hand of the tooth spiral and the hand of the feed spiral are opposite; there-

fore, according to Articles 275 and 276, the part of a complete circle between the intersecting points is $\frac{1}{1116.5}$. If a left-hand hob feeds down through the work (see Fig. 469), the work must revolve as shown by the arrow; and from this it can be seen that the work must make $1-\frac{1}{1116.5}$ or $\frac{1115.5}{1116.5}$ of a revolution while the hob makes 20 revolutions, because there are 20 teeth in the gear.

Fig. 483

The next step is to determine how many teeth the gears E, F, G, and H must have so that, when the table makes $\frac{1115.5}{1116.5}$ of a revolution, the hob will make 20 revolutions. When the hob makes 20 revolutions, the imaginary simple gear H′ will make $20\times\frac{80}{32}\times\frac{40}{60}\times\frac{24}{120}$ or 20 revolutions. When the work makes $\frac{1115.5}{1116.5}$ of a revolution, the imaginary simple gear E′ will make $\frac{1115.5}{1116.5}\times 60$ or $\frac{2231\times 60}{2233}$ revolutions.

If we know the number of revolutions the imaginary simple gears H′ and E′ must make, the number of teeth in them will be obtained by inverting the revolutions. (See Article 76.) Thus, if H′ makes 20 revolutions while E′ makes $\frac{2231\times 60}{2233}$ revolutions, H′ must have $\frac{2231\times 60}{2233}$ teeth, and E′ 20 teeth. By multiplying these by 2233 to clear of fractions, $H'=\frac{2231\times 60}{2233}\times 2233$ or 2231×60 teeth, and $E'=20\times 2233$ teeth. Divide these last numbers by 20, and $H'=2231\times 60\div 20$ or 6693 teeth, and $E'=2233\times 20\div 20$ or 2233 teeth. After factoring ($6693=97\times 69$; $2233=29\times 77$) and placing in proper compound order, E = 29 teeth, F = 97 teeth, G = 77 teeth, and H = 69 teeth.

282. Solution of the feed gears.

Solution 1: While the screw X makes 1 revolution, the hob feeds down $\frac{2}{5}$ inch or .400 inch; therefore, to feed the hob .040 inch, which is the feed to be used, the screw must make $.040\div .400$ or .1 of a revolution. While the screw makes .1 of a revolution, the imaginary simple gear A′ will make $.1\times 72$ or

7.2 revolutions. The hob feeds .040 inch while the table makes 1 revolution, or while the imaginary simple gear D′ makes $1\times 60\times\frac{2}{80}$ or 15 revolutions.

If we know the number of revolutions the imaginary simple gears A′ and D′ must make, the number of teeth in them will be obtained by inverting the revolutions. (See Article 76.) Thus, if A′ makes 7.2 revolutions while D′ makes 15 revolutions, A′ must have 15 teeth and D′ 7.2 teeth. By multiplying these by 10 to clear of fractions, A′ = 150 teeth and D′ = 72 teeth. After factoring ($150 = 10\times 15$; $72 = 8\times 9$) and placing in proper compound order, A = 15 teeth, B = 9 teeth, C = 10 teeth, and D = 8 teeth. If we multiply these by 3 for a better set of gears, A = 45 teeth, B = 27 teeth, C = 30 teeth, and D = 24 teeth. (See Article 74.)

Solution 2: When the feed gears A, B, C, and D all have the same number of teeth, the head carrying the hob will feed $60\times\frac{2}{80}\times\frac{1}{72}\times\frac{2}{5}$ or $\frac{1}{12}$ inch; therefore, the gear ratio for a .040-inch feed would be $\frac{\frac{1}{12}}{.040}=\frac{\frac{1}{12}}{\frac{1}{25}}=\frac{25}{12}$. Gears obtained from this ratio are the same as those obtained in Solution 1. Thus, $\frac{25}{12}=\frac{5\times 5}{3\times 4}=\frac{45\times 30}{27\times 24}$.

283. Method when using approximately the feed assumed:

Very often it is impossible to cut the exact lead of a spiral gear on an automatic gear-hobbing machine, but by allowing the feed to vary slightly, leads that are close enough for all practical purposes may be obtained.

Problem 2: In Fig. 482, if the ratio of the worm wheel on the table and the worm were 90 instead of 60, what feed and index gears would be used in cutting a 13-tooth spiral gear with a 10.2753-inch left-hand lead, if a left-hand hob with an approximate feed of .020 of an inch were used?

284. Solution of the index gears.

Solution: In Fig. 484, the feed spiral will be left-hand, since a left-hand hob is being used. (See Article 274.) In the distance of

the lead there will be $10.2753 \div .020$ or approximately 513 turns of the feed spiral. (See Article 275.) The hand of the tooth spiral and the hand of the feed spiral are the same; therefore, according to Article 276, the part of the complete circle between the intersecting points is $\frac{1}{512}$. If a left-hand hob feeds down through the work (see Fig. 469), the work must revolve as shown by the arrow, and from this it can be seen that the work must make $1+\frac{1}{512}$ or $\frac{513}{512}$ revolutions, while the hob makes 13 revolutions, because there are 13 teeth in the spiral gear.

Fig. 484

The next step is to determine how many teeth the gears E, F, G, and H must have so that, while the table makes $\frac{513}{512}$ revolutions, the hob will make 13 revolutions. When the table makes $\frac{513}{512}$ revolutions, the imaginary simple gear E′ will make $\frac{513}{512}\times 90$ revolutions, since the ratio of the worm wheel and worm is 90. While the hob makes 13 revolutions, the imaginary simple gear H′ will make $13\times\frac{80}{32}\times\frac{40}{20}\times\frac{24}{120}$ or 13 revolutions.

If we know the number of revolutions the imaginary simple gears E′ and H′ must make, we may obtain the number of teeth in them by inverting the revolutions. (See Article 76.) Thus, if E′ makes $\frac{513}{512}\times 90$ revolutions while H′ makes 13 revolutions, E′ must have 13 teeth and H′, $\frac{513}{512}\times 90$ teeth. If we multiply these by 512 to clear of fractions, $E' = 13\times 512 = 6656$ teeth, and $H' = \dfrac{513\times 90}{512}\times 512$ or 46170 teeth. The factors of these numbers are too large for compound gears; therefore, the numbers are reduced by the principle of continued fractions to other numbers from which suitable factors may be obtained.

Thus,

6656	46170	6
6234	39936	1
422	6234	14
326	5908	1
96	326	3
	288	

		6	1	14	1
1	0	1	1	15	16
0	1	6	7	104	111
		6	1	14	1

When reducing the numbers in this way, stop at a convergent where the larger number of the convergent is between the limits of 100 and 500. If will be close enough ordinarily and more readily factored.

Since $\frac{6656}{46170}$ = approximately $\frac{16}{111}$, E′ = 16 teeth and H′ = 111 teeth. After factoring (16 = 1×16; 111 = 3×37) and placing in proper compound order, E = 16 teeth, F = 3 teeth, G = 1 tooth, and H = 37 teeth. By multiplying these by 2 and 30, the following set of gears are obtained: E = 32 teeth, F = 90 teeth, G = 30 teeth, and H = 74 teeth. (See Article 74.)

285. Solution of the feed to be used with the above gears.

While the hob makes 13 revolutions, the imaginary simple gear H′ will make $13\times\frac{80}{32}\times\frac{40}{20}\times\frac{24}{120}$ or 13 revolutions. Therefore, when E = 32, F = 90, G = 30, and H = 74, the table will make $13\times\frac{90}{32}\times\frac{74}{30}\times\frac{1}{90}$ or $\frac{481}{480}$ revolutions, while the hob makes 13 revolutions. In other words, the gear blank makes $\frac{481}{480}$ revolutions while revolving from one intersecting point of the spirals to the next; or the part of a complete circle between the intersecting points of the spirals is $\frac{481}{480}-1$ or $\frac{1}{480}$.

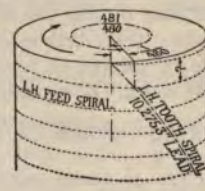


Fig. 485

If $\frac{1}{480}$ is the part of a complete circle between the intersecting points, and if the lead of the tooth spiral is 10.2753 inches, the distance of one turn of the feed spiral will be 10.2753 ÷ (480 + 1) or .021362 of an inch. (See Article 277, Problem 2.)

286. Solution of the feed gears.

When the gears A, B, C, and D all have the same number of teeth, the head carrying the hob will feed $90\times\frac{20}{80}\times\frac{1}{7}\frac{1}{2}\times\frac{2}{5}$ or .125 of an inch; therefore, the gear ratio for a .021362 of an inch feed will be $\frac{.125}{.021362}$ or $\frac{125000}{21362}$. The factors of these numbers are too

large for compound gears; therefore, the numbers are reduced by the principle of continued fractions to other numbers for which suitable factors may be obtained.

Thus,

21362	125000	5
18190	106810	1
3172	18190	5
2330	15860	1
842	2330	2
646	1684	1
	646	

		5	1	5	1	2	1
1/0	0/1	1/5	1/6	6/35	7/41	20/117	27/158
		5	1	5	1	2	1

When reducing the numbers in this way, stop at a convergent where the larger number of the convergent is between the limits of 100 and 500. It will be close enough ordinarily and more readily factored.

Since $\frac{125000}{21362}$ = approximately $\frac{158}{27}$, A′ = 158 teeth, and D′ = 27 teeth. After factoring (158 = 2×79; 27 = 1×27) and placing in proper compound order, A = 2 teeth, B = 1 tooth, C = 79 teeth, and D = 27 teeth. If we multiply A and B by 30, a better set of gears is obtained. Thus, A = 60 teeth, B = 30 teeth, C = 79 teeth, and D = 27 teeth. (See Article 74.) This set of gears will feed the hob $.125\times\frac{30\times27}{60\times79}$ or .0213607 of an inch. If one space between turns of the feed spiral measures .0213607 of an inch, 481 spaces will measure 481×.0213607 or 10.2745 inches; therefore, the given index and feed gears will cut a 10.2745-inch lead, which varies only .0008 of an inch from the true lead.

EXERCISE 67

1. In Fig. 482, what feed and index gears should be used to cut a 3-diametral pitch, 54-tooth, left-hand, 23° spiral gear, if a right-hand hob with a feed of exactly .022 of an inch is used, the ratio of the worm wheel and worm being 90 instead of 60?

2. If the automatic gear-hobbing machine (Fig. 482) using a left-hand hob for cutting a 27-tooth, left-hand spiral gear were geared so that the feed gears were A, 72 teeth; B, 30 teeth; C, 50 teeth; and D, 20 teeth, and if the index gears were E, 36 teeth; F, 74 teeth; G, 24 teeth; and H, 39 teeth; what lead would the gear being cut have, the ratio of the worm wheel and worm being 90 instead of 60?

TABLE OF TRIGONOMETRIC FUNCTIONS

Sines, Cosines, Tangents, Cotangents, Secants, and Cosecants of Angles from 0° to 90°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.00000	1.0000	.00000	Infinite	1.0000	Infinite	60
1	.00029	.0000	.00029	3437.7	.0000	3437.7	59
2	.00058	.0000	.00058	1718.9	.0000	1718.9	58
3	.00087	.0000	.00087	1145.9	.0000	1145.9	57
4	.00116	.0000	.00116	859.44	.0000	859.44	56
5	.00145	1.0000	.00145	687.55	1.0000	687.55	55
6	.00174	.0000	.00174	572.96	.0000	572.96	54
7	.00204	.0000	.00204	491.11	.0000	491.11	53
8	.00233	.0000	.00233	429.72	.0000	429.72	52
9	.00262	.0000	.00262	381.97	.0000	381.97	51
10	.00291	.99999	.00291	343.77	1.0000	343.77	50
11	.00320	.99999	.00320	312.52	.0000	312.52	49
12	.00349	.99999	.00349	286.48	.0000	286.48	48
13	.00378	.99999	.00378	264.44	.0000	264.44	47
14	.00407	.99999	.00407	245.55	.0000	245.55	46
15	.00436	.99999	.00436	229.18	1.0000	229.18	45
16	.00465	.99999	.00465	214.86	.0000	214.86	44
17	.00494	.99999	.00494	202.22	.0000	202.22	43
18	.00524	.99999	.00524	190.98	.0000	190.99	42
19	.00553	.99998	.00553	180.93	.0000	180.93	41
20	.00582	.99998	.00582	171.88	1.0000	171.89	40
21	.00611	.99998	.00611	163.70	.0000	163.70	39
22	.00640	.99998	.00640	156.26	.0000	156.26	38
23	.00669	.99998	.00669	149.46	.0000	149.47	37
24	.00698	.99997	.00698	143.24	.0000	143.24	36
25	.00727	.99997	.00727	137.51	1.0000	137.51	35
26	.00756	.99997	.00756	132.22	.0000	132.22	34
27	.00785	.99997	.00785	127.32	.0000	127.32	33
28	.00814	.99997	.00814	122.77	.0000	122.78	32
29	.00843	.99996	.00844	118.54	.0000	118.54	31
30	.00873	.99996	.00873	114.59	1.0000	114.59	30
31	.00902	.99996	.00902	110.89	.0000	110.90	29
32	.00931	.99996	.00931	107.43	.0000	107.43	28
33	.00960	.99995	.00960	104.17	.0000	104.17	27
34	.00989	.99995	.00989	101.11	.0000	101.11	26
35	.01018	.99995	.01018	98.218	1.0000	98.223	25
36	.01047	.99994	.01047	95.489	.0000	95.495	24
37	.01076	.99994	.01076	92.908	.0000	92.914	23
38	.01105	.99994	.01105	90.463	.0001	90.469	22
39	.01134	.99993	.01134	88.143	.0001	88.149	21
40	.01163	.99993	.01164	85.940	1.0001	85.946	20
41	.01193	.99993	.01193	83.843	.0001	83.849	19
42	.01222	.99992	.01222	81.847	.0001	81.853	18
43	.01251	.99992	.01251	79.943	.0001	79.950	17
44	.01280	.99992	.01280	78.126	.0001	78.133	16
45	.01309	.99991	.01309	76.390	1.0001	76.396	15
46	.01338	.99991	.01338	74.729	.0001	74.736	14
47	.01367	.99991	.01367	73.139	.0001	73.146	13
48	.01396	.99990	.01396	71.615	.0001	71.622	12
49	.01425	.99990	.01425	70.153	.0001	70.160	11
50	.01454	.99989	.01454	68.750	1.0001	68.757	10
51	.01483	.99989	.01484	67.402	.0001	67.409	9
52	.01512	.99988	.01513	66.105	.0001	66.113	8
53	.01542	.99988	.01542	64.858	.0001	64.866	7
54	.01571	.99988	.01571	63.657	.0001	63.664	6
55	.01600	.99987	.01600	62.499	1.0001	62.507	5
56	.01629	.99987	.01629	61.383	.0001	61.391	4
57	.01658	.99986	.01658	60.306	.0001	60.314	3
58	.01687	.99986	.01687	59.266	.0001	59.274	2
59	.01716	.99985	.01716	58.261	.0001	58.270	1
60	.01745	.99985	.01745	57.290	1.0001	57.299	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.00000	1.0000	.00000	Infinite	1.0000	Infinite	60
1	.00029	.0000	.00029	3437.7	.0000	3437.7	59
2	.00058	.0000	.00058	1718.9	.0000	1718.9	58
3	.00087	.0000	.00087	1145.9	.0000	1145.9	57
4	.00116	.0000	.00116	859.44	.0000	859.44	56
5	.00145	1.0000	.00145	687.55	1.0000	687.55	55
6	.00174	.0000	.00174	572.96	.0000	572.96	54
7	.00204	.0000	.00204	491.11	.0000	491.11	53
8	.00233	.0000	.00233	429.72	.0000	429.72	52
9	.00262	.0000	.00262	381.97	.0000	381.97	51
10	.00291	.99999	.00291	343.77	1.0000	343.77	50
11	.00320	.99999	.00320	312.52	.0000	312.52	49
12	.00349	.99999	.00349	286.48	.0000	286.48	48
13	.00378	.99999	.00378	264.44	.0000	264.44	47
14	.00407	.99999	.00407	245.55	.0000	245.55	46
15	.00436	.99999	.00436	229.18	1.0000	229.18	45
16	.00465	.99999	.00465	214.86	.0000	214.86	44
17	.00494	.99999	.00494	202.22	.0000	202.22	43
18	.00524	.99999	.00524	190.98	.0000	190.99	42
19	.00553	.99998	.00553	180.93	.0000	180.93	41
20	.00582	.99998	.00582	171.88	1.0000	171.89	40
21	.00611	.99998	.00611	163.70	.0000	163.70	39
22	.00640	.99998	.00640	156.26	.0000	156.26	38
23	.00669	.99998	.00669	149.46	.0000	149.47	37
24	.00698	.99997	.00698	143.24	.0000	143.24	36
25	.00727	.99997	.00727	137.51	1.0000	137.51	35
26	.00756	.99997	.00756	132.22	.0000	132.22	34
27	.00785	.99997	.00785	127.32	.0000	127.32	33
28	.00814	.99997	.00814	122.77	.0000	122.78	32
29	.00843	.99996	.00844	118.54	.0000	118.54	31
30	.00873	.99996	.00873	114.59	1.0000	114.59	30
31	.00902	.99996	.00902	110.89	.0000	110.90	29
32	.00931	.99996	.00931	107.43	.0000	107.43	28
33	.00960	.99995	.00960	104.17	.0000	104.17	27
34	.00989	.99995	.00989	101.11	.0000	101.11	26
35	.01018	.99995	.01018	98.218	1.0000	98.223	25
36	.01047	.99994	.01047	95.489	.0000	95.495	24
37	.01076	.99994	.01076	92.908	.0000	92.914	23
38	.01105	.99994	.01105	90.463	.0001	90.469	22
39	.01134	.99993	.01134	88.143	.0001	88.149	21
40	.01163	.99993	.01164	85.940	1.0001	85.946	20
41	.01193	.99993	.01193	83.843	.0001	83.849	19
42	.01222	.99992	.01222	81.847	.0001	81.853	18
43	.01251	.99992	.01251	79.943	.0001	79.950	17
44	.01280	.99992	.01280	78.126	.0001	78.133	16
45	.01309	.99991	.01309	76.390	1.0001	76.396	15
46	.01338	.99991	.01338	74.729	.0001	74.736	14
47	.01367	.99991	.01367	73.139	.0001	73.146	13
48	.01396	.99990	.01396	71.615	.0001	71.622	12
49	.01425	.99990	.01425	70.153	.0001	70.160	11
50	.01454	.99989	.01454	68.750	1.0001	68.757	10
51	.01483	.99989	.01484	67.402	.0001	67.409	9
52	.01512	.99988	.01513	66.105	.0001	66.113	8
53	.01542	.99988	.01542	64.858	.0001	64.866	7
54	.01571	.99988	.01571	63.657	.0001	63.664	6
55	.01600	.99987	.01600	62.499	1.0001	62.507	5
56	.01629	.99987	.01629	61.383	.0001	61.391	4
57	.01658	.99986	.01658	60.306	.0001	60.314	3
58	.01687	.99986	.01687	59.266	.0001	59.274	2
59	.01716	.99985	.01716	58.261	.0001	58.270	1
60	.01745	.99985	.01745	57.290	1.0001	57.299	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.01745	.99985	.01745	57.290	1.0001	57.299	60
1	.01774	.99984	.01775	56.350	.0001	56.359	59
2	.01803	.99984	.01804	55.441	.0001	55.450	58
3	.01832	.99983	.01833	54.561	.0002	54.570	57
4	.01861	.99983	.01862	53.708	.0002	53.718	56
5	.01891	.99982	.01891	52.882	1.0002	52.891	55
6	.01920	.99981	.01920	52.081	.0002	52.090	54
7	.01949	.99981	.01949	51.303	.0002	51.313	53
8	.01978	.99980	.01978	50.548	.0002	50.558	52
9	.02007	.99980	.02007	49.816	.0002	49.826	51
10	.02036	.99979	.02036	49.104	1.0002	49.114	50
11	.02065	.99979	.02066	48.412	.0002	48.422	49
12	.02094	.99978	.02095	47.739	.0002	47.750	48
13	.02123	.99977	.02124	47.085	.0002	47.096	47
14	.02152	.99977	.02153	46.449	.0002	46.460	46
15	.02181	.99976	.02182	45.829	1.0002	45.840	45
16	.02210	.99975	.02211	45.226	.0002	45.237	44
17	.02240	.99975	.02240	44.638	.0002	44.650	43
18	.02269	.99974	.02269	44.066	.0002	44.077	42
19	.02298	.99974	.02298	43.508	.0003	43.520	41
20	.02326	.99973	.02327	42.964	1.0003	42.976	40
21	.02356	.99972	.02357	42.433	.0003	42.445	39
22	.02385	.99971	.02386	41.916	.0003	41.928	38
23	.02414	.99971	.02415	41.410	.0003	41.423	37
24	.02443	.99970	.02444	40.917	.0003	40.930	36
25	.02472	.99969	.02473	40.436	1.0003	40.448	35
26	.02501	.99969	.02502	39.965	.0003	39.978	34
27	.02530	.99968	.02531	39.506	.0003	39.518	33
28	.02559	.99967	.02560	39.057	.0003	39.069	32
29	.02589	.99966	.02589	38.618	.0003	38.631	31
30	.02618	.99966	.02618	38.188	1.0003	38.201	30
31	.02647	.99965	.02648	37.769	.0003	37.782	29
32	.02676	.99964	.02677	37.358	.0003	37.371	28
33	.02705	.99963	.02706	36.956	.0004	36.969	27
34	.02734	.99963	.02735	36.563	.0004	36.576	26
35	.02763	.99962	.02764	36.177	1.0004	36.191	25
36	.02792	.99961	.02793	35.800	.0004	35.814	24
37	.02821	.99960	.02822	35.431	.0004	35.445	23
38	.02850	.99959	.02851	35.069	.0004	35.084	22
39	.02879	.99958	.02880	34.715	.0004	34.729	21
40	.02908	.99958	.02910	34.368	1.0004	34.382	20
41	.02937	.99957	.02939	34.027	.0004	34.042	19
42	.02967	.99956	.02968	33.693	.0004	33.708	18
43	.02996	.99955	.02997	33.366	.0004	33.381	17
44	.03025	.99954	.03026	33.045	.0004	33.060	16
45	.03054	.99953	.03055	32.730	1.0005	32.745	15
46	.03083	.99952	.03084	32.421	.0005	32.437	14
47	.03112	.99951	.03113	32.118	.0005	32.134	13
48	.03141	.99951	.03143	31.820	.0005	31.836	12
49	.03170	.99950	.03172	31.528	.0005	31.544	11
50	.03199	.99949	.03201	31.241	1.0005	31.257	10
51	.03228	.99948	.03230	30.960	.0005	30.976	9
52	.03257	.99947	.03259	30.683	.0005	30.699	8
53	.03286	.99946	.03288	30.411	.0005	30.428	7
54	.03315	.99945	.03317	30.145	.0005	30.161	6
55	.03344	.99944	.03346	29.882	1.0005	29.899	5
56	.03374	.99943	.03375	29.624	.0006	29.641	4
57	.03403	.99942	.03405	29.371	.0006	29.388	3
58	.03432	.99941	.03434	29.122	.0006	29.139	2
59	.03461	.99940	.03463	28.877	.0006	28.894	1
60	.03490	.99939	.03492	28.636	1.0006	28.654	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.03490	.99939	.03492	28.636	1.0006	28.654	60
1	.03519	.99938	.03521	28.399	.0006	28.417	59
2	.03548	.99937	.03550	28.166	.0006	28.184	58
3	.03577	.99936	.03579	27.937	.0006	27.955	57
4	.03606	.99935	.03608	27.712	.0006	27.730	56
5	.03635	.99934	.03638	27.490	1.0007	27.508	55
6	.03664	.99933	.03667	27.271	.0007	27.290	54
7	.03693	.99932	.03696	27.056	.0007	27.075	53
8	.03722	.99931	.03725	26.845	.0007	26.864	52
9	.03751	.99930	.03754	26.637	.0007	26.655	51
10	.03781	.99928	.03783	26.432	1.0007	26.450	50
11	.03810	.99927	.03812	26.230	.0007	26.249	49
12	.03839	.99926	.03842	26.031	.0007	26.050	48
13	.03868	.99925	.03871	25.835	.0007	25.854	47
14	.03897	.99924	.03900	25.642	.0008	25.661	46
15	.03926	.99923	.03929	25.452	1.0008	25.471	45
16	.03955	.99922	.03958	25.264	.0008	25.284	44
17	.03984	.99921	.03987	25.080	.0008	25.100	43
18	.04013	.99919	.04016	24.898	.0008	24.918	42
19	.04042	.99918	.04045	24.718	.0008	24.739	41
20	.04071	.99917	.04075	24.542	1.0008	24.562	40
21	.04100	.99916	.04104	24.367	.0008	24.388	39
22	.04129	.99915	.04133	24.196	.0008	24.216	38
23	.04158	.99913	.04162	24.026	.0009	24.047	37
24	.04187	.99912	.04191	23.859	.0009	23.880	36
25	.04217	.99911	.04220	23.694	1.0009	23.716	35
26	.04246	.99910	.04249	23.532	.0009	23.553	34
27	.04275	.99908	.04279	23.372	.0009	23.393	33
28	.04304	.99907	.04308	23.214	.0009	23.235	32
29	.04333	.99906	.04337	23.058	.0009	23.079	31
30	.04362	.99905	.04366	22.904	1.0009	22.925	30
31	.04391	.99903	.04395	22.752	.0010	22.774	29
32	.04420	.99902	.04424	22.602	.0010	22.624	28
33	.04449	.99901	.04453	22.454	.0010	22.476	27
34	.04478	.99900	.04483	22.308	.0010	22.330	26
35	.04507	.99898	.04512	22.164	1.0010	22.186	25
36	.04536	.99897	.04541	22.022	.0010	22.044	24
37	.04565	.99896	.04570	21.881	.0010	21.904	23
38	.04594	.99894	.04599	21.742	.0010	21.765	22
39	.04623	.99893	.04628	21.606	.0011	21.629	21
40	.04652	.99892	.04657	21.470	1.0011	21.494	20
41	.04681	.99890	.04687	21.337	.0011	21.360	19
42	.04711	.99889	.04716	21.205	.0011	21.228	18
43	.04740	.99888	.04745	21.075	.0011	21.098	17
44	.04769	.99886	.04774	20.946	.0011	20.970	16
45	.04798	.99885	.04803	20.819	1.0011	20.843	15
46	.04827	.99883	.04832	20.693	.0012	20.717	14
47	.04856	.99882	.04862	20.569	.0012	20.593	13
48	.04885	.99881	.04891	20.446	.0012	20.471	12
49	.04914	.99879	.04920	20.325	.0012	20.350	11
50	.04943	.99878	.04949	20.205	1.0012	20.230	10
51	.04972	.99876	.04978	20.087	.0012	20.112	9
52	.05001	.99875	.05007	19.970	.0012	19.995	8
53	.05030	.99873	.05037	19.854	.0013	19.880	7
54	.05059	.99872	.05066	19.740	.0013	19.766	6
55	.05088	.99870	.05095	19.627	1.0013	19.653	5
56	.05117	.99869	.05124	19.515	.0013	19.541	4
57	.05146	.99867	.05153	19.405	.0013	19.431	3
58	.05175	.99866	.05182	19.296	.0013	19.322	2
59	.05204	.99864	.05212	19.188	.0013	19.214	1
60	05234	.99863	.05241	19.081	1.0014	19.107	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.05234	.99863	.05241	19.081	1.0014	19.107	60
1	.05263	.99861	.05270	18.975	.0014	19.002	59
2	.05292	.99860	.05299	18.871	.0014	18.897	58
3	.05321	.99858	.05328	18.768	.0014	18.794	57
4	.05350	.99857	.05357	18.665	.0014	18.692	56
5	.05379	.99855	.05387	18.564	1.0014	18.591	55
6	.05408	.99854	.05416	18.464	.0015	18.491	54
7	.05437	.99852	.05445	18.365	.0015	18.393	53
8	.05466	.99850	.05474	18.268	.0015	18.295	52
9	.05495	.99849	.05503	18.171	.0015	18.198	51
10	.05524	.99847	.05532	18.075	1.0015	18.103	50
11	.05553	.99846	.05562	17.980	.0015	18.008	49
12	.05582	.99844	.05591	17.886	.0016	17.914	48
13	.05611	.99842	.05620	17.793	.0016	17.821	47
14	.05640	.99841	.05649	17.701	.0016	17.730	46
15	.05669	.99839	.05678	17.610	1.0016	17.639	45
16	.05698	.99837	.05707	17.520	.0016	17.549	44
17	.05727	.99836	.05737	17.431	.0016	17.460	43
18	.05756	.99834	.05766	17.343	.0017	17.372	42
19	.05785	.99832	.05795	17.256	.0017	17.285	41
20	.05814	.99831	.05824	17.169	1.0017	17.198	40
21	.05843	.99829	.05853	17.084	.0017	17.113	39
22	.05872	.99827	.05883	16.999	.0017	17.028	38
23	.05902	.99826	.05912	16.915	.0017	16.944	37
24	.05931	.99824	.05941	16.832	.0018	16.861	36
25	.05960	.99822	.05970	16.750	1.0018	16.779	35
26	.05989	.99820	.05999	16.668	.0018	16.698	34
27	.06018	.99819	.06029	16.587	.0018	16.617	33
28	.06047	.99817	.06058	16.507	.0018	16.538	32
29	.06076	.99815	.06087	16.428	.0018	16.459	31
30	.06105	.99813	.06116	16.350	1.0019	16.380	30
31	.06134	.99812	.06145	16.272	.0019	16.303	29
32	.06163	.99810	.06175	16.195	.0019	16.226	28
33	.06192	.99808	.06204	16.119	.0019	16.150	27
34	.06221	.99806	.06233	16.043	.0019	16.075	26
35	.06250	.99804	.06262	15.969	1.0019	16.000	25
36	.06279	.99803	.06291	15.894	.0020	15.926	24
37	.06308	.99801	.06321	15.821	.0020	15.853	23
38	.06337	.99799	.06350	15.748	.0020	15.780	22
39	.06366	.99797	.06379	15.676	.0020	15.708	21
40	.06395	.99795	.06408	15.605	1.0020	15.637	20
41	.06424	.99793	.06437	15.534	.0021	15.566	19
42	.06453	.99791	.06467	15.464	.0021	15.496	18
43	.06482	.99790	.06496	15.394	.0021	15.427	17
44	.06511	.99788	.06525	15.325	.0021	15.358	16
45	.06540	.99786	.06554	15.257	1.0021	15.290	15
46	.06569	.99784	.06583	15.189	.0022	15.222	14
47	.06598	.99782	.06613	15.122	.0022	15.155	13
48	.06627	.99780	.06642	15.056	.0022	15.089	12
49	.06656	.99778	.06671	14.990	.0022	15.023	11
50	.06685	.99776	.06700	14.924	1.0022	14.958	10
51	.06714	.99774	.06730	14.860	.0023	14.893	9
52	.06743	.99772	.06759	14.795	.0023	14.829	8
53	.06772	.99770	.06788	14.732	.0023	14.765	7
54	.06801	.99768	.06817	14.668	.0023	14.702	6
55	.06830	.99766	.06846	14.606	1.0023	14.640	5
56	.06859	.99764	.06876	14.544	.0024	14.578	4
57	.06888	.99762	.06905	14.482	.0024	14.517	3
58	.06918	.99760	.06934	14.421	.0024	14.456	2
59	.06947	.99758	.06963	14.361	.0024	14.395	1
60	.06976	.99756	.06993	14.301	1.0024	14.335	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.06976	.99756	.06993	14.301	1.0024	14.335	60
1	.07005	.99754	.07022	14.241	.0025	14.276	59
2	.07034	.99752	.07051	14.182	.0025	14.217	58
3	.07063	.99750	.07080	14.123	.0025	14.159	57
4	.07092	.99748	.07110	14.065	.0025	14.101	56
5	.07121	.99746	.07139	14.008	1.0025	14.043	55
6	.07150	.99744	.07168	13.951	.0026	13.986	54
7	.07179	.99742	.07197	13.894	.0026	13.930	53
8	.07208	.99740	.07226	13.838	.0026	13.874	52
9	.07237	.99738	.07256	13.782	.0026	13.818	51
10	.07266	.99736	.07285	13.727	1.0026	13.763	50
11	.07295	.99733	.07314	13.672	.0027	13.708	49
12	.07324	.99731	.07343	13.617	.0027	13.654	48
13	.07353	.99729	.07373	13.563	.0027	13.600	47
14	.07382	.99727	.07402	13.510	.0027	13.547	46
15	.07411	.99725	.07431	13.457	1.0027	13.494	45
16	.07440	.99723	.07460	13.404	.0028	13.441	44
17	.07469	.99721	.07490	13.351	.0028	13.389	43
18	.07498	.99718	.07519	13.299	.0028	13.337	42
19	.07527	.99716	.07548	13.248	.0028	13.286	41
20	.07556	.99714	.07577	13.197	1.0029	13.235	40
21	.07585	.99712	.07607	13.146	.0029	13.184	39
22	.07614	.99710	.07636	13.096	.0029	13.134	38
23	.07643	.99707	.07665	13.046	.0029	13.084	37
24	.07672	.99705	.07694	12.996	.0029	13.034	36
25	.07701	.99703	.07724	12.947	1.0030	12.985	35
26	.07730	.99701	.07753	12.898	.0030	12.937	34
27	.07759	.99698	.07782	12.849	.0030	12.888	33
28	.07788	.99696	.07812	12.801	.0030	12.840	32
29	.07817	.99694	.07841	12.754	.0031	12.793	31
30	.07846	.99692	.07870	12.706	1.0031	12.745	30
31	.07875	.99689	.07899	12.659	.0031	12.698	29
32	.07904	.99687	.07929	12.612	.0031	12.652	28
33	.07933	.99685	.07958	12.566	.0032	12.606	27
34	.07962	.99682	.07987	12.520	.0032	12.560	26
35	.07991	.99680	.08016	12.474	1.0032	12.514	25
36	.08020	.99678	.08046	12.429	.0032	12.469	24
37	.08049	.99675	.08075	12.384	.0032	12.424	23
38	.08078	.99673	.08104	12.339	.0033	12.379	22
39	.08107	.99671	.08134	12.295	.0033	12.335	21
40	.08136	.99668	.08163	12.250	1.0033	12.291	20
41	.08165	.99666	.08192	12.207	.0033	12.248	19
42	.08194	.99664	.08221	12.163	.0034	12.204	18
43	.08223	.99661	.08251	12.120	.0034	12.161	17
44	.08252	.99659	.08280	12.077	.0034	12.118	16
45	.08281	.99656	.08309	12.035	1.0034	12.076	15
46	.08310	.99654	.08339	11.992	.0035	12.034	14
47	.08339	.99652	.08368	11.950	.0035	11.992	13
48	.08368	.99649	.08397	11.909	.0035	11.950	12
49	.08397	.99647	.08426	11.867	.0035	11.909	11
50	.08426	.99644	.08456	11.826	1.0036	11.868	10
51	.08455	.99642	.08485	11.785	.0036	11.828	9
52	.08484	.99639	.08514	11.745	.0036	11.787	8
53	.08513	.99637	.08544	11.704	.0036	11.747	7
54	.08542	.99634	.08573	11.664	.0037	11.707	6
55	.08571	.99632	.08602	11.625	1.0037	11.668	5
56	.08600	.99629	.08632	11.585	.0037	11.628	4
57	.08629	.99627	.08661	11.546	.0037	11.589	3
58	.08658	.99624	.08690	11.507	.0038	11.550	2
59	.08687	.99622	.08719	11.468	.0038	11.512	1
60	.08715	.99619	.08749	11.430	1.0038	11.474	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.08715	.99619	.08749	11.430	1.0038	11.474	60
1	.08744	.99617	.08778	11.392	.0038	11.436	59
2	.08773	.99614	.08807	11.354	.0039	11.398	58
3	.08802	.99612	.08837	11.316	.0039	11.360	57
4	.08831	.99609	.08866	11.279	.0039	11.323	56
5	.08860	.99607	.08895	11.242	1.0039	11.286	55
6	.08889	.99604	.08925	11.205	.0040	11.249	54
7	.08918	.99601	.08954	11.168	.0040	11.213	53
8	.08947	.99599	.08983	11.132	.0040	11.176	52
9	.08976	.99596	.09013	11.095	.0040	11.140	51
10	.09005	.99594	.09042	11.059	1.0041	11.104	50
11	.09034	.99591	.09071	11.024	.0041	11.069	49
12	.09063	.99588	.09101	10.988	.0041	11.033	48
13	.09092	.99586	.09130	10.953	.0041	10.998	47
14	.09121	.99583	.09159	10.918	.0042	10.963	46
15	.09150	.99580	.09189	10.883	1.0042	10.929	45
16	.09179	.99578	.09218	10.848	.0042	10.894	44
17	.09208	.99575	.09247	10.814	.0043	10.860	43
18	.09237	.99572	.09277	10.780	.0043	10.826	42
19	.09266	.99570	.09306	10.746	.0043	10.792	41
20	.09295	.99567	.09335	10.712	1.0043	10.758	40
21	.09324	.99564	.09365	10.678	.0044	10.725	39
22	.09353	.99562	.09394	10.645	.0044	10.692	38
23	.09382	.99559	.09423	10.612	.0044	10.659	37
24	.09411	.99556	.09453	10.579	.0044	10.626	36
25	.09440	.99553	.09482	10.546	1.0045	10.593	35
26	.09469	.99551	.09511	10.514	.0045	10.561	34
27	.09498	.99548	.09541	10.481	.0045	10.529	33
28	.09527	.99545	.09570	10.449	.0046	10.497	32
29	.09556	.99542	.09599	10.417	.0046	10.465	31
30	.09584	.99540	.09629	10.385	1.0046	10.433	30
31	.09613	.99537	.09658	10.354	.0046	10.402	29
32	.09642	.99534	.09688	10.322	.0047	10.371	28
33	.09671	.99531	.09717	10.291	.0047	10.340	27
34	.09700	.99528	.09746	10.260	.0047	10.309	26
35	.09729	.99525	.09776	10.229	1.0048	10.278	25
36	.09758	.99523	.09805	10.199	.0048	10.248	24
37	.09787	.99520	.09834	10.168	.0048	10.217	23
38	.09816	.99517	.09864	10.138	.0048	10.187	22
39	.09845	.99514	.09893	10.108	.0049	10.157	21
40	.09874	.99511	.09922	10.078	1.0049	10.127	20
41	.09903	.99508	.09952	10.048	.0049	10.098	19
42	.09932	.99505	.09981	10.019	.0050	10.068	18
43	.09961	.99503	.10011	9.9893	.0050	10.039	17
44	.09990	.99500	.10040	9.9601	.0050	10.010	16
45	.10019	.99497	.10069	9.9310	1.0050	9.9812	15
46	.10048	.99494	.10099	9.9021	.0051	9.9525	14
47	.10077	.99491	.10128	9.8734	.0051	9.9239	13
48	.10106	.99488	.10158	9.8448	.0051	9.8955	12
49	.10134	.99485	.10187	9.8164	.0052	9.8672	11
50	.10163	.99482	.10216	9.7882	1.0052	9.8391	10
51	.10192	.99479	.10246	9.7601	.0052	9.8112	9
52	.10221	.99476	.10275	9.7322	.0053	9.7834	8
53	.10250	.99473	.10305	9.7044	.0053	9.7558	7
54	.10279	.99470	.10334	9.6768	.0053	9.7283	6
55	.10308	.99467	.10363	9.6493	1.0053	9.7010	5
56	.10337	.99464	.10393	9.6220	.0054	9.6739	4
57	.10366	.99461	.10422	9.5949	.0054	9.6469	3
58	.10395	.99458	.10452	9.5679	.0054	9.6200	2
59	.10424	.99455	.10481	9.5411	.0055	9.5933	1
60	.10453	.99452	.10510	9.5144	1.0055	9.5668	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.10453	.99452	.10510	9.5144	1.0055	9.5668	60
1	.10482	.99449	.10540	.4878	.0055	.5404	59
2	.10511	.99446	.10569	.4614	.0056	.5141	58
3	.10540	.99443	.10599	.4351	.0056	.4880	57
4	.10568	.99440	.10628	.4090	.0056	.4620	56
5	.10597	.99437	.10657	9.3831	1.0057	9.4362	55
6	.10626	.99434	.10687	.3572	.0057	.4105	54
7	.10655	.99431	.10716	.3315	.0057	.3850	53
8	.10684	.99428	.10746	.3060	.0057	.3596	52
9	.10713	.99424	.10775	.2806	.0058	.3343	51
10	.10742	.99421	.10805	9.2553	1.0058	9.3092	50
11	.10771	.99418	.10834	.2302	.0058	.2842	49
12	.10800	.99415	.10863	.2051	.0059	.2593	48
13	.10829	.99412	.10893	.1803	.0059	.2346	47
14	.10858	.99409	.10922	.1555	.0059	.2100	46
15	.10887	.99406	.10952	9.1309	1.0060	9.1855	45
16	.10916	.99402	.10981	.1064	.0060	.1612	44
17	.10944	.99399	.11011	.0821	.0060	.1370	43
18	.10973	.99396	.11040	.0579	.0061	.1129	42
19	.11002	.99393	.11069	.0338	.0061	.0890	41
20	.11031	.99390	.11099	9.0098	1.0061	9.0651	40
21	.11060	.99386	.11128	8.9860	.0062	.0414	39
22	.11089	.99383	.11158	.9623	.0062	.0179	38
23	.11118	.99380	.11187	.9387	.0062	8.9944	37
24	.11147	.99377	.11217	.9152	.0063	.9711	36
25	.11176	.99373	.11246	8.8918	1.0063	8.9479	35
26	.11205	.99370	.11276	.8686	.0063	.9248	34
27	.11234	.99367	.11305	.8455	.0064	.9018	33
28	.11262	.99364	.11335	.8225	.0064	.8790	32
29	.11291	.99360	.11364	.7996	.0064	.8563	31
30	.11320	.99357	.11393	8.7769	1.0065	8.8337	30
31	.11349	.99354	.11423	.7542	.0065	.8112	29
32	.11378	.99350	.11452	.7317	.0065	.7888	28
33	.11407	.99347	.11482	.7093	.0066	.7665	27
34	.11436	.99344	.11511	.6870	.0066	.7444	26
35	.11465	.99341	.11541	8.6648	1.0066	8.7223	25
36	.11494	.99337	.11570	.6427	.0067	.7004	24
37	.11523	.99334	.11600	.6208	.0067	.6786	23
38	.11551	.99330	.11629	.5989	.0067	.6569	22
39	.11580	.99327	.11659	.5772	.0068	.6353	21
40	.11609	.99324	.11688	8.5555	1.0068	8.6138	20
41	.11638	.99320	.11718	.5340	.0068	.5924	19
42	.11667	.99317	.11747	.5126	.0069	.5711	18
43	.11696	.99314	.11777	.4913	.0069	.5499	17
44	.11725	.99310	.11806	.4701	.0069	.5289	16
45	.11754	.99307	.11836	8.4489	1.0070	8.5079	15
46	.11783	.99303	.11865	.4279	.0070	.4871	14
47	.11811	.99300	.11895	.4070	.0070	.4663	13
48	.11840	.99296	.11924	.3862	.0071	.4457	12
49	.11869	.99293	.11954	.3655	.0071	.4251	11
50	.11898	.99290	.11983	8.3449	1.0071	8.4046	10
51	.11927	.99286	.12013	.3244	.0072	.3843	9
52	.11956	.99283	.12042	.3040	.0072	.3640	8
53	.11985	.99279	.12072	.2837	.0073	.3439	7
54	.12014	.99276	.12101	.2635	.0073	.3238	6
55	.12042	.99272	.12131	8.2434	1.0073	8.3039	5
56	.12071	.99269	.12160	.2234	.0074	.2840	4
57	.12100	.99265	.12190	.2035	.0074	.2642	3
58	.12129	.99262	.12219	.1837	.0074	.2446	2
59	.12158	.99258	.12249	.1640	.0075	.2250	1
60	.12187	.99255	.12278	8.1443	1.0075	8.2055	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.12187	.99255	.12278	8.1443	1.0075	8.2055	60
1	.12216	.99251	.12308	.1248	.0075	.1861	59
2	.12245	.99247	.12337	.1053	.0076	.1668	58
3	.12273	.99244	.12367	.0860	.0076	.1476	57
4	.12302	.99240	.12396	.0667	.0076	.1285	56
5	.12331	.99237	.12426	8.0476	1.0077	8.1094	55
6	.12360	.99233	.12456	.0285	.0077	.0905	54
7	.12389	.99229	.12485	.0095	.0078	.0717	53
8	.12418	.99226	.12515	7.9906	.0078	.0529	52
9	.12447	.99222	.12544	.9717	.0078	.0342	51
10	.12476	.99219	.12574	7.9530	1.0079	8.0156	50
11	.12504	.99215	.12603	.9344	.0079	7.9971	49
12	.12533	.99211	.12633	.9158	.0079	.9787	48
13	.12562	.99208	.12662	.8973	.0080	.9604	47
14	.12591	.99204	.12692	.8789	.0080	.9421	46
15	.12620	.99200	.12722	7.8606	1.0080	7.9240	45
16	.12649	.99197	.12751	.8424	.0081	.9059	44
17	.12678	.99193	.12781	.8243	.0081	.8879	43
18	.12706	.99189	.12810	.8062	.0082	.8700	42
19	.12735	.99186	.12840	.7882	.0082	.8522	41
20	.12764	.99182	.12869	7.7703	1.0082	7.8344	40
21	.12793	.99178	.12899	.7525	.0083	.8168	39
22	.12822	.99174	.12928	.7348	.0083	.7992	38
23	.12851	.99171	.12958	.7171	.0084	.7817	37
24	.12879	.99167	.12988	.6996	.0084	.7642	36
25	.12908	.99163	.13017	7.6821	1.0084	7.7469	35
26	.12937	.99160	.13047	.6646	.0085	.7296	34
27	.12966	.99156	.13076	.6473	.0085	.7124	33
28	.12995	.99152	.13106	.6300	.0085	.6953	32
29	.13024	.99148	.13136	.6129	.0086	.6783	31
30	.13053	.99144	.13165	7.5957	1.0086	7.6613	30
31	.13081	.99141	.13195	.5787	.0087	.6444	29
32	.13110	.99137	.13224	.5617	.0087	.6276	28
33	.13139	.99133	.13254	.5449	.0087	.6108	27
34	.13168	.99129	.13284	.5280	.0088	.5942	26
35	.13197	.99125	.13313	7.5113	1.0088	7.5776	25
36	.13226	.99121	.13343	.4946	.0089	.5611	24
37	.13254	.99118	.13372	.4780	.0089	.5446	23
38	.13283	.99114	.13402	.4615	.0089	.5282	22
39	.13312	.99110	.13432	.4451	.0090	.5119	21
40	.13341	.99106	.13461	7.4287	1.0090	7.4957	20
41	.13370	.99102	.13491	.4124	.0090	.4795	19
42	.13399	.99098	.13520	.3961	.0091	.4634	18
43	.13427	.99094	.13550	.3800	.0091	.4474	17
44	.13456	.99090	.13580	.3639	.0092	.4315	16
45	.13485	.99086	.13609	7.3479	1.0092	7.4156	15
46	.13514	.99083	.13639	.3319	.0092	.3998	14
47	.13543	.99079	.13669	.3160	.0093	.3840	13
48	.13571	.99075	.13698	.3002	.0093	.3683	12
49	.13600	.99071	.13728	.2844	.0094	.3527	11
50	.13629	.99067	.13757	7.2687	1.0094	7.3372	10
51	.13658	.99063	.13787	.2531	.0094	.3217	9
52	.13687	.99059	.13817	.2375	.0095	.3063	8
53	.13716	.99055	.13846	.2220	.0095	.2909	7
54	.13744	.99051	.13876	.2066	.0096	.2757	6
55	.13773	.99047	.13906	7.1912	1.0096	7.2604	5
56	.13802	.99043	.13935	.1759	.0097	.2453	4
57	.13831	.99039	.13965	.1607	.0097	.2302	3
58	.13860	.99035	.13995	.1455	.0097	.2152	2
59	.13888	.99031	.14024	.1304	.0098	.2002	1
60	.13917	.99027	.14054	7.1154	1.0098	7.1853	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.13917	.99027	.14054	7.1154	1.0098	7.1853	60
1	.13946	.99023	.14084	.1004	.0099	.1704	59
2	.13975	.99019	.14113	.0854	.0099	.1557	58
3	.14004	.99015	.14143	.0706	.0099	.1409	57
4	.14032	.99010	.14173	.0558	.0100	.1263	56
5	.14061	.99006	.14202	7.0410	1.0100	7.1117	55
6	.14090	.99002	.14232	.0264	.0101	.0972	54
7	.14119	.98998	.14262	.0117	.0101	.0827	53
8	.14148	.98994	.14291	6.9972	.0102	.0683	52
9	.14176	.98990	.14321	.9827	.0102	.0539	51
10	.14205	.98986	.14351	6.9682	1.0102	7.0396	50
11	.14234	.98982	.14380	.9538	.0103	.0254	49
12	.14263	.98978	.14410	.9395	.0103	.0112	48
13	.14292	.98973	.14440	.9252	.0104	6.9971	47
14	.14320	.98969	.14470	.9110	.0104	.9830	46
15	.14349	.98965	.14499	6.8969	1.0104	6.9690	45
16	.14378	.98961	.14529	.8828	.0105	.9550	44
17	.14407	.98957	.14559	.8687	.0105	.9411	43
18	.14436	.98952	.14588	.8547	.0106	.9273	42
19	.14464	.98948	.14618	.8408	.0106	.9135	41
20	.14493	.98944	.14648	6.8269	1.0107	6.8998	40
21	.14522	.98940	.14677	.8131	.0107	.8861	39
22	.14551	.98936	.14707	.7993	.0107	.8725	38
23	.14579	.98931	.14737	.7856	.0108	.8589	37
24	.14608	.98927	.14767	.7720	.0108	.8454	36
25	.14637	.98923	.14796	6.7584	1.0109	6.8320	35
26	.14666	.98919	.14826	.7448	.0109	.8185	34
27	.14695	.98914	.14856	.7313	.0110	.8052	33
28	.14723	.98910	.14886	.7179	.0110	.7919	32
29	.14752	.98906	.14915	.7045	.0111	.7787	31
30	.14781	.98901	.14945	6.6911	1.0111	6.7655	30
31	.14810	.98897	.14975	.6779	.0111	.7523	29
32	.14838	.98893	.15004	.6646	.0112	.7392	28
33	.14867	.98889	.15034	.6514	.0112	.7262	27
34	.14896	.98884	.15064	.6383	.0113	.7132	26
35	.14925	.98880	.15094	6.6252	1.0113	6.7003	25
36	.14953	.98876	.15123	.6122	.0114	.6874	24
37	.14982	.98871	.15153	.5992	.0114	.6745	23
38	.15011	.98867	.15183	.5863	.0115	.6617	22
39	.15040	.98862	.15213	.5734	.0115	.6490	21
40	.15068	.98858	.15243	6.5605	1.0115	6.6363	20
41	.15097	.98854	.15272	.5478	.0116	.6237	19
42	.15126	.98849	.15302	.5350	.0116	.6111	18
43	.15155	.98845	.15332	.5223	.0117	.5985	17
44	.15183	.98840	.15362	.5097	.0117	.5860	16
45	.15212	.98836	.15391	6.4971	1.0118	6.5736	15
46	.15241	.98832	.15421	.4845	.0118	.5612	14
47	.15270	.98827	.15451	.4720	.0119	.5488	13
48	.15298	.98823	.15481	.4596	.0119	.5365	12
49	.15328	.98818	.15511	.4472	.0119	.5243	11
50	.15356	.98814	.15540	6.4348	1.0120	6.5121	10
51	.15385	.98809	.15570	.4225	.0120	.4999	9
52	.15413	.98805	.15600	.4103	.0121	.4878	8
53	.15442	.98800	.15630	.3980	.0121	.4757	7
54	.15471	.98796	.15659	.3859	.0122	.4637	6
55	.15500	.98791	.15689	6.3737	1.0122	6.4517	5
56	.15528	.98787	.15719	.3616	.0123	.4398	4
57	.15557	.98782	.15749	.3496	.0123	.4279	3
58	.15586	.98778	.15779	.3376	.0124	.4160	2
59	.15615	.98773	.15809	.3257	.0124	.4042	1
60	.15643	.98769	.15838	6.3137	1.0125	6.3924	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.15643	.98769	.15838	6.3137	1.0125	6.3924	60
1	.15672	.98764	.15868	.3019	.0125	.3807	59
2	.15701	.98760	.15898	.2901	.0125	.3690	58
3	.15730	.98755	.15928	.2783	.0126	.3574	57
4	.15758	.98750	.15958	.2665	.0126	.3458	56
5	.15787	.98746	.15987	6.2548	1.0127	6.3343	55
6	.15816	.98741	.16017	.2432	.0127	.3228	54
7	.15844	.98737	.16047	.2316	.0128	.3113	53
8	.15873	.98732	.16077	.2200	.0128	.2999	52
9	.15902	.98727	.16107	.2085	.0129	.2885	51
10	.15931	.98723	.16137	6.1970	1.0129	6.2772	50
11	.15959	.98718	.16167	.1856	.0130	.2659	49
12	.15988	.98714	.16196	.1742	.0130	.2546	48
13	.16017	.98709	.16226	.1628	.0131	.2434	47
14	.16045	.98704	.16256	.1515	.0131	.2322	46
15	.16074	.98700	.16286	6.1402	1.0132	6.2211	45
16	.16103	.98695	.16316	.1290	.0132	.2100	44
17	.16132	.98690	.16346	.1178	.0133	.1990	43.
18	.16160	.98685	.16376	.1066	.0133	.1880	42
19	.16189	.98681	.16405	.0955	.0134	.1770	41
20	.16218	.98676	.16435	6.0844	1.0134	6.1661	40
21	.16246	.98671	.16465	.0734	.0135	.1552	39
22	.16275	.98667	.16495	.0624	.0135	.1443	38
23	.16304	.98662	.16525	.0514	.0136	.1335	37
24	.16333	.98657	.16555	.0405	.0136	.1227	36
25	.16361	.98652	.16585	6.0296	1.0136	6.1120	35
26	.16390	.98648	.16615	.0188	.0137	.1013	34
27	.16419	.98643	.16644	.0080	.0137	.0906	33
28	.16447	.98638	.16674	5.9972	.0138	.0800	32
29	.16476	.98633	.16704	.9865	.0138	.0694	31
30	.16505	.98628	.16734	5.9758	1.0139	6.0588	30
31	.16533	.98624	.16764	.9651	.0139	.0483	29
32	.16562	.98619	.16794	.9545	.0140	.0379	28
33	.16591	.98614	.16824	.9439	.0140	.0274	27
34	.16619	.98609	.16854	.9333	.0141	.0170	26
35	.16648	.98604	.16884	5.9228	1.0141	6.0066	25
36	.16677	.98600	.16914	.9123	.0142	5.9963	24
37	.16705	.98595	.16944	.9019	.0142	.9860	23
38	.16734	.98590	.16973	.8915	.0143	.9758	22
39	.16763	.98585	.17003	.8811	.0143	.9655	21
40	.16791	.98580	.17033	5.8708	1.0144	5.9554	20
41	.16820	.98575	.17063	.8605	.0144	.9452	19
42	.16849	.98570	.17093	.8502	.0145	.9351	18
43	.16878	.98565	.17123	.8400	.0145	.9250	17
44	.16906	.98560	.17153	.8298	.0146	.9150	16
45	.16935	.98556	.17183	5.8196	1.0146	5.9049	15
46	.16964	.98551	.17213	.8095	.0147	.8950	14
47	.16992	.98546	.17243	.7994	.0147	.8850	13
48	.17021	.98541	.17273	.7894	.0148	.8751	12
49	.17050	98536	.17303	. 7794	.0148	.8652	11
50	.17078	.98531	.17333	5.7694	1.0149	5.8554	10
51	.17107	.98526	.17363	.7594	.0150	.8456	9
52	.17136	.98521	.17393	.7495	.0150	.8358	8
53	.17164	.98516	.17423	.7396	.0151	.8261	7
54	.17193	.98511	.17453	.7297	.0151	.8163	6
55	.17221	.98506	.17483	5.7199	1.0152	5.8067	5
56	.17250	.98501	.17513	.7101	.0152	.7970	4
57	.17279	.98496	.17543	.7004	.0153	.7874	3
58	.17307	.98491	.17573	.6906	.0153	.7778	2
59	.17336	.98486	.17603	.6809	.0154	.7683	1
60	.17365	.98481	.17633	5.6713	1.0154	5.7588	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.17365	.98481	.17633	5.6713	1.0154	5.7588	60
1	.17393	.98476	.17663	.6616	.0155	.7493	59
2	.17422	.98471	.17693	.6520	.0155	.7398	58
3	.17451	.98465	.17723	.6425	.0156	.7304	57
4	.17479	.98460	.17753	.6329	.0156	.7210	56
5	.17508	.98455	.17783	5.6234	1.0157	5.7117	55
6	.17537	.98450	.17813	.6140	.0157	.7023	54
7	.17565	.98445	.17843	.6045	.0158	.6930	53
8	.17594	.98440	.17873	.5951	.0158	.6838	52
9	.17622	.98435	.17903	.5857	.0159	.6745	51
10	.17651	.98430	.17933	5.5764	1.0159	5.6653	50
11	.17680	.98425	.17963	.5670	.0160	.6561	49
12	.17708	.98419	.17993	.5578	.0160	.6470	48
13	.17737	.98414	.18023	.5485	.0161	.6379	47
14	.17766	.98409	.18053	.5393	.0162	.6288	46
15	.17794	.98404	.18083	5.5301	1.0162	5.6197	45
16	.17823	.98399	.18113	.5209	.0163	.6107	44
17	.17852	.98394	.18143	.5117	.0163	.6017	43
18	.17880	.98388	.18173	.5026	.0164	.5928	42
19	.17909	.98383	.18203	.4936	.0164	.5838	41
20	.17937	.98378	.18233	5.4845	1.0165	5.5749	40
21	.17966	.98373	.18263	.4755	.0165	.5660	39
22	.17995	.98368	.18293	.4665	.0166	.5572	38
23	.18023	.98362	.18323	.4575	.0166	.5484	37
24	.18052	.98357	.18353	.4486	.0167	.5396	36
25	.18080	.98352	.18383	5.4396	1.0167	5.5308	35
26	.18109	.98347	.18413	.4308	.0168	.5221	34
27	.18138	.98341	.18444	.4219	.0169	.5134	33
28	.18166	.98336	.18474	.4131	.0169	.5047	32
29	.18195	.98331	.18504	.4043	.0170	.4960	31
30	.18223	.98325	.18534	5.3955	1.0170	5.4874	30
31	.18252	.98320	.18564	.3868	.0171	.4788	29
32	.18281	.98315	.18594	.3780	.0171	.4702	28
33	.18309	.98309	.18624	.3694	.0172	.4617	27
34	.18338	.98304	.18654	.3607	.0172	.4532	26
35	.18366	.98299	.18684	5.3521	1.0173	5.4447	25
36	.18395	.98293	.18714	.3434	.0174	.4362	24
37	.18424	.98288	.18745	.3349	.0174	.4278	23
38	.18452	.98283	.18775	.3263	.0175	.4194	22
39	.18481	.98277	.18805	.3178	.0175	.4110	21
40	.18509	.98272	.18835	5.3093	1.0176	5.4026	20
41	.18538	.98267	.18865	.3008	.0176	.3943	19
42	.18567	.98261	.18895	.2923	.0177	.3860	18
43	.18595	.98256	.18925	.2839	.0177	.3777	17
44	.18624	.98250	.18955	.2755	.0178	.3695	16
45	.18652	.98245	.18985	5.2671	1.0179	5.3612	15
46	.18681	.98240	.19016	.2588	.0179	.3530	14
47	.18709	.98234	.19046	.2505	.0180	.3449	13
48	.18738	.98229	.19076	.2422	.0180	.3367	12
49	.18767	.98223	.19106	.2339	.0181	.3286	11
50	.18795	.98218	.19136	5.2257	1.0181	5.3205	10
51	.18824	.98212	.19166	.2174	.0182	.3124	9
52	.18852	.98207	.19197	.2092	.0182	.3044	8
53	.18881	.98201	.19227	.2011	.0183	.2963	7
54	.18909	.98196	.19257	.1929	.0184	.2883	6
55	.18938	.98190	.19287	5.1848	1.0184	5.2803	5
56	.18967	.98185	.19317	.1767	.0185	.2724	4
57	.18995	.98179	.19347	.1686	.0185	.2645	3
58	.19024	.98174	.19378	.1606	.0186	.2566	2
59	.19052	.98168	.19408	.1525	.0186	.2487	1
60	.19081	.98163	.19438	5.1445	1.0187	5.2408	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.19081	.98163	.19438	5.1445	1.0187	5.2408	60
1	.19109	.98157	.19468	.1366	.0188	.2330	59
2	.19138	.98152	.19498	.1286	.0188	.2252	58
3	.19166	.98146	.19529	.1207	.0189	.2174	57
4	.19195	.98140	.19559	.1128	.0189	.2097	56
5	.19224	.98135	.19589	5.1049	1.0190	5.2019	55
6	.19252	.98129	.19619	.0970	.0191	.1942	54
7	.19281	.98124	.19649	.0892	.0191	.1865	53
8	.19309	.98118	.19680	.0814	.0192	.1788	52
9	.19338	.98112	.19710	.0736	.0192	.1712	51
10	.19366	.98107	.19740	5.0658	1.0193	5.1636	50
11	.19395	.98101	.19770	.0581	.0193	.1560	49
12	.19423	.98095	.19800	.0504	.0194	.1484	48
13	.19452	.98090	.19831	.0427	0195	.1409	47
14	.19480	.98084	.19861	.0350	.0195	.1333	46
15	.19509	.98078	.19891	5.0273	1.0196	5.1258	45
16	.19537	.98073	.19921	.0197	.0196	.1183	44
17	.19566	.98067	.19952	.0121	.0197	.1109	43
18	.19595	.98061	.19982	.0045	.0198	.1034	42
19	.19623	.98056	.20012	4.9969	.0198	.0960	41
20	.19652	.98050	.20042	4.9894	1.0199	5.0886	40
21	.19680	.98044	.20073	.9819	0199	.0812	39
22	.19709	.98039	.20103	.9744	.0200	.0739	38
23	.19737	.98033	.20133	.9669	.0201	.0666	37
24	.19766	.98027	.20163	.9594	.0201	.0593	36
25	.19794	.98021	.20194	4.9520	1.0202	5.0520	35
26	.19823	.98016	.20224	.9446	.0202	.0447	34
27	.19851	.98010	.20254	9372	.0203	.0375	33
28	.19880	.98004	.20285	.9298	.0204	.0302	32
29	.19908	.97998	.20315	.9225	.0204	.0230	31
30	.19937	.97992	.20345	4.9151	1.0205	5.0158	30
31	.19965	.97987	.20375	.9078	.0205	.0087	29
32	.19994	.97981	.20406	.9006	.0206	.0015	28
33	.20022	.97975	.20436	.8933	.0207	4.9944	27
34	.20051	.97969	.20466	.8860	.0207	.9873	26
35	.20079	.97963	.20497	4.8788	1.0208	4.9802	25
36	.20108	.97957	.20527	.8716	.0208	.9732	24
37	.20136	.97952	.20557	.8644	.0209	.9661	23
38	.20165	.97946	.20588	.8573	.0210	.9591	22
39	.20193	.97940	.20618	.8501	.0210	.9521	21
40	.20222	.97934	.20648	4.8430	1.0211	4.9452	20
41	.20250	.97928	.20679	.8359	.0211	.9382	19
42	.20279	.97922	.20709	.8288	.0212	.9313	18
43	.20307	.97916	.20739	.8217	.0213	.9243	17
44	.20336	.97910	.20770	.8147	.0213	.9175	16
45	.20364	.97904	.20800	4.8077	1.0214	4.9106	15
46	.20393	.97899	.20830	.8007	.0215	.9037	14
47	.20421	.97893	.20861	.7937	.0215	.8969	13
48	.20450	.97887	.20891	.7867	.0216	.8901	12
49	.20478	.97881	.20921	.7798	.0216	.8833	11
50	.20506	.97875	.20952	4.7728	1.0217	4.8765	10
51	.20535	.97869	.20982	7659	.0218	.8697	9
52	.20563	.97863	.21012	.7591	.0218	.8630	8
53	.20592	.97857	.21043	.7522	.0219	.8563	7
54	.20620	.97851	.21073	.7453	.0220	.8496	6
55	.20649	.97845	.21104	4.7385	1.0220	4.8429	5
56	.20677	.97839	.21134	.7317	.0221	.8362	4
57	.20706	.97833	.21164	.7249	.0221	.8296	3
58	.20734	.97827	.21195	.7181	.0222	.8229	2
59	.20763	.97821	.21225	.7114	.0223	.8163	1
60	.20791	.97815	.21256	4.7046	1.0223	4.8097	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.20791	.97815	.21256	4.7046	1.0223	4.8097	60
1	.20820	.97809	.21286	.6979	.0224	.8032	59
2	.20848	.97803	.21316	.6912	.0225	.7966	58
3	.20876	.97797	.21347	.6845	.0225	.7901	57
4	.20905	.97790	.21377	.6778	.0226	.7835	56
5	.20933	.97784	.21408	4.6712	1.0226	4.7770	55
6	.20962	.97778	.21438	.6646	.0227	.7706	54
7	.20990	.97772	.21468	.6580	.0228	.7641	53
8	.21019	.97766	.21499	.6514	.0228	.7576	52
9	.21047	.97760	.21529	.6448	.0229	.7512	51
10	.21076	.97754	.21560	4.6382	1.0230	4.7448	50
11	.21104	.97748	.21590	.6317	.0230	.7384	49
12	.21132	.97741	.21621	.6252	.0231	.7320	48
13	.21161	.97735	.21651	.6187	.0232	.7257	47
14	.21189	.97729	.21682	.6122	.0232	.7193	46
15	.21218	.97723	.21712	4.6057	1.0233	4.7130	45
16	.21246	.97717	.21742	.5993	.0234	.7067	44
17	.21275	.97711	.21773	.5928	.0234	.7004	43
18	.21303	.97704	.21803	.5864	.0235	.6942	42
19	.21331	.97698	.21834	.5800	.0235	.6879	41
20	.21360	.97692	.21864	4.5736	1.0236	4.6817	40
21	.21388	.97686	.21895	.5673	.0237	.6754	39
22	.21417	.97680	.21925	.5609	.0237	.6692	38
23	.21445	.97673	.21956	.5546	.0238	.6631	37
24	.21473	.97667	.21986	.5483	.0239	.6569	36
25	.21502	.97661	.22017	4.5420	1.0239	4.6507	35
26	.21530	.97655	.22047	.5357	.0240	.6446	34
27	.21559	.97648	.22078	.5294	.0241	.6385	33
28	.21587	.97642	.22108	.5232	.0241	.6324	32
29	.21615	.97636	.22139	.5169	.0242	.6263	31
30	.21644	.97630	.22169	4.5107	1.0243	4.6201	30
31	.21672	.97623	.22200	.5045	.0243	.6142	29
32	.21701	.97617	.22230	.4983	.0244	.6081	28
33	.21729	.97611	.22261	.4921	.0245	.6021	27
34	.21757	.97604	.22291	.4860	.0245	.5961	26
35	.21786	.97598	.22322	4.4799	1.0246	4.5901	25
36	.21814	.97592	.22353	.4737	.0247	.5841	24
37	.21843	.97585	.22383	.4676	.0247	.5782	23
38	.21871	.97579	.22414	.4615	.0248	.5722	22
39	.21899	.97573	.22444	.4555	.0249	.5663	21
40	.21928	.97566	.22475	4.4494	1.0249	4.5604	20
41	.21956	.97560	.22505	.4434	.0250	.5545	19
42	.21985	.97553	.22536	.4373	.0251	.5486	18
43	.22013	.97547	.22566	.4313	.0251	.5428	17
44	.22041	.97541	.22597	.4253	.0252	.5369	16
45	.22070	.97534	.22628	4.4194	1.0253	4.5311	15
46	.22098	.97528	.22658	.4134	.0253	.5253	14
47	.22126	.97521	.22689	.4074	.0254	.5195	13
48	.22155	.97515	.22719	.4015	.0255	.5137	12
49	.22183	.97508	.22750	.3956	.0255	.5079	11
50	.22211	.97502	.22781	4.3897	1.0256	4.5021	10
51	.22240	.97495	.22811	.3838	.0257	.4964	9
52	.22268	.97489	.22842	.3779	.0257	.4907	8
53	.22297	.97483	.22872	.3721	.0258	.4850	7
54	.22325	.97476	.22903	.3662	.0259	.4793	6
55	.22353	.97470	.22934	4.3604	1.0260	4.4736	5
56	.22382	.97463	.22964	.3546	.0260	.4679	4
57	.22410	.97457	.22995	.3488	.0261	.4623	3
58	.22438	.97450	.23025	.3430	.0262	.4566	2
59	.22467	.97443	.23056	.3372	.0262	.4510	1
60	.22495	.97437	.23087	4.3315	1.0263	4.4454	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.22495	.97437	.23087	4.3315	1.0263	4.4454	60
1	.22523	.97430	.23117	.3257	.0264	.4398	59
2	.22552	.97424	.23148	.3200	.0264	.4342	58
3	.22580	.97417	.23179	.3143	.0265	.4287	57
4	.22608	.97411	.23209	.3086	.0266	.4231	56
5	.22637	.97404	.23240	4.3029	1.0266	4.4176	55
6	.22665	.97398	.23270	.2972	.0267	.4121	54
7	.22693	.97391	.23301	.2916	.0268	.4065	53
8	.22722	.97384	.23332	.2859	.0268	.4011	52
9	.22750	.97378	.23363	.2803	.0269	.3956	51
10	.22778	.97371	.23393	4.2747	1.0270	4.3901	50
11	.22807	.97364	.23424	.2691	.0271	.3847	49
12	.22835	.97358	.23455	.2635	.0271	.3792	48
13	.22863	.97351	.23485	.2579	.0272	.3738	47
14	.22892	.97344	.23516	.2524	.0273	.3684	46
15	.22920	.97338	.23547	4.2468	1.0273	4.3630	45
16	.22948	.97331	.23577	.2413	.0274	.3576	44
17	.22977	.97324	.23608	.2358	.0275	.3522	43
18	.23005	.97318	.23639	.2303	.0276	.3469	42
19	.23033	.97311	.23670	.2248	.0276	.3415	41
20	.23061	.97304	.23700	4.2193	1.0277	4.3362	40
21	.23090	.97298	.23731	.2139	.0278	.3309	39
22	.23118	.97291	.23762	.2084	.0278	.3256	38
23	.23146	.97284	.23793	.2030	.0279	.3203	37
24	.23175	.97277	.23823	.1976	.0280	.3150	36
25	.23203	.97271	.23854	4.1921	1.0280	4.3098	35
26	.23231	.97264	.23885	.1867	.0281	.3045	34
27	.23260	.97257	.23916	.1814	.0282	.2993	33
28	.23288	.97250	.23946	.1760	.0283	.2941	32
29	.23316	.97244	.23977	.1706	.0283	.2888	31
30	.23344	.97237	.24008	4.1653	1.0284	4.2836	30
31	.23373	.97230	.24039	.1600	.0285	.2785	29
32	.23401	.97223	.24069	.1546	.0285	.2733	28
33	.23429	.97216	.24100	.1493	.0286	.2681	27
34	.23458	.97210	.24131	.1440	.0287	.2630	26
35	.23486	.97203	.24162	4.1388	1.0288	4.2579	25
36	.23514	.97196	.24192	.1335	.0288	.2527	24
37	.23542	.97189	.24223	.1282	.0289	.2476	23
38	.23571	.97182	.24254	.1230	.0290	.2425	22
39	.23599	.97175	.24285	.1178	.0291	.2375	21
40	.23627	.97169	.24316	4.1126	1.0291	4.2324	20
41	.23655	.97162	.24346	.1073	.0292	.2273	19
42	.23684	.97155	.24377	.1022	.0293	.2223	18
43	.23712	.97148	.24408	.0970	.0293	.2173	17
44	.23740	.97141	.24439	.0918	.0294	.2122	16
45	.23768	.97134	.24470	4.0867	1.0295	4.2072	15
46	.23797	.97127	.24501	.0815	.0296	.2022	14
47	.23825	.97120	.24531	.0764	.0296	.1972	13
48	.23853	.97113	.24562	.0713	.0297	.1923	12
49	.23881	.97106	.24593	.0662	.0298	.1873	11
50	.23910	.97099	.24624	4.0611	1.0299	4.1824	10
51	.23938	.97092	.24655	.0560	.0299	.1774	9
52	.23966	.97086	.24686	.0509	.0300	.1725	8
53	.23994	.97079	.24717	.0458	.0301	.1676	7
54	.24023	.97072	.24747	.0408	.0302	.1627	6
55	.24051	.97065	.24778	4.0358	1.0302	4.1578	5
56	.24079	.97058	.24809	.0307	.0303	.1529	4
57	.24107	.97051	.24840	.0257	.0304	.1481	3
58	.24136	.97044	.24871	.0207	.0305	.1432	2
59	.24164	.97037	.24902	.0157	.0305	.1384	1
60	.24192	.97029	.24933	4.0108	1.0306	4.1336	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.24192	.97029	.24933	4.0108	1.0306	4.1336	60
1	.24220	.97022	.24964	.0058	.0307	.1287	59
2	.24249	.97015	.24995	.0009	.0308	.1239	58
3	.24277	.97008	.25025	3.9959	.0308	.1191	57
4	.24305	.97001	.25056	.9910	.0309	.1144	56
5	.24333	.96994	.25087	3.9861	1.0310	4.1096	55
6	.24361	.96987	.25118	.9812	.0311	.1048	54
7	.24390	.96980	.25149	.9763	.0311	.1001	53
8	.24418	.96973	.25180	.9714	.0312	.0953	52
9	.24446	.96966	.25211	.9665	.0313	.0906	51
10	.24474	.96959	.25242	3.9616	1.0314	4.0859	50
11	.24502	.96952	.25273	.9568	.0314	.0812	49
12	.24531	.96944	.25304	.9520	.0315	.0765	48
13	.24559	.96937	.25335	.9471	.0316	.0718	47
14	.24587	.96930	.25366	.9423	.0317	.0672	46
15	.24615	.96923	.25397	3.9375	1.0317	4.0625	45
16	.24643	.96916	.25428	.9327	.0318	.0579	44
17	.24672	.96909	.25459	.9279	.0319	.0532	43
18	.24700	.96901	.25490	.9231	.0320	.0486	42
19	.24728	.96894	.25521	.9184	.0320	.0440	41
20	.24756	.96887	.25552	3.9136	1.0321	4.0394	40
21	.24784	.96880	.25583	.9089	.0322	.0348	39
22	.24813	.96873	.25614	.9042	.0323	.0302	38
23	.24841	.96865	.25645	.8994	.0323	.0256	37
24	.24869	.96858	.25676	.8947	.0324	.0211	36
25	.24897	.96851	.25707	3.8900	1.0325	4.0165	35
26	.24925	.96844	.25738	.8853	.0326	.0120	34
27	.24953	.96836	.25769	.8807	.0327	.0074	33
28	.24982	.96829	.25800	.8760	.0327	.0029	32
29	.25010	.96822	.25831	.8713	.0328	3.9984	31
30	.25038	.96815	.25862	3.8667	1.0329	3.9939	30
31	.25066	.96807	.25893	.8621	.0330	.9894	29
32	.25094	.96800	.25924	.8574	.0330	.9850	28
33	.25122	.96793	.25955	.8528	.0331	.9805	27
34	.25151	.96785	.25986	.8482	.0332	.9760	26
35	.25179	.96778	.26017	3.8436	1.0333	3.9716	25
36	.25207	.96771	.26048	.8390	.0334	.9672	24
37	.25235	.96763	.26079	.8345	.0334	.9627	23
38	.25263	.96756	.26110	.8299	.0335	.9583	22
39	.25291	.96749	.26141	.8254	.0336	.9539	21
40	.25319	.96741	.26172	3.8208	1.0337	3.9495	20
41	.25348	.96734	.26203	.8163	.0338	.9451	19
42	.25376	.96727	.26234	.8118	.0338	.9408	18
43	.25404	.96719	.26266	.8073	.0339	.9364	17
44	.25432	.96712	.26297	.8027	.0340	.9320	16
45	.25460	.96704	.26328	3.7983	1.0341	3.9277	15
46	.25488	.96697	.26359	.7938	.0341	.9234	14
47	.25516	.96690	.26390	.7893	.0342	.9190	13
48	.25544	.96682	.26421	.7848	.0343	.9147	12
49	.25573	.96675	.26452	.7804	.0344	.9104	11
50	.25601	.96667	.26483	3.7759	1.0345	3.9061	10
51	.25629	.96660	.26514	.7715	.0345	.9018	9
52	.25657	.96652	.26546	.7671	.0346	.8976	8
53	.25685	.96645	.26577	.7627	.0347	.8933	7
54	.25713	.96638	.26608	.7583	.0348	.8890	6
55	.25741	.96630	.26639	3.7539	1.0349	3.8848	5
56	.25769	.96623	.26670	.7495	.0349	.8805	4
57	.25798	.96615	.26701	.7451	.0350	.8763	3
58	.25826	.96608	.26732	.7407	.0351	.8721	2
59	.25854	.96600	.26764	.7364	.0352	.8679	1
60	.25882	.96592	.26795	3.7320	1.0353	3.8637	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.25882	.96592	.26795	3.7320	1.0353	3.8637	60
1	.25910	.96585	.26826	.7277	.0353	.8595	59
2	.25938	.96577	.26857	.7234	.0354	.8553	58
3	.25966	.96570	.26888	.7191	.0355	.8512	57
4	.25994	.96562	.26920	.7147	.0356	.8470	56
5	.26022	.96555	.26951	3.7104	1.0357	3.8428	55
6	.26050	.96547	.26982	.7062	.0358	.8387	54
7	.26078	.96540	.27013	.7019	.0358	.8346	53
8	.26107	.96532	.27044	.6976	.0359	.8304	52
9	.26135	.96524	.27076	.6933	.0360	.8263	51
10	.26163	.96517	.27107	3.6891	1.0361	3.8222	50
11	.26191	.96509	.27138	.6848	.0362	.8181	49
12	.26219	.96502	.27169	.6806	.0362	.8140	48
13	.26247	.96494	.27201	.6764	.0363	.8100	47
14	.26275	.96486	.27232	.6722	.0364	.8059	46
15	.26303	.96479	.27263	3.6679	1.0365	3.8018	45
16	.26331	.96471	.27294	.6637	.0366	.7978	44
17	.26359	.96463	.27326	.6596	.0367	.7937	43
18	.26387	.96456	.27357	.6554	.0367	.7897	42
19	.26415	.96448	.27388	.6512	.0368	.7857	41
20	.26443	.96440	.27419	3.6470	1.0369	3.7816	40
21	.26471	.96433	.27451	.6429	.0370	.7776	39
22	.26499	.96425	.27482	.6387	.0371	.7736	38
23	.26527	.96417	.27513	.6346	.0371	.7697	37
24	.26556	.96409	.27544	.6305	.0372	.7657	36
25	.26584	.96402	.27576	3.6263	1.0373	3.7617	35
26	.26612	.96394	.27607	.6222	.0374	.7577	34
27	.26640	.96386	.27638	.6181	.0375	.7538	33
28	.26668	.96378	.27670	.6140	.0376	.7498	32
29	.26696	.96371	.27701	.6100	.0376	.7459	31
30	.26724	.96363	.27732	3.6059	1.0377	3.7420	30
31	.26752	.96355	.27764	.6018	.0378	.7380	29
32	.26780	.96347	.27795	.5977	.0379	.7341	28
33	.26808	.96340	.27826	.5937	.0380	.7302	27
34	.26836	.96332	.27858	.5896	.0381	.7263	26
35	.26864	.96324	.27889	3.5856	1.0382	3.7224	25
36	.26892	.96316	.27920	.5816	.0382	.7186	24
37	.26920	.96308	.27952	.5776	.0383	.7147	23
38	.26948	.96301	.27983	.5736	.0384	.7108	22
39	.26976	.96293	.28014	.5696	.0385	.7070	21
40	.27004	.96285	.28046	3.5656	1.0386	3.7031	20
41	.27032	.96277	.28077	.5616	.0387	.6993	19
42	.27060	.96269	.28109	.5576	.0387	.6955	18
43	.27088	.96261	.28140	.5536	.0388	.6917	17
44	.27116	.96253	.28171	.5497	.0389	.6878	16
45	.27144	.96245	.28203	3.5457	1.0390	3.6840	15
46	.27172	.96238	.28234	.5418	.0391	.6802	14
47	.27200	.96230	.28266	.5378	.0392	.6765	13
48	.27228	.96222	.28297	.5339	.0393	.6727	12
49	.27256	.96214	.28328	.5300	.0393	.6689	11
50	.27284	.96206	.28360	3.5261	1.0394	3.6651	10
51	.27312	.96198	.28391	.5222	.0395	.6614	9
52	.27340	.96190	.28423	.5183	.0396	.6576	8
53	.27368	.96182	.28454	.5144	.0397	.6539	7
54	.27396	.96174	.28486	.5105	.0398	.6502	6
55	.27424	.96166	.28517	3.5066	1.0399	3.6464	5
56	.27452	.96158	.28549	.5028	.0399	.6427	4
57	.27480	.96150	.28580	.4989	.0400	.6390	3
58	.27508	.96142	.28611	.4951	.0401	.6353	2
59	.27536	.96134	.28643	.4912	.0402	.6316	1
60	.27564	.96126	.28674	3.4874	1.0403	3.6279	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.27564	.96126	.28674	3.4874	1.0403	3.6279	60
1	.27592	.96118	.28706	.4836	.0404	.6243	59
2	.27620	.96110	.28737	.4798	.0405	.6206	58
3	.27648	.96102	.28769	.4760	.0406	.6169	57
4	.27675	.96094	.28800	.4722	.0406	.6133	56
5	.27703	.96086	.28832	3.4684	1.0407	3.6096	55
6	.27731	.96078	.28863	.4646	.0408	.6060	54
7	.27759	.96070	.28895	.4608	.0409	.6024	53
8	.27787	.96062	.28926	.4570	.0410	.5987	52
9	.27815	.96054	.28958	.4533	.0411	.5951	51
10	.27843	.96045	.28990	3.4495	1.0412	3.5915	50
11	.27871	.96037	.29021	.4458	.0413	.5879	49
12	.27899	.96029	.29053	.4420	.0413	.5843	48
13	.27927	.96021	.29084	.4383	.0414	.5807	47
14	.27955	.96013	.29116	.4346	.0415	.5772	46
15	.27983	.96005	.29147	3.4308	1.0416	3.5736	45
16	.28011	.95997	.29179	.4271	.0417	.5700	44
17	.28039	.95989	.29210	.4234	.0418	.5665	43
18	.28067	.95980	.29242	.4197	.0419	.5629	42
19	.28094	.95972	.29274	.4160	.0420	.5594	41
20	.28122	.95964	.29305	3.4124	1.0420	3.5559	40
21	.28150	.95956	.29337	.4087	.0421	.5523	39
22	.28178	.95948	.29368	.4050	.0422	.5488	38
23	.28206	.95940	.29400	.4014	.0423	.5453	37
24	.28234	.95931	.29432	.3977	.0424	.5418	36
25	.28262	.95923	.29463	3.3941	1.0425	3.5383	35
26	.28290	.95915	.29495	.3904	.0426	.5348	34
27	.28318	.95907	.29526	.3868	.0427	.5313	33
28	.28346	.95898	.29558	.3832	.0428	.5279	32
29	.28374	.95890	.29590	.3795	.0428	.5244	31
30	.28401	.95882	.29621	3.3759	1.0429	3.5209	30
31	.28429	.95874	.29653	.3723	.0430	.5175	29
32	.28457	.95865	.29685	.3687	.0431	.5140	28
33	.28485	.95857	.29716	.3651	.0432	.5106	27
34	.28513	.95849	.29748	.3616	.0433	.5072	26
35	.28541	.95840	.29780	3.3580	1.0434	3.5037	25
36	.28569	.95832	.29811	.3544	.0435	.5003	24
37	.28597	.95824	.29843	.3509	.0436	.4969	23
38	.28624	.95816	.29875	.3473	.0437	.4935	22
39	.28652	.95807	.29906	.3438	.0438	.4901	21
40	.28680	.95799	.29938	3.3402	1.0438	3.4867	20
41	.28708	.95791	.29970	.3367	.0439	.4833	19
42	.28736	.95782	.30001	.3332	.0440	.4799	18
43	.28764	.95774	.30033	.3296	.0441	.4766	17
44	.28792	.95765	.30065	.3261	.0442	.4732	16
45	.28820	.95757	.30096	3.3226	1.0443	3.4698	15
46	.28847	.95749	.30128	.3191	.0444	.4665	14
47	.28875	.95740	.30160	.3156	.0445	.4632	13
48	.28903	.95732	.30192	.3121	.0446	.4598	12
49	.28931	.95723	.30223	.3087	.0447	.4565	11
50	.28959	.95715	.30255	3.3052	1.0448	3.4532	10
51	.28987	.95707	.30287	.3017	.0448	.4498	9
52	.29014	.95698	.30319	.2983	.0449	.4465	8
53	.29042	.95690	.30350	.2948	.0450	.4432	7
54	.29070	.95681	.30382	.2914	.0451	.4399	6
55	.29098	.95673	.30414	3.2879	1.0452	3.4366	5
56	.29126	.95664	.30446	.2845	.0453	.4334	4
57	.29154	.95656	.30478	.2811	.0454	.4301	3
58	.29181	.95647	.30509	.2777	.0455	.4268	2
59	.29209	.95639	.30541	.2742	.0456	.4236	1
60	.29237	.95630	.30573	3.2708	1.0457	3.4203	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.29237	.95630	.30573	3.2708	1.0457	3.4203	60
1	.29265	.95622	.30605	.2674	.0458	.4170	59
2	.29293	.95613	.30637	.2640	.0459	.4138	58
3	.29321	.95605	.30668	.2607	.0460	.4106	57
4	.29348	.95596	.30700	.2573	.0461	.4073	56
5	.29376	.95588	.30732	3.2539	1.0461	3.4041	55
6	.29404	.95579	.30764	.2505	.0462	.4009	54
7	.29432	.95571	.30796	.2472	.0463	.3977	53
8	.29460	.95562	.30828	.2438	.0464	.3945	52
9	.29487	.95554	.30859	.2405	.0465	.3913	51
10	.29515	.95545	.30891	3.2371	1.0466	3.3881	50
11	.29543	.95536	.30923	.2338	.0467	.3849	49
12	.29571	.95528	.30955	.2305	.0468	.3817	48
13	.29598	.95519	.30987	.2271	.0469	.3785	47
14	.29626	.95511	.31019	.2238	.0470	.3754	46
15	.29654	.95502	.31051	3.2205	1.0471	3.3722	45
16	.29682	.95493	.31083	.2172	.0472	.3690	44
17	.29710	.95485	.31115	.2139	.0473	.3659	43
18	.29737	.95476	.31146	.2106	.0474	.3627	42
19	.29765	.95467	.31178	.2073	.0475	.3596	41
20	.29793	.95459	.31210	3.2041	1.0476	3.3565	40
21	.29821	.95450	.31242	.2008	.0477	.3534	39
22	.29848	.95441	.31274	.1975	.0478	.3502	38
23	.29876	.95433	.31306	.1942	.0478	.3471	37
24	.29904	.95424	.31338	.1910	.0479	.3440	36
25	.29932	.95415	.31370	3.1877	1.0480	3.3409	35
26	.29959	.95407	.31402	.1845	.0481	.3378	34
27	.29987	.95398	.31434	.1813	.0482	.3347	33
28	.30015	.95389	.31466	.1780	.0483	.3316	32
29	.30043	.95380	.31498	.1748	.0484	.3286	31
30	.30070	.95372	.31530	3.1716	1.0485	3.3255	30
31	.30098	.95363	.31562	.1684	.0486	.3224	29
32	.30126	.95354	.31594	.1652	.0487	.3194	28
33	.30154	.95345	.31626	.1620	.0488	.3163	27
34	.30 81	.95337	.31658	.1588	.0489	.3133	26
35	.30209	.95328	.31690	3.1556	1.0490	3.3102	25
36	.30237	.95319	.31722	.1524	.0491	.3072	24
37	.30265	.95310	.31754	.1492	.0492	.3042	23
38	.30292	.95301	.31786	.1460	.0493	.3011	22
39	.30320	.95293	.31818	.1429	.0494	.2981	21
40	.30348	.95284	.31850	3.1397	1.0495	3.2951	20
41	.30375	.95275	.31882	.1366	.0496	.2921	19
42	.30403	.95266	.31914	.1334	.0497	.2891	18
43	.30431	.95257	.31946	.1303	.0498	.2861	17
44	.30459	.95248	.31978	.1271	.0499	.2831	16
45	.30486	.95239	.32010	3.1240	1.0500	3.2801	15
46	.30514	.95231	.32042	.1209	.0501	.2772	14
47	.30542	.95222	.32074	.1177	.0502	.2742	13
48	.30569	.95213	.32106	.1146	.0503	.2712	12
49	.30597	.95204	.32138	.1115	.0504	.2683	11
50	.30625	.95195	.32171	3.1084	1.0505	3.2653	10
51	.30653	.95186	.32203	.1053	.0506	.2624	9
52	.30680	.95177	.32235	.1022	.0507	.2594	8
53	.30708	.95168	.32267	.0991	.0508	.2565	7
54	.30736	.95159	.32299	.0960	.0509	.2535	6
55	.30763	.95150	.32331	3.0930	1.0510	3.2506	5
56	.30791	.95141	.32363	.0899	.0511	.2477	4
57	.30819	.95132	.32395	.0868	.0512	.2448	3
58	.30846	.95124	.32428	.0838	.0513	.2419	2
59	.30874	.95115	.32460	.0807	.0514	.2390	1
60	.30902	.95106	.32492	3.0777	1.0515	3.2361	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.30902	.95106	.32492	3.0777	1.0515	3.2361	60
1	.30929	.95097	.32524	.0746	.0516	.2332	59
2	.30957	.95088	.32556	.0716	.0517	.2303	58
3	.30985	.95079	.32588	.0686	.0518	.2274	57
4	.31012	.95070	.32621	.0655	.0519	.2245	56
5	.31040	.95061	.32653	3.0625	1.0520	3.2216	55
6	.31068	.95051	.32685	.0595	.0521	.2188	54
7	.31095	.95042	.32717	.0565	.0522	.2159	53
8	.31123	.95033	.32749	.0535	.0523	.2131	52
9	.31150	.95024	.32782	.0505	.0524	.2102	51
10	.31178	.95015	.32814	3.0475	1.0525	3.2074	50
11	.31206	.95006	.32846	.0445	.0526	.2045	49
12	.31233	.94997	.32878	.0415	.0527	.2017	48
13	.31261	.94988	.32910	.0385	.0528	.1989	47
14	.31289	.94979	.32943	.0356	.0529	.1960	46
15	.31316	.94970	.32975	3.0326	1.0530	3.1932	45
16	.31344	.94961	.33007	.0296	.0531	.1904	44
17	.31372	.94952	.33039	.0267	.0532	.1876	43
18	.31399	.94942	.33072	.0237	.0533	.1848	42
19	.31427	.94933	.33104	.0208	.0534	.1820	41
20	.31454	.94924	.33136	3.0178	1.0535	3.1792	40
21	.31482	.94915	.33169	.0149	.0536	.1764	39
22	.31510	.94906	.33201	.0120	.0537	.1736	38
23	.31537	.94897	.33233	.0090	.0538	.1708	37
24	.31565	.94888	.33265	.0061	.0539	.1681	36
25	.31592	.94878	.33298	3.0032	1.0540	3.1653	35
26	.31620	.94869	.33330	.0003	.0541	.1625	34
27	.31648	.94860	.33362	2.9974	.0542	.1598	33
28	.31675	.94851	.33395	.9945	.0543	.1570	32
29	.31703	.94841	.33427	.9916	.0544	.1543	31
30	.31730	.94832	.33459	2.9887	1.0545	3.1515	30
31	.31758	.94823	.33492	.9858	.0546	.1488	29
32	.31786	.94814	.33524	.9829	.0547	.1461	28
33	.31813	.94805	.33557	.9800	.0548	.1433	27
34	.31841	.94795	.33589	.9772	.0549	.1406	26
35	.31868	.94786	.33621	2.9743	1.0550	3.1379	25
36	.31896	.94777	.33654	.9714	.0551	.1352	24
37	.31923	.94767	.33686	.9686	.0552	.1325	23
38	.31951	.94758	.33718	.9657	.0553	.1298	22
39	.31978	.94749	.33751	.9629	.0554	.1271	21
40	.32006	.94740	.33783	2.9600	1.0555	3.1244	20
41	.32034	.94730	.33816	.9572	.0556	.1217	19
42	.32061	.94721	.33848	.9544	.0557	.1190	18
43	.32089	.94712	.33880	.9515	.0558	.1163	17
44	.32116	.94702	.33913	.9487	.0559	.1137	16
45	.32144	.94693	.33945	2.9459	1.0560	3.1110	15
46	.32171	.94684	.33978	.9431	.0561	.1083	14
47	.32199	.94674	.34010	.9403	.0562	.1057	13
48	.32226	.94665	.34043	.9375	.0563	.1030	12
49	.32254	.94655	.34075	.9347	.0565	.1004	11
50	.32282	.94646	.34108	2.9319	1.0566	3.0977	10
51	.32309	.94637	.34140	.9291	.0567	.0951	9
52	.32337	.94627	.34173	.9263	.0568	.0925	8
53	.32364	.94618	.34205	.9235	.0569	.0898	7
54	.32392	.94608	.34238	.9208	.0570	.0872	6
55	.32419	.94599	.34270	2.9180	1.0571	3.0846	5
56	.32447	.94590	.34303	.9152	.0572	.0820	4
57	.32474	.94580	.34335	.9125	.0573	.0793	3
58	.32502	.94571	.34368	.9097	.0574	.0767	2
59	.32529	.94561	.34400	.9069	.0575	.0741	1
60	.32557	.94552	.34433	2.9042	1.0576	3.0715	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.32557	.94552	.34433	2.9042	1.0576	3.0715	60
1	.32584	.94542	.34465	.9015	.0577	.0690	59
2	.32612	.94533	.34498	.8987	.0578	.0664	58
3	.32639	.94523	.34530	.8960	.0579	.0638	57
4	.32667	.94514	.34563	.8933	.0580	.0612	56
5	.32694	.94504	.34595	2.8905	1.0581	3.0586	55
6	.32722	.94495	.34628	.8878	.0582	.0561	54
7	.32749	.94485	.34661	.8851	.0584	.0535	53
8	.32777	.94476	.34693	.8824	.0585	.0509	52
9	.32804	.94466	.34726	.8797	.0586	.0484	51
10	.32832	.94457	.34758	2.8770	1.0587	3.0458	50
11	.32859	.94447	.34791	.8743	.0588	.0433	49
12	.32887	.94438	.34824	.8716	.0589	.0407	48
13	.32914	.94428	.34856	.8689	.0590	.0382	47
14	.32942	.94418	.34889	.8662	.0591	.0357	46
15	.32969	.94409	.34921	2.8636	1.0592	3.0331	45
16	.32996	.94399	.34954	.8609	.0593	.0306	44
17	.33024	.94390	.34987	.8582	.0594	.0281	43
18	.33051	.94380	.35019	.8555	.0595	.0256	42
19	.33079	.94370	.35052	.8529	.0596	.0231	41
20	.33106	.94361	.35085	2.8502	1.0598	3.0206	40
21	.33134	.94351	.35117	.8476	.0599	.0181	39
22	.33161	.94341	.35150	.8449	.0600	.0156	38
23	.33189	.94332	.35183	.8423	.0601	.0131	37
24	.33216	.94322	.35215	.8396	.0602	.0106	36
25	.33243	.94313	.35248	2.8370	1.0603	3.0081	35
26	.33271	.94303	.35281	.8344	.0604	.0056	34
27	.33298	.94293	.35314	.8318	.0605	.0031	33
28	.33326	.94283	.35346	.8291	.0606	.0007	32
29	.33353	.94274	.35379	.8265	.0607	2.9982	31
30	.33381	.94264	.35412	2.8239	1.0608	2.9957	30
31	.33408	.94254	.35445	.8213	.0609	.9933	29
32	.33435	.94245	.35477	.8187	.0611	.9908	28
33	.33463	.94235	.35510	.8161	.0612	.9884	27
34	.33490	.94225	.35543	.8135	.0613	.9859	26
35	.33518	.94215	.35576	2.8109	1.0614	2.9835	25
36	.33545	.94206	.35608	.8083	.0615	.9810	24
37	.33572	.94196	.35641	.8057	.0616	.9786	23
38	.33600	.94186	.35674	.8032	.0617	.9762	22
39	.33627	.94176	.35707	.8006	.0618	.9738	21
40	.33655	.94167	.35739	2.7980	1.0619	2.9713	20
41	.33682	.94157	.35772	.7954	.0620	.9689	19
42	.33709	.94147	.35805	.7929	.0622	.9665	18
43	.33737	.94137	.35838	.7903	.0623	.9641	17
44	.33764	.94127	.35871	.7878	.0624	.9617	16
45	.33792	.94118	.35904	2.7852	1.0625	2.9593	15
46	.33819	.94108	.35936	.7827	.0626	.9569	14
47	.33846	.94098	.35969	.7801	.0627	.9545	13
48	.33874	.94088	.36002	.7776	.0628	.9521	12
49	.33901	.94078	.36035	.7751	.0629	.9497	11
50	.33928	.94068	.36068	2.7725	1.0630	2.9474	10
51	.33956	.94058	.36101	.7700	.0632	.9450	9
52	.33983	.94049	.36134	.7675	.0633	.9426	8
53	.34011	.94039	.36167	.7650	.0634	.9402	7
54	.34038	.94029	.36199	.7625	.0635	.9379	6
55	.34065	.94019	.36232	2.7600	1.0636	2.9355	5
56	.34093	.94009	.36265	.7575	.0637	.9332	4
57	.34120	.93999	.36298	.7550	.0638	.9308	3
58	.34147	.93989	.36331	.7525	.0639	.9285	2
59	.34175	.93979	.36364	.7500	.0641	.9261	1
60	.34202	.93969	.36397	2.7475	1.0642	2.9238	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.34202	.93969	.36397	2.7475	1.0642	2.9238	60
1	.34229	.93959	.36430	.7450	.0643	.9215	59
2	.34257	.93949	.36463	.7425	.0644	.9191	58
3	.34284	.93939	.36496	.7400	.0645	.9168	57
4	.34311	.93929	.36529	.7376	.0646	.9145	56
5	.34339	.93919	.36562	2.7351	1.0647	2.9122	55
6	.34366	.93909	.36595	.7326	.0648	.9098	54
7	.34393	.93899	.36628	.7302	.0650	.9075	53
8	.34421	.93889	.36661	.7277	.0651	.9052	52
9	.34448	.93879	.36694	.7252	.0652	.9029	51
10	.34475	.93869	.36727	2.7228	1.0653	2.9006	50
11	.34502	.93859	.36760	.7204	.0654	.8983	49
12	.34530	.93849	.36793	.7179	.0655	.8960	48
13	.34557	.93839	.36826	.7155	.0656	.8937	47
14	.34584	.93829	.36859	.7130	.0658	.8915	46
15	.34612	.93819	.36892	2.7106	1.0659	2.8892	45
16	.34639	.93809	.36925	.7082	.0660	.8869	44
17	.34666	.93799	.36958	.7058	.0661	.8846	43
18	.34693	.93789	.36991	.7033	.0662	.8824	42
19	.34721	.93779	.37024	.7009	.0663	.8801	41
20	.34748	.93769	.37057	2.6985	1.0664	2.8778	40
21	.34775	.93758	.37090	.6961	.0666	.8756	39
22	.34803	.93748	.37123	.6937	.0667	.8733	38
23	.34830	.93738	.37156	.6913	.0668	.8711	37
24	.34857	.93728	.37190	.6889	.0669	.8688	36
25	.34884	.93718	.37223	2.6865	1.0670	2.8666	35
26	.34912	.93708	.37256	.6841	.0671	.8644	34
27	.34939	.93698	.37289	.6817	.0673	.8621	33
28	.34966	.93687	.37322	.6794	.0674	.8599	32
29	.34993	.93677	.37355	.6770	.0675	.8577	31
30	.35021	.93667	.37388	2.6746	1.0676	2.8554	30
31	.35048	.93657	.37422	.6722	.0677	.8532	29
32	.35075	.93647	.37455	.6699	.0678	.8510	28
33	.35102	.93637	.37488	.6675	.0679	.8488	27
34	.35130	.93626	.37521	.6652	.0681	.8466	26
35	.35157	.93616	.37554	2.6628	1.0682	2.8444	25
36	.35184	.93606	.37587	.6604	.0683	.8422	24
37	.35211	.93596	.37621	.6581	.0684	.8400	23
38	.35239	.93585	.37654	.6558	.0685	.8378	22
39	.35266	.93575	.37687	.6534	.0686	.8356	21
40	.35293	.93565	.37720	2.6511	1.0688	2.8334	20
41	.35320	.93555	.37754	.6487	.0689	.8312	19
42	.35347	.93544	.37787	.6464	.0690	.8290	18
43	.35375	.93534	.37820	.6441	.0691	.8269	17
44	.35402	.93524	.37853	.6418	.0692	.8247	16
45	.35429	.93513	.37887	2.6394	1.0694	2.8225	15
46	.35456	.93503	.37920	.6371	.0695	.8204	14
47	.35483	.93493	.37953	.6348	.0696	.8182	13
48	.35511	.93482	.37986	.6325	.0697	.8160	12
49	.35538	.93472	.38020	.6302	.0698	.8139	11
50	.35565	.93462	.38053	2.6279	1.0699	2.8117	10
51	.35592	.93451	.38086	.6256	.0701	.8096	9
52	.35619	.93441	.38120	.6233	.0702	.8074	8
53	.35647	.93431	.38153	.6210	.0703	.8053	7
54	.35674	.93420	.38186	.6187	.0704	.8032	6
55	.35701	.93410	.38220	2.6164	1.0705	2.8010	5
56	.35728	.93400	.38253	.6142	.0707	.7989	4
57	.35755	.93389	.38286	.6119	.0708	.7968	3
58	.35782	.93379	.38320	.6096	.0709	.7947	2
59	.35810	.93368	.38353	.6073	.0710	.7925	1
60	.35837	.93358	.38386	2.6051	1.0711	2.7904	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.35837	.93358	.38386	2.6051	1.0711	2.7904	60
1	.35864	.93348	.38420	.6028	.0713	.7883	59
2	.35891	.93337	.38453	.6006	.0714	.7862	58
3	.35918	.93327	.38486	.5983	.0715	.7841	57
4	.35945	.93316	.38520	.5960	.0716	.7820	56
5	.35972	.93306	.38553	2.5938	1.0717	2.7799	55
6	.36000	.93295	.38587	.5916	.0719	.7778	54
7	.36027	.93285	.38620	.5893	.0720	.7757	53
8	.36054	.93274	.38654	.5871	.0721	.7736	52
9	.36081	.93264	.38687	.5848	.0722	.7715	51
10	.36108	.93253	.38720	2.5826	1.0723	2.7694	50
11	.36135	.93243	.38754	.5804	.0725	.7674	49
12	.36162	.93232	.38787	.5781	.0726	.7653	48
13	.36189	.93222	.38821	.5759	.0727	.7632	47
14	.36217	.93211	.38854	.5737	.0728	.7611	46
15	.36244	.93201	.38888	2.5715	1.0729	2.7591	45
16	.36271	.93190	.38921	.5693	.0731	.7570	44
17	.36298	.93180	.38955	.5671	.0732	.7550	43
18	.36325	.93169	.38988	.5649	.0733	.7529	42
19	.36352	.93158	.39022	.5627	.0734	.7509	41
20	.36379	.93148	.39055	2.5605	1.0736	2.7488	40
21	.36406	.93137	.39089	.5583	.0737	.7468	39
22	.36433	.93127	.39122	.5561	.0738	.7447	38
23	.36460	.93116	.39156	.5539	.0739	.7427	37
24	.36488	.93105	.39189	.5517	.0740	.7406	36
25	.36515	.93095	.39223	2.5495	1.0742	2.7386	35
26	.36542	.93084	.39257	.5473	.0743	.7366	34
27	.36569	.93074	.39290	.5451	.0744	.7346	33
28	.36596	.93063	.39324	.5430	.0745	.7325	32
29	.36623	.93052	.39357	.5408	.0747	.7305	31
30	.36650	.93042	.39391	2.5386	1.0748	2.7285	30
31	.36677	.93031	.39425	.5365	.0749	.7265	29
32	.36704	.93020	.39458	.5343	.0750	.7245	28
33	.36731	.93010	.39492	.5322	.0751	.7225	27
34	.36758	.92999	.39525	.5300	.0753	.7205	26
35	.36785	.92988	.39559	2.5278	1.0754	2.7185	25
36	.36812	.92978	.39593	.5257	.0755	.7165	24
37	.36839	.92967	.39626	.5236	.0756	.7145	23
38	.36866	.92956	.39660	.5214	.0758	.7125	22
39	.36893	.92945	.39694	.5193	.0759	.7105	21
40	.36921	.92935	.39727	2.5171	1.0760	2.7085	20
41	.36948	.92924	.39761	.5150	.0761	.7065	19
42	.36975	.92913	.39795	.5129	.0763	.7045	18
43	.37002	.92902	.39828	.5108	.0764	.7026	17
44	.37029	.92892	.39862	.5086	.0765	.7006	16
45	.37056	.92881	.39896	2.5065	1.0766	2.6986	15
46	.37083	.92870	.39930	.5044	.0768	.6967	14
47	.37110	.92859	.39963	.5023	.0769	.6947	13
48	.37137	.92848	.39997	.5002	.0770	.6927	12
49	.37164	.92838	.40031	.4981	.0771	.6908	11
50	.37191	.92827	.40065	2.4960	1.0773	2.6888	10
51	.37218	.92816	.40098	.4939	.0774	.6869	9
52	.37245	.92805	.40132	.4918	.0775	.6849	8
53	.37272	.92794	.40166	.4897	.0776	.6830	7
54	.37299	.92784	.40200	.4876	.0778	.6810	6
55	.37326	.92773	.40233	2.4855	1.0779	2.6791	5
56	.37353	.92762	.40267	.4834	.0780	.6772	4
57	.37380	.92751	.40301	.4813	.0781	.6752	3
58	.37407	.92740	.40335	.4792	.0783	.6733	2
59	.37434	.92729	.40369	.4772	.0784	.6714	1
60	.37461	.92718	.40403	2.4751	1.0785	2.6695	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.37461	.92718	.40403	2.4751	1.0785	2.6695	60
1	.37488	.92707	.40436	.4730	.0787	.6675	59
2	.37514	.92696	.40470	.4709	.0788	.6656	58
3	.37541	.92686	.40504	.4689	.0789	.6637	57
4	.37568	.92675	.40538	.4668	.0790	.6618	56
5	.37595	.92664	.40572	2.4647	1.0792	2.6599	55
6	.37622	.92653	.40606	.4627	.0793	.6580	54
7	.37649	.92642	.40640	.4606	.0794	.6561	53
8	.37676	.92631	.40673	.4586	.0795	.6542	52
9	.37703	.92620	.40707	.4565	.0797	.6523	51
10	.37730	.92609	.40741	2.4545	1.0798	2.6504	50
11	.37757	.92598	.40775	.4525	.0799	.6485	49
12	.37784	.92587	.40809	.4504	.0801	.6466	48
13	.37811	.92576	.40843	.4484	.0802	.6447	47
14	.37838	.92565	.40877	.4463	.0803	.6428	46
15	.37865	.92554	.40911	2.4443	1.0804	2.6410	45
16	.37892	.92543	.40945	.4423	.0806	.6391	44
17	.37919	.92532	.40979	.4403	.0807	.6372	43
18	.37946	.92521	.41013	.4382	.0808	.6353	42
19	.37972	.92510	.41047	.4362	.0810	.6335	41
20	.37999	.92499	.41081	2.4342	1.0811	2.6316	40
21	.38026	.92488	.41115	.4322	.0812	.6297	39
22	.38053	.92477	.41149	.4302	.0813	.6279	38
23	.38080	.92466	.41183	.4282	.0815	.6260	37
24	.38107	.92455	.41217	.4262	.0816	.6242	36
25	.38134	.92443	.41251	2.4242	1.0817	2.6223	35
26	.38161	.92432	.41285	.4222	.0819	.6205	34
27	.38188	.92421	.41319	.4202	.0820	.6186	33
28	.38214	.92410	.41353	.4182	.0821	.6168	32
29	.38241	.92399	.41387	.4162	.0823	.6150	31
30	.38268	.92388	.41421	2.4142	1.0824	2.6131	30
31	.38295	.92377	.41455	.4122	.0825	.6113	29
32	.38322	.92366	.41489	.4102	.0826	.6095	28
33	.38349	.92354	.41524	.4083	.0828	.6076	27
34	.38376	.92343	.41558	.4063	.0829	.6058	26
35	.38403	.92332	.41592	2.4043	1.0830	2.6040	25
36	.38429	.92321	.41626	.4023	.0832	.6022	24
37	.38456	.92310	.41660	.4004	.0833	.6003	23
38	.38483	.92299	.41694	.3984	.0834	.5985	22
39	.38510	.92287	.41728	.3964	.0836	.5967	21
40	.38537	.92276	.41762	2.3945	1.0837	2.5949	20
41	.38564	.92265	.41797	.3925	.0838	.5931	19
42	.38591	.92254	.41831	.3906	.0840	.5913	18
43	.38617	.92242	.41865	.3886	.0841	.5895	17
44	.38644	.92231	.41899	.3867	.0842	.5877	16
45	.38671	.92220	.41933	2.3847	1.0844	2.5859	15
46	.38698	.92209	.41968	.3828	.0845	.5841	14
47	.38725	.92197	.42002	.3808	.0846	.5823	13
48	.38751	.92186	.42036	.3789	.0847	.5805	12
49	.38778	.92175	.42070	.3770	.0849	.5787	11
50	.38805	.92164	.42105	2.3750	1.0850	2.5770	10
51	.38832	.92152	.42139	.3731	.0851	.5752	9
52	.38859	.92141	.42173	.3712	.0853	.5734	8
53	.38886	.92130	.42207	.3692	.0854	.5716	7
54	.38912	.92118	.42242	.3673	.0855	.5699	6
55	.38939	.92107	.42276	2.3654	1.0857	2.5681	5
56	.38966	.92096	.42310	.3635	.0858	.5663	4
57	.38993	.92084	.42344	.3616	.0859	.5646	3
58	.39019	.92073	.42379	.3597	.0861	.5628	2
59	.39046	.92062	.42413	.3577	.0862	.5610	1
60	.39073	.92050	.42447	2.3558	1.0864	2.5593	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.39073	.92050	.42447	2.3558	1.0864	2.5593	60
1	.39100	.92039	.42482	.3539	.0865	.5575	59
2	.39126	.92028	.42516	.3520	.0866	.5558	58
3	.39153	.92016	.42550	.3501	.0868	.5540	57
4	.39180	.92005	.42585	.3482	.0869	.5523	56
5	.39207	.91993	.42619	2.3463	1.0870	2.5506	55
6	.39234	.91982	.42654	.3445	.0872	.5488	54
7	.39260	.91971	.42688	.3426	.0873	.5471	53
8	.39287	.91959	.42722	.3407	.0874	.5453	52
9	.39314	.91948	.42757	.3388	.0876	.5436	51
10	.39341	.91936	.42791	2.3369	1.0877	2.5419	50
11	.39367	.91925	.42826	.3350	.0878	.5402	49
12	.39394	.91913	.42860	.3332	.0880	.5384	48
13	.39421	.91902	.42894	.3313	.0881	.5367	47
14	.39448	.91891	.42929	.3294	.0882	.5350	46
15	.39474	.91879	.42963	2.3276	1.0884	2.5333	45
16	.39501	.91868	42998	.3257	.0885	.5316	44
17	.39528	.91856	43032	.3238	.0886	.5299	43
18	.39554	.91845	.43067	.3220	.0888	.5281	42
19	.39581	.91833	.43101	.3201	.0889	.5264	41
20	.39608	.91822	.43136	2.3183	1.0891	2.5247	40
21	.39635	.91810	.43170	.3164	.0892	.5230	39
22	.39661	.91798	.43205	.3145	.0893	.5213	38
23	.39688	.91787	.43239	.3127	.0895	.5196	37
24	.39715	.91775	.43274	.3109	.0896	.5179	36
25	.39741	.91764	.43308	2.3090	1.0897	2.5163	35
26	.39768	.91752	.43343	.3072	.0899	.5146	34
27	.39795	.91741	.43377	.3053	.0900	.5129	33
28	.39821	.91729	.43412	.3035	.0902	.5112	32
29	.39848	.91718	.43447	.3017	.0903	.5095	31
30	.39875	.91706	.43481	2.2998	1.0904	2.5078	30
31	.39901	.91694	.43516	.2980	.0906	.5062	29
32	.39928	.91683	.43550	.2962	.0907	.5045	28
33	.39955	.91671	.43585	.2944	.0908	.5028	27
34	.39981	.91659	.43620	.2925	.0910	.5011	26
35	.40008	.91648	.43654	2.2907	1.0911	2.4995	25
36	.40035	.91636	.43689	.2889	.0913	.4978	24
37	.40061	.91625	.43723	.2871	.0914	.4961	23
38	.40088	.91613	.43758	.2853	.0915	.4945	22
39	.40115	.91601	.43793	.2835	.0917	.4928	21
40	.40141	.91590	.43827	2.2817	1.0918	2.4912	20
41	.40168	.91578	.43862	.2799	.0920	.4895	19
42	.40195	.91566	.43897	.2781	.0921	.4879	18
43	.40221	.91554	.43932	.2763	.0922	.4862	17
44	.40248	.91543	.43966	.2745	.0924	.4846	16
45	.40275	.91531	.44001	2.2727	1.0925	2.4829	15
46	.40301	.91519	.44036	.2709	.0927	.4813	14
47	.40328	.91508	.44070	.2691	.0928	.4797	13
48	.40354	.91496	.44105	.2673	.0929	.4780	12
49	.40381	.91484	.44140	.2655	.0931	.4764	11
50	.40408	.91472	.44175	2.2637	1.0932	2.4748	10
51	.40434	.91461	.44209	.2619	.0934	.4731	9
52	.40461	.91449	.44244	.2602	.0935	.4715	8
53	.40487	.91437	.44279	.2584	.0936	.4699	7
54	.40514	.91425	.44314	.2566	.0938	.4683	6
55	.40541	.91414	.44349	2.2548	1.0939	2.4666	5
56	.40567	.91402	.44383	.2531	.0941	.4650	4
57	.40594	.91390	.44418	.2513	.0942	.4634	3
58	.40620	.91378	.44453	.2495	.0943	.4618	2
59	.40647	.91366	.44488	.2478	.0945	.4602	1
60	.40674	.91354	.44523	2.2460	1.0946	2.4586	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.40674	.91354	.44523	2.2460	1.0946	2.4586	60
1	.40700	.91343	.44558	.2443	.0948	.4570	59
2	.40727	.91331	.44593	.2425	.0949	.4554	58
3	.40753	.91319	.44627	.2408	.0951	.4538	57
4	.40780	.91307	.44662	.2390	.0952	.4522	56
5	.40806	.91295	.44697	2.2373	1.0953	2.4506	55
6	.40833	.91283	.44732	.2355	.0955	.4490	54
7	.40860	.91271	.44767	.2338	.0956	.4474	53
8	.40886	.91260	.44802	.2320	.0958	.4458	52
9	.40913	.91248	.44837	.2303	.0959	.4442	51
10	.40939	.91236	.44872	2.2286	1.0961	2.4426	50
11	.40966	.91224	.44907	.2268	.0962	.4411	49
12	.40992	.91212	.44942	.2251	.0963	.4395	48
13	.41019	.91200	.44977	.2 34	.0965	.4379	47
14	.41045	.91188	.45012	.2216	.0966	.4363	46
15	.41072	.91176	.45047	2.2199	1.0968	2.4347	45
16	.41098	.91164	.45082	.2182	.0969	.4332	44
17	.41125	.91152	.45117	.2165	.0971	.4316	43
18	.41151	.91140	.45152	.2147	.0972	.4300	42
19	.41178	.91128	.45187	.2130	.0973	.4285	41
20	.41204	.91116	.45222	2.2113	1.0975	2.4269	40
21	.41231	.91104	.45257	.2096	.0976	.4254	39
22	.41257	.91092	.45292	.2079	.0978	.4238	38
23	.41284	.91080	.45327	.2062	.0979	.4222	37
24	.41310	.91068	.45362	.2045	.0981	.4207	36
25	.41337	.91056	.45397	2.2028	1.0982	2.4191	35
26	.41363	.91044	.45432	.2011	.0984	.4176	34
27	.41390	.91032	.45467	.1994	.0985	.4160	33
28	.41416	.91020	.45502	.1977	.0986	.4145	32
29	.41443	.91008	.45537	.1960	.0988	.4130	31
30	.41469	.90996	.45573	2.1943	1.0989	2.4114	30
31	.41496	.90984	.45608	.1926	.0991	.4099	29
32	.41522	.90972	.45643	.1909	.0992	.4083	28
33	.41549	.90960	.45678	.1892	.0994	.4068	27
34	.41575	.90948	.45713	.1875	.0995	.4053	26
35	.41602	.90936	.45748	2.1859	1.0997	2.4037	25
36	.41628	.90924	.45783	.1842	.0998	.4022	24
37	.41654	.90911	.45819	.1825	.1000	.4007	23
38	.41681	.90899	.45854	.1808	.1001	.3992	22
39	.41707	.90887	.45889	.1792	.1003	.3976	21
40	.41734	.90875	.45924	2.1775	1.1004	2.3961	20
41	.41760	.90863	.45960	.1758	.1005	.3946	19
42	.41787	.90851	.45995	.1741	.1007	.3931	18
43	.41813	.90839	.46030	.1725	.1008	.3916	17
44	.41839	.90826	.46065	.1708	.1010	.3901	16
45	.41866	.90814	.46101	2.1692	1.1011	2.3886	15
46	.41892	.90802	.46136	.1675	.1013	.3871	14
47	.41919	.90790	.46171	.1658	.1014	.3856	13
48	.41945	.90778	.46206	.1642	.1016	.3841	12
49	.41972	.90765	.46242	.1625	.1017	.3826	11
50	.41998	.90753	.46277	2.1609	1.1019	2.3811	10
51	.42024	.90741	.46312	.1592	.1020	.3796	9
52	.42051	.90729	.46348	.1576	.1022	.3781	8
53	.42077	.90717	.46383	.1559	.1023	.3766	7
54	.42103	.90704	.46418	.1543	.1025	.3751	6
55	.42130	.90692	.46454	2.1527	1.1026	2.3736	5
56	.42156	.90680	.46489	.1510	.1028	.3721	4
57	.42183	.90668	.46524	.1494	.1029	.3706	3
58	.42209	.90655	.46560	.1478	.1031	.3691	2
59	.42235	.90643	.46595	.1461	.1032	.3677	1
60	.42262	.90631	.46631	2.1445	1.1034	2.3662	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

25°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.42262	.90631	.46631	2.1445	1.1034	2.3662	60
1	.42288	.90618	.46666	.1429	.1035	.3647	59
2	.42314	.90606	.46702	.1412	.1037	.3632	58
3	.42341	.90594	.46737	.1396	.1038	.3618	57
4	.42367	.90581	.46772	.1380	.1040	.3603	56
5	.42394	.90569	.46808	2.1364	1.1041	2.3588	55
6	.42420	.90557	.46843	.1348	.1043	.3574	54
7	.42446	.90544	.46879	.1331	.1044	.3559	53
8	.42473	.90532	.46914	.1315	.1046	.3544	52
9	.42499	.90520	.46950	.1299	.1047	.3530	51
10	.42525	.90507	.46985	2.1283	1.1049	2.3515	50
11	.42552	.90495	.47021	.1267	.1050	.3501	49
12	.42578	.90483	.47056	.1251	.1052	.3486	48
13	.42604	.90470	.47092	.1235	.1053	.3472	47
14	.42630	.90458	.47127	.1219	.1055	.3457	46
15	.42657	.90445	.47163	2.1203	1.1056	2.3443	45
16	.42683	.90433	.47199	.1187	.1058	.3428	44
17	.42709	.90421	.47234	.1171	.1059	.3414	43
18	.42736	.90408	.47270	.1155	.1061	.3399	42
19	.42762	.90396	.47305	.1139	.1062	.3385	41
20	.42788	.90383	.47341	2.1123	1.1064	2.3371	40
21	.42815	.90371	.47376	.1107	.1065	.3356	39
22	.42841	.90358	.47412	.1092	.1067	.3342	38
23	.42867	.90346	.47448	.1076	.1068	.3328	37
24	.42893	.90333	.47483	.1060	.1070	.3313	36
25	.42920	.90321	.47519	2.1044	1.1072	2.3299	35
26	.42946	.90308	.47555	.1028	.1073	.3285	34
27	.42972	.90296	.47590	.1013	.1075	.3271	33
28	.42998	.90283	.47626	.0997	.1076	.3256	32
29	.43025	.90271	.47662	.0981	.1078	.3242	31
30	.43051	.90258	.47697	2.0965	1.1079	2.3228	30
31	.43077	.90246	.47733	.0950	.1081	.3214	29
32	.43104	.90233	.47769	.0934	.1082	.3200	28
33	.43130	.90221	.47805	.0918	.1084	.3186	27
34	.43156	.90208	.47840	.0903	.1085	.3172	26
35	.43182	.90196	.47876	2.0887	1.1087	2.3158	25
36	.43208	.90183	.47912	.0872	.1088	.3143	24
37	.43235	.90171	.47948	.0856	.1090	.3129	23
38	.43261	.90158	.47983	.0840	.1092	.3115	22
39	.43287	.90145	.48019	.0825	.1093	.3101	21
40	.43313	.90133	.48055	2.0809	1.1095	2.3087	20
41	.43340	.90120	.48091	.0794	.1096	.3073	19
42	.43366	.90108	.48127	.0778	.1098	.3059	18
43	.43392	.90095	.48162	.0763	.1099	.3046	17
44	.43418	.90082	.48198	.0747	.1101	.3032	16
45	.43444	.90070	.48234	2.0732	1.1102	2.3018	15
46	.43471	.90057	.48270	.0717	.1104	.3004	14
47	.43497	.90044	.48306	.0701	.1106	.2990	13
48	.43523	.90032	.48342	.0686	.1107	.2976	12
49	.43549	.90019	.48378	.0671	.1109	.2962	11
50	.43575	.90006	.48414	2.0655	1.1110	2.2949	10
51	.43602	.89994	.48449	.0640	.1112	.2935	9
52	.43628	.89981	.48485	.0625	.1113	.2921	8
53	.43654	.89968	.48521	.0609	.1115	.2907	7
54	.43680	.89956	.48557	.0594	.1116	.2894	6
55	.43706	.89943	.48593	2.0579	1.1118	2.2880	5
56	.43732	.89930	.48629	.0564	.1120	.2866	4
57	.43759	.89918	.48665	.0548	.1121	.2853	3
58	.43785	.89905	.48701	.0533	.1123	.2839	2
59	.43811	.89892	.48737	.0518	.1124	.2825	1
60	.43837	.89879	.48773	2.0503	1.1126	2.2812	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.43837	.89879	.48773	2.0503	1.1126	2.2812	60
1	.43863	.89867	.48809	.0488	.1127	.2798	59
2	.43889	.89854	.48845	.0473	.1129	.2784	58
3	.43915	.89841	.48881	.0458	.1131	.2771	57
4	.43942	.89828	.48917	.0443	.1132	.2757	56
5	.43968	.89815	.48953	2.0427	1.1134	2.2744	55
6	.43994	.89803	.48989	.0412	.1135	.2730	54
7	.44020	.89790	.49025	.0397	.1137	.2717	53
8	.44046	.89777	.49062	.0382	.1139	.2703	52
9	.44072	.89764	.49098	.0367	.1140	.2690	51
10	.44098	.89751	.49134	2.0352	1.1142	2.2676	50
11	.44124	.89739	.49170	.0338	.1143	.2663	49
12	.44150	.89726	.49206	.0323	.1145	.2650	48
13	.44177	.89713	.49242	.0308	.1147	.2636	47
14	.44203	.89700	.49278	.0293	.1148	.2623	46
15	.44229	.89687	.49314	2.0278	1.1150	2.2610	45
16	.44255	.89674	.49351	.0263	.1151	.2596	44
17	.44281	.89661	.49387	.0248	.1153	.2583	43
18	.44307	.89649	.49423	.0233	.1155	.2570	42
19	.44333	.89636	.49459	.0219	.1156	.2556	41
20	.44359	.89623	.49495	2.0204	1.1158	2.2543	40
21	.44385	.89610	.49532	.0189	.1159	.2530	39
22	.44411	.89597	.49568	.0174	.1161	.2517	38
23	.44437	.89584	.49604	.0159	.1163	.2503	37
24	.44463	.89571	.49640	.0145	.1164	.2490	36
25	.44489	.89558	.49677	2.0130	1.1166	2.2477	35
26	.44516	.89545	.49713	.0115	.1167	.2464	34
27	.44542	.89532	.49749	.0101	.1169	.2451	33
28	.44568	.89519	.49785	.0086	.1171	.2438	32
29	.44594	.89506	.49822	.0071	.1172	.2425	31
30	.44620	.89493	.49858	2.0057	1.1174	2.2411	30
31	.44646	.89480	.49894	.0042	.1176	.2398	29
32	.44672	.89467	.49931	.0028	.1177	.2385	28
33	.44698	.89454	.49967	.0013	.1179	.2372	27
34	.44724	.89441	.50003	1.9998	.1180	.2359	26
35	.44750	.89428	.50040	1.9984	1.1182	2.2346	25
36	.44776	.89415	.50076	.9969	.1184	.2333	24
37	.44802	.89402	.50113	.9955	.1185	.2320	23
38	.44828	.89389	.50149	.9940	.1187	.2307	22
39	.44854	.89376	.50185	.9926	.1189	.2294	21
40	.44880	.89363	.50222	1.9912	1.1190	2.2282	20
41	.44906	.89350	.50258	.9897	.1192	.2269	19
42	.44932	.89337	.50295	.9883	.1193	.2256	18
43	.44958	.89324	.50331	.9868	.1195	.2243	17
44	.44984	.89311	.50368	.9854	.1197	.2230	16
45	.45010	.89298	.50404	1.9840	1.1198	2.2217	15
46	.45036	.89285	.50441	.9825	.1200	.2204	14
47	.45062	.89272	.50477	.9811	.1202	.2192	13
48	.45088	.89258	.50514	.9797	.1203	.2179	12
49	.45114	.89245	.50550	.9782	.1205	.2166	11
50	.45140	.89232	.50587	1.9768	1.1207	2.2153	10
51	.45166	.89219	.50623	.9754	.1208	.2141	9
52	.45191	.89206	.50660	.9739	.1210	.2128	8
53	.45217	.89193	.50696	.9725	.1212	.2115	7
54	.45243	.89180	.50733	.9711	.1213	.2103	6
55	.45269	.89166	.50769	1.9697	1.1215	2.2090	5
56	.45295	.89153	.50806	.9683	.1217	.2077	4
57	.45321	.89140	.50843	.9668	.1218	.2065	3
58	.45347	.89127	.50879	.9654	.1220	.2052	2
59	.45373	.89114	.50916	.9640	.1222	.2039	1
60	.45399	.89101	.50952	1.9626	1.1223	2.2027	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.42262	.90631	.46631	2.1445	1.1034	2.3662	60
1	.42288	.90618	.46666	.1429	.1035	.3647	59
2	.42314	.90606	.46702	.1412	.1037	.3632	58
3	.42341	.90594	.46737	.1396	.1038	.3618	57
4	.42367	.90581	.46772	.1380	.1040	.3603	56
5	.42394	.90569	.46808	2.1364	1.1041	2.3588	55
6	.42420	.90557	.46843	.1348	.1043	.3574	54
7	.42446	.90544	.46879	.1331	.1044	.3559	53
8	.42473	.90532	.46914	.1315	.1046	.3544	52
9	.42499	.90520	.46950	.1299	.1047	.3530	51
10	.42525	.90507	.46985	2.1283	1.1049	2.3515	50
11	.42552	.90495	.47021	.1267	.1050	.3501	49
12	.42578	.90483	.47056	.1251	.1052	.3486	48
13	.42604	.90470	.47092	.1235	.1053	.3472	47
14	.42630	.90458	.47127	.1219	.1055	.3457	46
15	.42657	.90445	.47163	2.1203	1.1056	2.3443	45
16	.42683	.90433	.47199	.1187	.1058	.3428	44
17	.42709	.90421	.47234	.1171	.1059	.3414	43
18	.42736	.90408	.47270	.1155	.1061	.3399	42
19	.42762	.90396	.47305	.1139	.1062	.3385	41
20	.42788	.90383	.47341	2.1123	1.1064	2.3371	40
21	.42815	.90371	.47376	.1107	.1065	.3356	39
22	.42841	.90358	.47412	.1092	.1067	.3342	38
23	.42867	.90346	.47448	.1076	.1068	.3328	37
24	.42893	.90333	.47483	.1060	.1070	.3313	36
25	.42920	.90321	.47519	2.1044	1.1072	2.3299	35
26	.42946	.90308	.47555	.1028	.1073	.3285	34
27	.42972	.90296	.47590	.1013	.1075	.3271	33
28	.42998	.90283	.47626	.0997	.1076	.3256	32
29	.43025	.90271	.47662	.0981	.1078	.3242	31
30	.43051	.90258	.47697	2.0965	1.1079	2.3228	30
31	.43077	.90246	.47733	.0950	.1081	.3214	29
32	.43104	.90233	.47769	.0934	.1082	.3200	28
33	.43130	.90221	.47805	.0918	.1084	.3186	27
34	.43156	.90208	.47840	.0903	.1085	.3172	26
35	.43182	.90196	.47876	2.0887	1.1087	2.3158	25
36	.43208	.90183	.47912	.0872	.1088	.3143	24
37	.43235	.90171	.47948	.0856	.1090	.3129	23
38	.43261	.90158	.47983	.0840	.1092	.3115	22
39	.43287	.90145	.48019	.0825	.1093	.3101	21
40	.43313	.90133	.48055	2.0809	1.1095	2.3087	20
41	.43340	.90120	.48091	.0794	.1096	.3073	19
42	.43366	.90108	.48127	.0778	.1098	.3059	18
43	.43392	.90095	.48162	.0763	.1099	.3046	17
44	.43418	.90082	.48198	.0747	.1101	.3032	16
45	.43444	.90070	.48234	2.0732	1.1102	2.3018	15
46	.43471	.90057	.48270	.0717	.1104	.3004	14
47	.43497	.90044	.48306	.0701	.1106	.2990	13
48	.43523	.90032	.48342	.0686	.1107	.2976	12
49	.43549	.90019	.48378	.0671	.1109	.2962	11
50	.43575	.90006	.48414	2.0655	1.1110	2.2949	10
51	.43602	.89994	.48449	.0640	.1112	.2935	9
52	.43628	.89981	.48485	.0625	.1113	.2921	8
53	.43654	.89968	.48521	.0609	.1115	.2907	7
54	.43680	.89956	.48557	.0594	.1116	.2894	6
55	.43706	.89943	.48593	2.0579	1.1118	2.2880	5
56	.43732	.89930	.48629	.0564	.1120	.2866	4
57	.43759	.89918	.48665	.0548	.1121	.2853	3
58	.43785	.89905	.48701	.0533	.1123	.2839	2
59	.43811	.89892	.48737	.0518	.1124	.2825	1
60	.43837	.89879	.48773	2.0503	1.1126	2.2812	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.43837	.89879	.48773	2.0503	1.1126	2.2812	60
1	.43863	.89867	.48809	.0488	.1127	.2798	59
2	.43889	.89854	.48845	.0473	.1129	.2784	58
3	.43915	.89841	.48881	.0458	.1131	.2771	57
4	.43942	.89828	.48917	.0443	.1132	.2757	56
5	.43968	.89815	.48953	2.0427	1.1134	2.2744	55
6	.43994	.89803	.48989	.0412	.1135	.2730	54
7	.44020	.89790	.49025	.0397	.1137	.2717	53
8	.44046	.89777	.49062	.0382	.1139	.2703	52
9	.44072	.89764	.49098	.0367	.1140	.2690	51
10	.44098	.89751	.49134	2.0352	1.1142	2.2676	50
11	.44124	.89739	.49170	.0338	.1143	.2663	49
12	.44150	.89726	.49206	.0323	.1145	.2650	48
13	.44177	.89713	.49242	.0308	.1147	.2636	47
14	.44203	.89700	.49278	.0293	.1148	.2623	46
15	.44229	.89687	.49314	2.0278	1.1150	2.2610	45
16	.44255	.89674	.49351	.0263	.1151	.2596	44
17	.44281	.89661	.49387	.0248	.1153	.2583	43
18	.44307	.89649	.49423	.0233	.1155	.2570	42
19	.44333	.89636	.49459	.0219	.1156	.2556	41
20	.44359	.89623	.49495	2.0204	1.1158	2.2543	40
21	.44385	.89610	.49532	.0189	.1159	.2530	39
22	.44411	.89597	.49568	.0174	.1161	.2517	38
23	.44437	.89584	.49604	.0159	.1163	.2503	37
24	.44463	.89571	.49640	.0145	.1164	.2490	36
25	.44489	.89558	.49677	2.0130	1.1166	2.2477	35
26	.44516	.89545	.49713	.0115	.1167	.2464	34
27	.44542	.89532	.49749	.0101	.1169	.2451	33
28	.44568	.89519	.49785	.0086	.1171	.2438	32
29	.44594	.89506	.49822	.0071	.1172	.2425	31
30	.44620	.89493	.49858	2.0057	1.1174	2.2411	30
31	.44646	.89480	.49894	.0042	.1176	.2398	29
32	.44672	.89467	.49931	.0028	.1177	.2385	28
33	.44698	.89454	.49967	.0013	.1179	.2372	27
34	.44724	.89441	.50003	1.9998	.1180	.2359	26
35	.44750	.89428	.50040	1.9984	1.1182	2.2346	25
36	.44776	.89415	.50076	.9969	.1184	.2333	24
37	.44802	.89402	.50113	.9955	.1185	.2320	23
38	.44828	.89389	.50149	.9940	.1187	.2307	22
39	.44854	.89376	.50185	.9926	.1189	.2294	21
40	.44880	.89363	.50222	1.9912	1.1190	2.2282	20
41	.44906	.89350	.50258	.9897	.1192	.2269	19
42	.44932	.89337	.50295	.9883	.1193	.2256	18
43	.44958	.89324	.50331	.9868	.1195	.2243	17
44	.44984	.89311	.50368	.9854	.1197	.2230	16
45	.45010	.89298	.50404	1.9840	1.1198	2.2217	15
46	.45036	.89285	.50441	.9825	.1200	.2204	14
47	.45062	.89272	.50477	.9811	.1202	.2192	13
48	.45088	.89258	.50514	.9797	.1203	.2179	12
49	.45114	.89245	.50550	.9782	.1205	.2166	11
50	.45140	.89232	.50587	1.9768	1.1207	2.2153	10
51	.45166	.89219	.50623	.9754	.1208	.2141	9
52	.45191	.89206	.50660	.9739	.1210	.2128	8
53	.45217	.89193	.50696	.9725	.1212	.2115	7
54	.45243	.89180	.50733	.9711	.1213	.2103	6
55	.45269	.89166	.50769	1.9697	1.1215	2.2090	5
56	.45295	.89153	.50806	.9683	.1217	.2077	4
57	.45321	.89140	.50843	.9668	.1218	.2065	3
58	.45347	.89127	.50879	.9654	.1220	.2052	2
59	.45373	.89114	.50916	.9640	.1222	.2039	1
60	.45399	.89101	.50952	1.9626	1.1223	2.2027	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.45399	.89101	.50952	1.9626	1.1223	2.2027	60
1	.45425	.89087	.50989	.9612	.1225	.2014	59
2	.45451	.89074	.51026	.9598	.1226	.2002	58
3	.45477	.89061	.51062	.9584	.1228	.1989	57
4	.45503	.89048	.51099	.9570	.1230	.1977	56
5	.45528	.89034	.51136	1.9556	1.1231	2.1964	55
6	.45554	.89021	.51172	.9542	.1233	.1952	54
7	.45580	.89008	.51209	.9528	.1235	.1939	53
8	.45606	.88995	.51246	.9514	.1237	.1927	52
9	.45632	.88981	.51283	.9500	.1238	.1914	51
10	.45658	.88968	.51319	1.9486	1.1240	2.1902	50
11	.45684	.88955	.51356	.9472	.1242	.1889	49
12	.45710	.88942	.51393	.9458	.1243	.1877	48
13	.45736	.88928	.51430	.9444	.1245	.1865	47
14	.45761	.88915	.51466	.9430	.1247	.1852	46
15	.45787	.88902	.51503	1.9416	1.1248	2.1840	45
16	.45813	.88888	.51540	.9402	.1250	.1828	44
17	.45839	.88875	.51577	.9388	.1252	.1815	43
18	.45865	.88862	.51614	.9375	.1253	.1803	42
19	.45891	.88848	.51651	.9361	.1255	.1791	41
20	.45917	.88835	.51687	1.9347	1.1257	2.1778	40
21	.45942	.88822	.51724	.9333	.1258	.1766	39
22	.45968	.88808	.51761	.9319	.1260	.1754	38
23	.45994	.88795	.51798	.9306	.1262	.1742	37
24	.46020	.88781	.51835	.9292	.1264	.1730	36
25	.46046	.88768	.51872	1.9278	1.1265	2.1717	35
26	.46072	.88755	.51909	.9264	.1267	.1705	34
27	.46097	.88741	.51946	.9251	.1269	.1693	33
28	.46123	.88728	.51983	.9237	.1270	.1681	32
29	.46149	.88714	.52020	.9223	.1272	.1669	31
30	.46175	.88701	.52057	1.9210	1.1274	2.1657	30
31	.46201	.88688	.52094	.9196	.1275	.1645	29
32	.46226	.88674	.52131	.9182	.1277	.1633	28
33	.46252	.88661	.52168	.9169	.1279	.1620	27
34	.46278	.88647	.52205	.9155	.1281	.1608	26
35	.46304	.88634	.52242	1.9142	1.1282	2.1596	25
36	.46330	.88620	.52279	.9128	.1284	.1584	24
37	.46355	.88607	.52316	.9115	.1286	.1572	23
38	.46381	.88593	.52353	.9101	.1287	.1560	22
39	.46407	.88580	.52390	.9088	.1289	.1548	21
40	.46433	.88566	.52427	1.9074	1.1291	2.1536	20
41	.46458	.88553	.52464	.9061	.1293	.1525	19
42	.46484	.88539	.52501	.9047	.1294	.1513	18
43	.46510	.88526	.52538	.9034	.1296	.1501	17
44	.46536	.88512	.52575	.9020	.1298	.1489	16
45	.46561	.88499	.52612	1.9007	1.1299	2.1477	15
46	.46587	.88485	.52650	.8993	.1301	.1465	14
47	.46613	.88472	.52687	.8980	.1303	.1453	13
48	.46639	.88458	.52724	.8967	.1305	.1441	12
49	.46664	.88444	.52761	.8953	.1306	.1430	11
50	.46690	.88431	.52798	1.8940	1.1308	2.1418	10
51	.46716	.88417	.52836	.8927	.1310	.1406	9
52	.46741	.88404	.52873	.8913	.1312	.1394	8
53	.46767	.88390	.52910	.8900	.1313	.1382	7
54	.46793	.88376	.52947	.8887	.1315	.1371	6
55	.46819	.88363	.52984	1.8873	1.1317	2.1359	5
56	.46844	.88349	.53022	.8860	.1319	.1347	4
57	.46870	.88336	.53059	.8847	.1320	.1335	3
58	.46896	.88322	.53096	.8834	.1322	.1324	2
59	.46921	.88308	.53134	.8820	.1324	.1312	1
60	.46947	.88295	.53171	1.8807	1.1326	2.1300	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan	Secant	Cosec.	M
0	.46947	.88295	.53171	1.8807	1.1326	2.1300	60
1	.46973	.88281	.53208	.8794	.1327	.1289	59
2	.46998	.88267	.53245	.8781	.1329	.1277	58
3	.47024	.88254	.53283	.8768	.1331	.1266	57
4	.47050	.88240	.53320	.8754	.1333	.1254	56
5	.47075	.88226	.53358	1.8741	1.1334	2.1242	55
6	.47101	.88213	.53395	.8728	.1336	.1231	54
7	.47127	.88199	.53432	.8715	.1338	.1219	53
8	.47152	.88185	.53470	.8702	.1340	.1208	52
9	.47178	.88171	.53507	.8689	.1341	.1196	51
10	.47204	.88158	.53545	1.8676	1.1343	2.1185	50
11	.47229	.88144	.53582	.8663	.1345	.1173	49
12	.47255	.88130	.53619	.8650	.1347	.1162	48
13	.47281	.88117	.53657	.8637	.1349	.1150	47
14	.47306	.88103	.53694	.8624	.1350	.1139	46
15	.47332	.88089	.53732	1.8611	1.1352	2.1127	45
16	.47357	.88075	.53769	.8598	.1354	.1116	44
17	.47383	.88061	.53807	.8585	.1356	.1104	43
18	.47409	.88048	.53844	.8572	.1357	.1093	42
19	.47434	.88034	.53882	.8559	.1359	.1082	41
20	.47460	.88020	.53919	1.8546	1.1361	2.1070	40
21	.47486	.88006	.53957	.8533	.1363	.1059	39
22	.47511	.87992	.53995	.8520	.1365	.1048	38
23	.47537	.87979	.54032	.8507	.1366	.1036	37
24	.47562	.87965	.54070	.8495	.1368	.1025	36
25	.47588	.87951	.54107	1.8482	1.1370	2.1014	35
26	.47613	.87937	.54145	.8469	.1372	.1002	34
27	.47639	.87923	.54183	.8456	.1373	.0991	33
28	.47665	.87909	.54220	.8443	.1375	.0980	32
29	.47690	.87895	.54258	.8430	.1377	.0969	31
30	.47716	.87882	.54295	1.8418	1.1379	2.0957	30
31	.47741	.87868	.54333	.8405	.1381	.0946	29
32	.47767	.87854	.54371	.8392	.1382	.0935	28
33	.47792	.87840	.54409	.8379	.1384	.0924	27
34	.47818	.87826	.54446	.8367	.1386	.0912	26
35	.47844	.87812	.54484	1.8354	1.1388	2.0901	25
36	.47869	.87798	.54522	.8341	.1390	.0890	24
37	.47895	.87784	.54559	.8329	.1391	.0879	23
38	.47920	.87770	.54597	.8316	.1393	.0868	22
39	.47946	.87756	.54635	.8303	.1395	.0857	21
40	.47971	.87742	.54673	1.8291	1.1397	2.0846	20
41	.47997	.87728	.54711	.8278	.1399	.0835	19
42	.48022	.87715	.54748	.8265	.1401	.0824	18
43	.48048	.87701	.54786	.8253	.1402	.0812	17
44	.48073	.87687	.54824	.8240	.1404	.0801	16
45	.48099	.87673	.54862	1.8227	1.1406	2.0790	15
46	.48124	.87659	.54900	.8215	.1408	.0779	14
47	.48150	.87645	.54937	.8202	.1410	.0768	13
48	.48175	.87631	.54975	.8190	.1411	.0757	12
49	.48201	.87617	.55013	.8177	.1413	.0746	11
50	.48226	.87603	.55051	1.8165	1.1415	2.0735	10
51	.48252	.87588	.55089	.8152	.1417	.0725	9
52	.48277	.87574	.55127	.8140	.1419	.0714	8
53	.48303	.87560	.55165	.8127	.1421	.0703	7
54	.48328	.87546	.55203	.8115	.1422	.0692	6
55	.48354	.87532	.55241	1.8102	1.1424	2.0681	5
56	.48379	.87518	.55279	.8090	.1426	.0670	4
57	.48405	.87504	.55317	.8078	.1428	.0659	3
58	.48430	.87490	.55355	.8065	.1430	.0648	2
59	.48455	.87476	.55393	.8053	.1432	.0637	1
60	.48481	.87462	.55431	1.8040	1.1433	2.0627	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.48481	.87462	.55431	1.8040	1.1433	2.0627	60
1	.48506	.87448	.55469	.8028	.1435	.0616	59
2	.48532	.87434	.55507	.8016	.1437	.0605	58
3	.48557	.87420	.55545	.8003	.1439	.0594	57
4	.48583	.87406	.55583	.7991	.1441	.0583	56
5	.48608	.87391	.55621	1.7979	1.1443	2.0573	55
6	.48633	.87377	.55659	.7966	.1445	.0562	54
7	.48659	.87363	.55697	.7954	.1446	.0551	53
8	.48684	.87349	.55735	.7942	.1448	.0540	52
9	.48710	.87335	.55774	.7930	.1450	.0530	51
10	.48735	.87320	.55812	1.7917	1.1452	2.0519	50
11	.48760	.87306	.55850	.7905	.1454	.0508	49
12	.48786	.87292	.55888	.7893	.1456	.0498	48
13	.48811	.87278	.55926	.7881	.1458	.0487	47
14	.48837	.87264	.55964	.7868	.1459	.0476	46
15	.48862	.87250	.56003	1.7856	1.1461	2.0466	45
16	.48887	.87235	.56041	.7844	.1463	.0455	44
17	.48913	.87221	.56079	.7832	.1465	.0444	43
18	.48938	.87207	.56117	.7820	.1467	.0434	42
19	.48964	.87193	.56156	.7808	.1469	.0423	41
20	.48989	.87178	.56194	1.7795	1.1471	2.0413	40
21	.49014	.87164	.56232	.7783	.1473	.0402	39
22	.49040	.87150	.56270	.7771	.1474	.0392	38
23	.49065	.87136	.56309	.7759	.1476	.0381	37
24	.49090	.87121	.56347	.7747	.1478	.0370	36
25	.49116	.87107	.56385	1.7735	1.1480	2.0360	35
26	.49141	.87093	.56424	.7723	.1482	.0349	34
27	.49166	.87078	.56462	.7711	.1484	.0339	33
28	.49192	.87064	.56500	.7699	.1486	.0329	32
29	.49217	.87050	.56539	.7687	.1488	.0318	31
30	.49242	.87035	.56577	1.7675	1.1489	2.0308	30
31	.49268	.87021	.56616	.7663	.1491	.0297	29
32	.49293	.87007	.56654	.7651	.1493	.0287	28
33	.49318	.86992	.56692	.7639	.1495	.0276	27
34	.49343	.86978	.56731	.7627	.1497	.0266	26
35	.49369	.86964	.56769	1.7615	1.1499	2.0256	25
36	.49394	.86949	.56808	.7603	.1501	.0245	24
37	.49419	.86935	.56846	.7591	.1503	.0235	23
38	.49445	.86921	.56885	.7579	.1505	.0224	22
39	.49470	.86906	.56923	.7567	.1507	.0214	21
40	.49495	.86892	.56962	1.7555	1.1508	2.0204	20
41	.49521	.86877	.57000	.7544	.1510	.0194	19
42	.49546	.86863	.57039	.7532	.1512	.0183	18
43	.49571	.86849	.57077	.7520	.1514	.0173	17
44	.49596	.86834	.57116	.7508	.1516	.0163	16
45	.49622	.86820	.57155	1.7496	1.1518	2.0152	15
46	.49647	.86805	.57193	.7484	.1520	.0142	14
47	.49672	.86791	.57232	.7473	.1522	.0132	13
48	.49697	.86776	.57270	.7461	.1524	.0122	12
49	.49723	.86762	.57309	.7449	.1526	.0111	11
50	.49748	.86748	.57348	1.7437	1.1528	2.0101	10
51	.49773	.86733	.57386	.7426	.1530	.0091	9
52	.49798	.86719	.57425	.7414	.1531	.0081	8
53	.49823	.86704	.57464	.7402	.1533	.0071	7
54	.49849	.86690	.57502	.7390	.1535	.0061	6
55	.49874	.86675	.57541	1.7379	1.1537	2.0050	5
56	.49899	.86661	.57580	.7367	.1539	.0040	4
57	.49924	.86646	.57619	.7355	.1541	.0030	3
58	.49950	.86632	.57657	.7344	.1543	.0020	2
59	.49975	.86617	.57696	.7332	.1545	.0010	1
60	.50000	.86603	.57735	1.7320	1.1547	2.0000	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.50000	.86603	.57735	1.7320	1.1547	2.0000	60
1	.50025	.86588	.57774	.7309	.1549	1.9990	59
2	.50050	.86573	.57813	.7297	.1551	.9980	58
3	.50075	.86559	.57851	.7286	.1553	.9970	57
4	.50101	.86544	.57890	.7274	.1555	.9960	56
5	.50126	.86530	.57929	1.7262	1.1557	1.9950	55
6	.50151	.86515	.57968	.7251	.1559	.9940	54
7	.50176	.86500	.58007	.7239	.1561	.9930	53
8	.50201	.86486	.58046	.7228	.1562	.9920	52
9	.50226	.86471	.58085	.7216	.1564	.9910	51
10	.50252	.86457	.58123	1.7205	1.1566	1.9900	50
11	.50277	.86442	.58162	.7193	.1568	.9890	49
12	.50302	.86427	.58201	.7182	.1570	.9880	48
13	.50327	.86413	.58240	.7170	.1572	.9870	47
14	.50352	.86398	.58279	.7159	.1574	.9860	46
15	.50377	.86383	.58318	1.7147	1.1576	1.9850	45
16	.50402	.86369	.58357	.7136	.1578	.9840	44
17	.50428	.86354	.58396	.7124	.1580	.9830	43
18	.50453	.86339	.58435	.7113	.1582	.9820	42
19	.50478	.86325	.58474	.7101	.1584	.9811	41
20	.50503	.86310	.58513	1.7090	1.1586	1.9801	40
21	.50528	.86295	.58552	.7079	.1588	.9791	39
22	.50553	.86281	.58591	.7067	.1590	.9781	38
23	.50578	.86266	.58630	.7056	.1592	.9771	37
24	.50603	.86251	.58670	.7044	.1594	.9761	36
25	.50628	.86237	.58709	1.7033	1.1596	1.9752	35
26	.50653	.86222	.58748	.7022	.1598	.9742	34
27	.50679	.86207	.58787	.7010	.1600	.9732	33
28	.50704	.86192	.58826	.6999	.1602	.9722	32
29	.50729	.86178	.58865	.6988	.1604	.9713	31
30	.50754	.86163	.58904	1.6977	1.1606	1.9703	30
31	.50779	.86148	.58944	.6965	.1608	.9693	29
32	.50804	.86133	.58983	.6954	.1610	.9683	28
33	.50829	.86118	.59022	.6943	.1612	.9674	27
34	.50854	.86104	.59061	.6931	.1614	.9664	26
35	.50879	.86089	.59100	1.6920	1.1616	1.9654	25
36	.50904	.86074	.59140	.6909	.1618	.9645	24
37	.50929	.86059	.59179	.6898	.1620	.9635	23
38	.50954	.86044	.59218	.6887	.1622	.9625	22
39	.50979	.86030	.59258	.6875	.1624	.9616	21
40	.51004	.86015	.59297	1.6864	1.1626	1.9606	20
41	.51029	.86000	.59336	.6853	.1628	.9596	19
42	.51054	.85985	.59376	.6842	.1630	.9587	18
43	.51079	.85970	.59415	.6831	.1632	.9577	17
44	.51104	.85955	.59454	.6820	.1634	.9568	16
45	.51129	.85941	.59494	1.6808	1.1636	1.9558	15
46	.51154	.85926	.59533	.6797	.1638	.9549	14
47	.51179	.85911	.59572	.6786	.1640	.9539	13
48	.51204	.85896	.59612	.6775	.1642	.9530	12
49	.51229	.85881	.59651	.6764	.1644	.9520	11
50	.51254	.85866	.59691	1.6753	1.1646	1.9510	10
51	.51279	.85851	.59730	.6742	.1648	.9501	9
52	.51304	.85836	.59770	.6731	.1650	.9491	8
53	.51329	.85821	.59809	.6720	.1652	.9482	7
54	.51354	.85806	.59849	.6709	.1654	.9473	6
55	.51379	.85791	.59888	1.6698	1.1656	1.9463	5
56	.51404	.85777	.59928	.6687	.1658	.9454	4
57	.51429	.85762	.59967	.6676	.1660	.9444	3
58	.51454	.85747	.60007	.6665	.1662	.9435	2
59	.51479	.85732	.60046	.6654	.1664	.9425	1
60	.51504	.85717	.60086	1.6643	1.1666	1.9416	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.51504	.85717	.60086	1.6643	1.1666	1.9416	60
1	.51529	.85702	.60126	.6632	.1668	.9407	59
2	.51554	.85687	.60165	.6621	.1670	.9397	58
3	.51578	.85672	.60205	.6610	.1672	.9388	57
4	.51603	.85657	.60244	.6599	.1674	.9378	56
5	.51628	.85642	.60284	1.6588	1.1676	1.9369	55
6	.51653	.85627	.60324	.6577	.1678	.9360	54
7	.51678	.85612	.60363	.6566	.1681	.9350	53
8	.51703	.85597	.60403	.6555	.1683	.9341	52
9	.51728	.85582	.60443	.6544	.1685	.9332	51
10	.51753	.85566	.60483	1.6534	1.1687	1.9322	50
11	.51778	.85551	.60522	.6523	.1689	.9313	49
12	.51803	.85536	.60562	.6512	.1691	.9304	48
13	.51827	.85521	.60602	.6501	.1693	.9295	47
14	.51852	.85506	.60642	.6490	.1695	.9285	46
15	.51877	.85491	.60681	1.6479	1.1697	1.9276	45
16	.51902	.85476	.60721	.6469	.1699	.9267	44
17	.51927	.85461	.60761	.6458	.1701	.9258	43
18	.51952	.85446	.60801	.6447	.1703	.9248	42
19	.51977	.85431	.60841	.6436	.1705	.9239	41
20	.52002	.85416	.60881	1.6425	1.1707	1.9230	40
21	.52026	.85400	.60920	.6415	.1709	.9221	39
22	.52051	.85385	.60960	.6404	.1712	.9212	38
23	.52076	.85370	.61000	.6393	.1714	.9203	37
24	.52101	.85355	.61040	.6383	.1716	.9193	36
25	.52126	.85340	.61080	1.6372	1.1718	1.9184	35
26	.52151	.85325	.61120	.6361	.1720	.9175	34
27	.52175	.85309	.61160	.6350	.1722	.9166	33
28	.52200	.85294	.61200	.6340	.1724	.9157	32
29	.52225	.85279	.61240	.6329	.1726	.9148	31
30	.52250	.85264	.61280	1.6318	1.1728	1.9139	30
31	.52275	.85249	.61320	.6308	.1730	.9130	29
32	.52299	.85234	.61360	.6297	.1732	.9121	28
33	.52324	.85218	.61400	.6286	.1734	.9112	27
34	.52349	.85203	.61440	.6276	.1737	.9102	26
35	.52374	.85188	.61480	1.6265	1.1739	1.9093	25
36	.52398	.85173	.61520	.6255	.1741	.9084	24
37	.52423	.85157	.61560	.6244	.1743	.9075	23
38	.52448	.85142	.61601	.6233	.1745	.9066	22
39	.52473	.85127	.61641	.6223	.1747	.9057	21
40	.52498	.85112	.61681	1.6212	1.1749	1.9048	20
41	.52522	.85096	.61721	.6202	.1751	.9039	19
42	.52547	.85081	.61761	.6191	.1753	.9030	18
43	.52572	.85066	.61801	.6181	.1756	.9021	17
44	.52597	.85050	.61842	.6170	.1758	.9013	16
45	.52621	.85035	.61882	1.6160	1.1760	1.9004	15
46	.52646	.85020	.61922	.6149	.1762	.8995	14
47	.52671	.85004	.61962	.6139	.1764	.8986	13
48	.52695	.84989	.62003	.6128	.1766	.8977	12
49	.52720	.84974	.62043	.6118	.1768	.8968	11
50	.52745	.84959	.62083	1.6107	1.1770	1.8959	10
51	.52770	.84943	.62123	.6097	.1772	.8950	9
52	.52794	.84928	.62164	.6086	.1775	.8941	8
53	.52819	.84912	.62204	.6076	.1777	.8932	7
54	.52844	.84897	.62244	.6066	.1779	.8924	6
55	.52868	.84882	.62285	1.6055	1.1781	1.8915	5
56	.52893	.84866	.62325	.6045	.1783	.8906	4
57	.52918	.84851	.62366	.6034	.1785	.8897	3
58	.52942	.84836	.62406	.6024	.1787	.8888	2
59	.52967	.84820	.62446	.6014	.1790	.8879	1
60	.52992	.84805	.62487	1.6003	1.1792	1.8871	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.52992	.84805	.62487	1.6003	1.1792	1.8871	60
1	.53016	.84789	.62527	.5993	.1794	.8862	59
2	.53041	.84774	.62568	.5983	.1796	.8853	58
3	.53066	.84758	.62608	.5972	.1798	.8844	57
4	.53090	.84743	.62649	.5962	.1800	.8836	56
5	.53115	.84728	.62689	1.5952	1.1802	1.8827	55
6	.53140	.84712	.62730	.5941	.1805	.8818	54
7	.53164	.84697	.62770	.5931	.1807	.8809	53
8	.53189	.84681	.62811	.5921	.1809	.8801	52
9	.53214	.84666	.62851	.5910	.1811	.8792	51
10	.53238	.84650	.62892	1.5900	1.1813	1.8783	50
11	.53263	.84635	.62933	.5890	.1815	.8775	49
12	.53288	.84619	.62973	.5880	.1818	.8766	48
13	.53312	.84604	.63014	.5869	.1820	.8757	47
14	.53337	.84588	.63055	.5859	.1822	.8749	46
15	.53361	.84573	.63095	1.5849	1.1824	1.8740	45
16	.53386	.84557	.63136	.5839	.1826	.8731	44
17	.53411	.84542	.63177	.5829	.1828	.8723	43
18	.53435	.84526	.63217	.5818	.1831	.8714	42
19	.53460	.84511	.63258	.5808	.1833	.8706	41
20	.53484	.84495	.63299	1.5798	1.1835	1.8697	40
21	.53509	.84479	.63339	.5788	.1837	.8688	39
22	.53533	.84464	.63380	.5778	.1839	.8680	38
23	.53558	.84448	.63421	.5768	.1841	.8671	37
24	.53583	.84433	.63462	.5757	.1844	.8663	36
25	.53607	.84417	.63503	1.5747	1.1846	1.8654	35
26	.53632	.84402	.63543	.5737	.1848	.8646	34
27	.53656	.84386	.63584	.5727	.1850	.8637	33
28	.53681	.84370	.63625	.5717	.1852	.8629	32
29	.53705	.84355	.63666	.5707	.1855	.8620	31
30	.53730	.84339	.63707	1.5697	1.1857	1.8611	30
31	.53754	.84323	.63748	.5687	.1859	.8603	29
32	.53779	.84308	.63789	.5677	.1861	.8595	28
33	.53803	.84292	.63830	.5667	.1863	.8586	27
34	.53828	.84276	.63871	.5657	.1866	.8578	26
35	.53852	.84261	.63912	1.5646	1.1868	1.8569	25
36	.53877	.84245	.63953	.5636	.1870	.8561	24
37	.53901	.84229	.63994	.5626	.1872	.8552	23
38	.53926	.84214	.64035	.5616	.1874	.8544	22
39	.53950	.84198	.64076	.5606	.1877	.8535	21
40	.53975	.84182	.64117	1.5596	1.1879	1.8527	20
41	.53999	.84167	.64158	.5586	.1881	.8519	19
42	.54024	.84151	.64199	.5577	.1883	.8510	18
43	.54048	.84135	.64240	.5567	.1886	.8502	17
44	.54073	.84120	.64281	.5557	.1888	.8493	16
45	.54097	.84104	.64322	1.5547	1.1890	1.8485	15
46	.54122	.84088	.64363	.5537	.1892	.8477	14
47	.54146	.84072	.64404	.5527	.1894	.8468	13
48	.54171	.84057	.64446	.5517	.1897	.8460	12
49	.54195	.84041	.64487	.5507	.1899	.8452	11
50	.54220	.84025	.64528	1.5497	1.1901	1.8443	10
51	.54244	.84009	.64569	.5487	.1903	.8435	9
52	.54268	.83993	.64610	.5477	.1906	.8427	8
53	.54293	.83978	.64652	.5467	.1908	.8418	7
54	.54317	.83962	.64693	.5458	.1910	.8410	6
55	.54342	.83946	.64734	1.5448	1.1912	1.8402	5
56	.54366	.83930	.64775	.5438	.1915	.8394	4
57	.54391	.83914	.64817	.5428	.1917	.8385	3
58	.54415	.83899	.64858	.5418	.1919	.8377	2
59	.54439	.83883	.64899	.5408	.1921	.8369	1
60	.54464	.83867	.64941	1.5399	1.1924	1.8361	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.54464	.83867	.64941	1.5399	1.1924	1.8361	60
1	.54488	.83851	.64982	.5389	.1926	.8352	59
2	.54513	.83835	.65023	.5379	.1928	.8344	58
3	.54537	.83819	.65065	.5369	.1930	.8336	57
4	.54561	.83804	.65106	.5359	.1933	.8328	56
5	.54586	.83788	.65148	1.5350	1.1935	1.8320	55
6	.54610	.83772	.65189	.5340	.1937	.8311	54
7	.54634	.83756	.65231	.5330	.1939	.8303	53
8	.54659	.83740	.65272	.5320	.1942	.8295	52
9	.54683	.83724	.65314	.5311	.1944	.8287	51
10	.54708	.83708	.65355	1.5301	1.1946	1.8279	50
11	.54732	.83692	.65397	.5291	.1948	.8271	49
12	.54756	.83676	.65438	.5282	.1951	.8263	48
13	.54781	.83660	.65480	.5272	.1953	.8255	47
14	.54805	.83644	.65521	.5262	.1955	.8246	46
15	.54829	.83629	.65563	1.5252	1.1958	1.8238	45
16	.54854	.83613	.65604	.5243	.1960	.8230	44
17	.54878	.83597	.65646	.5233	.1962	.8222	43
18	.54902	.83581	.65688	.5223	.1964	.8214	42
19	.54926	.83565	.65729	.5214	.1967	.8206	41
20	.54951	.83549	.65771	1.5204	1.1969	1.8198	40
21	.54975	.83533	.65813	.5195	.1971	.8190	39
22	.54999	.83517	.65854	.5185	.1974	.8182	38
23	.55024	.83501	.65896	.5175	.1976	.8174	37
24	.55048	.83485	.65938	.5166	.1978	.8166	36
25	.55072	.83469	.65980	1.5156	1.1980	1.8158	35
26	.55097	.83453	.66021	.5147	.1983	.8150	34
27	.55121	.83437	.66063	.5137	.1985	.8142	33
28	.55145	.83421	.66105	.5127	.1987	.8134	32
29	.55169	.83405	.66147	.5118	.1990	.8126	31
30	.55194	.83388	.66188	1.5108	1.1992	1.8118	30
31	.55218	.83372	.66230	.5099	.1994	.8110	29
32	.55242	.83356	.66272	.5089	.1997	.8102	28
33	.55266	.83340	.66314	.5080	.1999	.8094	27
34	.55291	.83324	.66356	.5070	.2001	.8086	26
35	.55315	.83308	.66398	1.5061	1.2004	1.8078	25
36	.55339	.83292	.66440	.5051	.2006	.8070	24
37	.55363	.83276	.66482	.5042	.2008	.8062	23
38	.55388	.83260	.66524	.5032	.2010	.8054	22
39	.55412	.83244	.66566	.5023	.2013	.8047	21
40	.55436	.83228	.66608	1.5013	1.2015	1.8039	20
41	.55460	.83211	.66650	.5004	.2017	.8031	19
42	.55484	.83195	.66692	.4994	.2020	.8023	18
43	.55509	.83179	.66734	.4985	.2022	.8015	17
44	.55533	.83163	.66776	.4975	.2024	.8007	16
45	.55557	.83147	.66818	1.4966	1.2027	1.7999	15
46	.55581	.83131	.66860	.4957	.2029	.7992	14
47	.55605	.83115	.66902	.4947	.2031	.7984	13
48	.55629	.83098	.66944	.4938	.2034	.7976	12
49	.55654	.83082	.66986	.4928	.2036	.7968	11
50	.55678	.83066	.67028	1.4919	1.2039	1.7960	10
51	.55702	.83050	.67071	.4910	.2041	.7953	9
52	.55726	.83034	.67113	.4900	.2043	.7945	8
53	.55750	.83017	.67155	.4891	.2046	.7937	7
54	.55774	.83001	.67197	.4881	.2048	.7929	6
55	.55799	.82985	.67239	1.4872	1.2050	1.7921	5
56	.55823	.82969	.67282	.4863	.2053	.7914	4
57	.55847	.82952	.67324	.4853	.2055	.7906	3
58	.55871	.82936	.67366	.4844	.2057	.7898	2
59	.55895	.82920	.67408	.4835	.2060	.7891	1
60	.55919	.82904	.67451	1.4826	1.2062	1.7883	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.55919	.82904	.67451	1.4826	1.2062	1.7883	60
1	.55943	.82887	.67493	.4816	.2064	.7875	59
2	.55967	.82871	.67535	.4807	.2067	.7867	58
3	.55992	.82855	.67578	.4798	.2069	.7860	57
4	.56016	.82839	.67620	.4788	.2072	.7852	56
5	.56040	.82822	.67663	1.4779	1.2074	1.7844	55
6	.56064	.82806	.67705	.4770	.2076	.7837	54
7	.56088	.82790	.67747	.4761	.2079	.7829	53
8	.56112	.82773	.67790	.4751	.2081	.7821	52
9	.56136	.82757	.67832	.4742	.2083	.7814	51
10	.56160	.82741	.67875	1.4733	1.2086	1.7806	50
11	.56184	.82724	.67917	.4724	.2088	.7798	49
12	.56208	.82708	.67960	.4714	.2091	.7791	48
13	.56232	.82692	.68002	.4705	.2093	.7783	47
14	.56256	.82675	.68045	.4696	.2095	.7776	46
15	.56280	.82659	.68087	1.4687	1.2098	1.7768	45
16	.56304	.82643	.68130	.4678	.2100	.7760	44
17	.56328	.82626	.68173	.4669	.2103	.7753	43
18	.56353	.82610	.68215	.4659	.2105	.7745	42
19	.56377	.82593	.68258	.4650	.2107	.7738	41
20	.56401	.82577	.68301	1.4641	1.2110	1.7730	40
21	.56425	.82561	.68343	.4632	.2112	.7723	39
22	.56449	.82544	.68386	.4623	.2115	.7715	38
23	.56473	.82528	.68429	.4614	.2117	.7708	37
24	.56497	.82511	.68471	.4605	.2119	.7700	36
25	.56521	.82495	.68514	1.4595	1.2122	1.7693	35
26	.56545	.82478	.68557	.4586	.2124	.7685	34
27	.56569	.82462	.68600	.4577	.2127	.7678	33
28	.56593	.82445	.68642	.4568	.2129	.7670	32
29	.56617	.82429	.68685	.4559	.2132	.7663	31
30	.56641	.82413	.68728	1.4550	1.2134	1.7655	30
31	.56664	.82396	.68771	.4541	.2136	.7648	29
32	.56688	.82380	.68814	.4532	.2139	.7640	28
33	.56712	.82363	.68857	.4523	.2141	.7633	27
34	.56736	.82347	.68899	.4514	.2144	.7625	26
35	.56760	.82330	.68942	1.4505	1.2146	1.7618	25
36	.56784	.82314	.68985	.4496	.2149	.7610	24
37	.56808	.82297	.69028	.4487	.2151	.7603	23
38	.56832	.82280	.69071	.4478	.2153	.7596	22
39	.56856	.82264	.69114	.4469	.2156	.7588	21
40	.56880	.82247	.69157	1.4460	1.2158	1.7581	20
41	.56904	.82231	.69200	.4451	.2161	.7573	19
42	.56928	.82214	.69243	.4442	.2163	.7566	18
43	.56952	.82198	.69286	.4433	.2166	.7559	17
44	.56976	.82181	.69329	.4424	.2168	.7551	16
45	.57000	.82165	.69372	1.4415	1.2171	1.7544	15
46	.57023	.82148	.69415	.4406	.2173	.7537	14
47	.57047	.82131	.69459	.4397	.2175	.7529	13
48	.57071	.82115	.69502	.4388	.2178	.7522	12
49	.57095	.82098	.69545	.4379	.2180	.7514	11
50	.57119	.82082	.69588	1.4370	1.2183	1.7507	10
51	.57143	.82065	.69631	.4361	.2185	.7500	9
52	.57167	.82048	.69674	.4352	.2188	.7493	8
53	.57191	.82032	.69718	.4343	.2190	.7485	7
54	.57214	.82015	.69761	.4335	.2193	.7478	6
55	.57238	.81998	.69804	1.4326	1.2195	1.7471	5
56	.57262	.81982	.69847	.4317	.2198	.7463	4
57	.57286	.81965	.69891	.4308	.2200	.7456	3
58	.57310	.81948	.69934	.4299	.2203	.7449	2
59	.57334	.81932	.69977	.4290	.2205	.7442	1
60	.57358	.81915	.70021	1.4281	1.2208	1.7434	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.57358	.81915	.70021	1.4281	1.2208	1.7434	60
1	.57381	.81898	.70064	.4273	.2210	.7427	59
2	.57405	.81882	.70107	.4264	.2213	.7420	58
3	.57429	.81865	.70151	.4255	.2215	.7413	57
4	.57453	.81848	.70194	.4246	.2218	.7405	56
5	.57477	.81832	.70238	1.4237	1.2220	1.7398	55
6	.57500	.81815	.70281	.4228	.2223	.7391	54
7	.57524	.81798	.70325	.4220	.2225	.7384	53
8	.57548	.81781	.70368	.4211	.2228	.7377	52
9	.57572	.81765	.70412	.4202	.2230	.7369	51
10	.57596	.81748	.70455	1.4193	1.2233	1.7362	50
11	.57619	.81731	.70499	.4185	.2235	.7355	49
12	.57643	.81714	.70542	.4176	.2238	.7348	48
13	.57667	.81698	.70586	.4167	.2240	.7341	47
14	.57691	.81681	.70629	.4158	.2243	.7334	46
15	.57714	.81664	.70673	1.4150	1.2245	1.7327	45
16	.57738	.81647	.70717	.4141	.2248	.7319	44
17	.57762	.81630	.70760	.4132	.2250	.7312	43
18	.57786	.81614	.70804	.4123	.2253	.7305	42
19	.57809	.81597	.70848	.4115	.2355	.7298	41
20	.57833	.81580	.70891	1.4106	1.2258	1.7291	40
21	.57857	.81563	.70935	.4097	.2260	.7284	39
22	.57881	.81546	.70979	.4089	.2263	.7277	38
23	.57904	.81530	.71022	.4080	.2265	.7270	37
24	.57928	.81513	.71066	.4071	.2268	.7263	36
25	.57952	.81496	.71110	1.4063	1.2270	1.7256	35
26	.57975	.81479	.71154	.4054	.2273	.7249	34
27	.57999	.81462	.71198	.4045	.2276	.7242	33
28	.58023	.81445	.71241	.4037	.2278	.7234	32
29	.58047	.81428	.71285	.4028	.2281	.7227	31
30	.58070	.81411	.71329	1.4019	1.2283	1.7220	30
31	.58094	.81395	.71373	.4011	.2286	.7213	29
32	.58118	.81378	.71417	.4002	.2288	.7206	28
33	.58141	.81361	.71461	.3994	.2291	.7199	27
34	.58165	.81344	.71505	.3985	.2293	.7192	26
35	.58189	.81327	.71549	1.3976	1.2296	1.7185	25
36	.58212	.81310	.71593	.3968	.2298	.7178	24
37	.58236	.81293	.71637	.3959	.2301	.7171	23
38	.58259	.81276	.71681	.3951	.2304	.7164	22
39	.58283	.81259	.71725	.3942	.2306	.7157	21
40	.58307	.81242	.71769	1.3933	1.2309	1.7151	20
41	.58330	.81225	.71813	.3925	.2311	.7144	19
42	.58354	.81208	.71857	.3916	.2314	.7137	18
43	.58378	.81191	.71901	.3908	.2316	.7130	17
44	.58401	.81174	.71945	.3899	.2319	.7123	16
45	.58425	.81157	.71990	1.3891	1.2322	1.7116	15
46	.58448	.81140	.72034	.3882	.2324	.7109	14
47	.58472	.81123	.72078	.3874	.2327	.7102	13
48	.58496	.81106	.72122	.3865	.2329	.7095	12
49	.58519	.81089	.72166	.3857	.2332	.7088	11
50	.58543	.81072	.72211	1.3848	1.2335	1.7081	10
51	.58566	.81055	.72255	.3840	.2337	.7075	9
52	.58590	.81038	.72299	.3831	.2340	.7068	8
53	.58614	.81021	.72344	.3823	.2342	.7061	7
54	.58637	.81004	.72388	.3814	.2345	.7054	6
55	.58661	.80987	.72432	1.3806	1.2348	1.7047	5
56	.58684	.80970	.72477	.3797	.2350	.7040	4
57	.58708	.80953	.72521	.3789	.2353	.7033	3
58	.58731	.80936	.72565	.3781	.2355	.7027	2
59	.58755	.80919	.72610	.3772	.2358	.7020	1
60	.58778	.80902	.72654	1.3764	1.2361	1.7013	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.58778	.80902	.72654	1.3764	1.2361	1.7013	60
1	.58802	.80885	.72699	.3755	.2363	.7006	59
2	.58825	.80867	.72743	.3747	.2366	.6999	58
3	.58849	.80850	.72788	.3738	.2368	.6993	57
4	.58873	.80833	.72832	.3730	.2371	.6986	56
5	.58896	.80816	.72877	1.3722	1.2374	1.6979	55
6	.58920	.80799	.72921	.3713	.2376	.6972	54
7	.58943	.80782	.72966	.3705	.2379	.6965	53
8	.58967	.80765	.73010	.3697	.2382	.6959	52
9	.58990	.80747	.73055	.3688	.2384	.6952	51
10	.59014	.80730	.73100	1.3680	1.2387	1.6945	50
11	.59037	.80713	.73144	.3672	.2389	.6938	49
12	.59060	.80696	.73189	.3663	.2392	.6932	48
13	.59084	.80679	.73234	.3655	.2395	.6925	47
14	.59107	.80662	.73278	.3647	.2397	.6918	46
15	.59131	.80644	.73323	1.3638	1.2400	1.6912	45
16	.59154	.80627	.73368	.3630	.2403	.6905	44
17	.59178	.80610	.73412	.3622	.2405	.6898	43
18	.59201	.80593	.73457	.3613	.2408	.6891	42
19	.59225	.80576	.73502	.3605	.2411	.6885	41
20	.59248	.80558	.73547	1.3597	1.2413	1.6878	40
21	.59272	.80541	.73592	.3588	.2416	.6871	39
22	.59295	.80524	.73637	.3580	.2419	.6865	38
23	.59318	.80507	.73681	.3572	.2421	.6858	37
24	.59342	.80489	.73726	.3564	.2424	.6851	36
25	.59365	.80472	.73771	1.3555	1.2427	1.6845	35
26	.59389	.80455	.73816	.3547	.2429	.6838	34
27	.59412	.80437	.73861	.3539	.2432	.6831	33
28	.59435	.80420	.73906	.3531	.2435	.6825	32
29	.59459	.80403	.73951	.3522	.2437	.6818	31
30	.59482	.80386	.73996	1.3514	1.2440	1.6812	30
31	.59506	.80368	.74041	.3506	.2443	.6805	29
32	.59529	.80351	.74086	.3498	.2445	.6798	28
33	.59552	.80334	.74131	.3489	.2448	.6792	27
34	.59576	.80316	.74176	.3481	.2451	.6785	26
35	.59599	.80299	.74221	1.3473	1.2453	1.6779	25
36	.59622	.80282	.74266	.3465	.2456	.6772	24
37	.59646	.80264	.74312	.3457	.2459	.6766	23
38	.59669	.80247	.74357	.3449	.2461	.6759	22
39	.59692	.80230	.74402	.3440	.2464	.6752	21
40	.59716	.80212	.74447	1.3432	1.2467	1.6746	20
41	.59739	.80195	.74492	.3424	.2470	.6739	19
42	.59762	.80177	.74538	.3416	.2472	.6733	18
43	.59786	.80160	.74583	.3408	.2475	.6726	17
44	.59809	.80143	.74628	.3400	.2478	.6720	16
45	.59832	.80125	.74673	1.3392	1.2480	1.6713	15
46	.59856	.80108	.74719	.3383	.2483	.6707	14
47	.59879	.80090	.74764	.3375	.2486	.6700	13
48	.59902	.80073	.74809	.3367	.2488	.6694	12
49	.59926	.80056	.74855	.3359	.2491	.6687	11
50	.59949	.80038	.74900	1.3351	1.2494	1.6681	10
51	.59972	.80021	.74946	.3343	.2497	.6674	9
52	.59995	.80003	.74991	.3335	.2499	.6668	8
53	.60019	.79986	.75037	.3327	.2502	.6661	7
54	.60042	.79968	.75082	.3319	.2505	.6655	6
55	.60065	.79951	.75128	1.3311	1.2508	1.6648	5
56	.60088	.79933	.75173	.3303	.2510	.6642	4
57	.60112	.79916	.75219	.3294	.2513	.6636	3
58	.60135	.79898	.75264	.3286	.2516	.6629	2
59	.60158	.79881	.75310	.3278	.2519	.6623	1
60	.60181	.79863	.75355	1.3270	1.2521	1.6616	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.60181	.79863	.75355	1.3270	1.2521	1.6616	60
1	.60205	.79846	.75401	.3262	.2524	.6610	59
2	.60228	.79828	.75447	.3254	.2527	.6603	58
3	.60251	.79811	.75492	.3246	.2530	.6597	57
4	.60274	.79793	.75538	.3238	.2532	.6591	56
5	.60298	.79776	.75584	1.3230	1.2535	1.6584	55
6	.60320	.79758	.75629	.3222	.2538	.6578	54
7	.60344	.79741	.75675	.3214	.2541	.6572	53
8	.60367	.79723	.75721	.3206	.2543	.6565	52
9	.60390	.79706	.75767	.3198	.2546	.6559	51
10	.60413	.79688	.75812	1.3190	1.2549	1.6552	50
11	.60437	.79670	.75858	.3182	.2552	.6546	49
12	.60460	.79653	.75904	.3174	.2554	.6540	48
13	.60483	.79635	.75950	.3166	.2557	.6533	47
14	.60506	.79618	.75996	.3159	.2560	.6527	46
15	.60529	.79600	.76042	1.3151	1.2563	1.6521	45
16	.60552	.79582	.76088	.3143	.2565	.6514	44
17	.60576	.79565	.76134	.3135	.2568	.6508	43
18	.60599	.79547	.76179	.3127	.2571	.6502	42
19	.60622	.79530	.76225	.3119	.2574	.6496	41
20	.60645	.79512	.76271	1.3111	1.2577	1.6489	40
21	.60668	.79494	.76317	.3103	.2579	.6483	39
22	.60691	.79477	.76364	.3095	.2582	.6477	38
23	.60714	.79459	.76410	.3087	.2585	.6470	37
24	.60737	.79441	.76456	.3079	.2588	.6464	36
25	.60761	.79424	.76502	1.3071	1.2591	1.6458	35
26	.60784	.79406	.76548	.3064	.2593	.6452	34
27	.60807	.79388	.76594	.3056	.2596	.6445	33
28	.60830	.79371	.76640	.3048	.2599	.6439	32
29	.60853	.79353	.76686	.3040	.2602	.6433	31
30	.60876	.79335	.76733	1.3032	1.2605	1.6427	30
31	.60899	.79318	.76779	.3024	.2607	.6420	29
32	.60922	.79300	.76825	.3016	.2610	.6414	28
33	.60945	.79282	.76871	.3009	.2613	.6408	27
34	.60968	.79264	.76918	.3001	.2616	.6402	26
35	.60991	.79247	.76964	1.2993	1.2619	1.6396	25
36	.61014	.79229	.77010	.2985	.2622	.6389	24
37	.61037	.79211	.77057	.2977	.2624	.6383	23
38	.61061	.79193	.77103	.2970	.2627	.6377	22
39	.61084	.79176	.77149	.2962	.2630	.6371	21
40	.61107	.79158	.77196	1.2954	1.2633	1.6365	20
41	.61130	.79140	.77242	.2946	.2636	.6359	19
42	.61153	.79122	.77289	.2938	.2639	.6352	18
43	.61176	.79104	.77335	.2931	.2641	.6346	17
44	.61199	.79087	.77382	.2923	.2644	.6340	16
45	.61222	.79069	.77428	1.2915	1.2647	1.6334	15
46	.61245	.79051	.77475	.2907	.2650	.6328	14
47	.61268	.79033	.77521	.2900	.2653	.6322	13
48	.61290	.79015	.77568	.2892	.2656	.6316	12
49	.61314	.78998	.77614	.2884	.2659	.6309	11
50	.61337	.78980	.77661	1.2876	1.2661	1.6303	10
51	.61360	.78962	.77708	.2869	.2664	.6297	9
52	.61383	.78944	.77754	.2861	.2667	.6291	8
53	.61405	.78926	.77801	.2853	.2670	.6285	7
54	.61428	.78908	.77848	.2845	.2673	.6279	6
55	.61451	.78890	.77895	1.2838	1.2676	1.6273	5
56	.61474	.78873	.77941	.2830	.2679	.6267	4
57	.61497	.78855	.77988	.2822	.2681	.6261	3
58	.61520	.78837	.78035	.2815	.2684	.6255	2
59	.61543	.78819	.78082	.2807	.2687	.6249	1
60	.61566	.78801	.78128	1.2799	1.2690	1.6243	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.61566	.78801	.78128	1.2799	1.2690	1.6243	60
1	.61589	.78783	.78175	.2792	.2693	.6237	59
2	.61612	.78765	.78222	.2784	.2696	.6231	58
3	.61635	.78747	.78269	.2776	.2699	.6224	57
4	.61658	.78729	.78316	.2769	.2702	.6218	56
5	.61681	.78711	.78363	1.2761	1.2705	1.6212	55
6	.61703	.78693	.78410	.2753	.2707	.6205	54
7	.61726	.78675	.78457	.2746	.2710	.6200	53
8	.61749	.78657	.78504	.2738	.2713	.6194	52
9	.61772	.78640	.78551	.2730	.2716	.6188	51
10	.61795	.78622	.78598	1.2723	1.2719	1.6182	50
11	.61818	.78604	.78645	.2715	.2722	.6176	49
12	.61841	.78586	.78692	.2708	.2725	.6170	48
13	.61864	.78568	.78739	.2700	.2728	.6164	47
14	.61886	.78550	.78786	.2692	.2731	.6159	46
15	.61909	.78532	.78834	1.2685	1.2734	1.6153	45
16	.61932	.78514	.78881	.2677	.2737	.6147	44
17	.61955	.78496	.78928	.2670	.2739	.6141	43
18	.61978	.78478	.78975	.2662	.2742	.6135	42
19	.62001	.78460	.79022	.2655	.2745	.6129	41
20	.62023	.78441	.79070	1.2647	1.2748	1.6123	40
21	.62046	.78423	.79117	.2639	.2751	.6117	39
22	.62069	.78405	.79164	.2632	.2754	.6111	38
23	.62092	.78387	.79212	.2624	.2757	.6105	37
24	.62115	.78369	.79259	.2617	.2760	.6099	36
25	.62137	.78351	.79306	1.2609	1.2763	1.6093	35
26	.62160	.78333	.79354	.2602	.2766	.6087	34
27	.62183	.78315	.79401	.2594	.2769	.6081	33
28	.62206	.78297	.79449	.2587	.2772	.6077	32
29	.62229	.78279	.79496	.2579	.2775	.6070	31
30	.62251	.78261	.79543	1.2572	1.2778	1.6064	30
31	.62274	.78243	.79591	.2564	.2781	.6058	29
32	.62297	.78224	.79639	.2557	.2784	.6052	28
33	.62320	.78206	.79686	.2549	.2787	.6046	27
34	.62342	.78188	.79734	.2542	.2790	.6040	26
35	.62365	.78170	.79781	1.2534	1.2793	1.6034	25
36	.62388	.78152	.79829	.2527	.2795	.6029	24
37	.62411	.78134	.79876	.2519	.2798	.6023	23
38	.62433	.78116	.79924	.2512	.2801	.6017	22
39	.62456	.78097	.79972	.2504	.2804	.6011	21
40	.62479	.78079	.80020	1.2497	1.2807	1.6005	20
41	.62501	.78061	.80067	.2489	.2810	.6000	19
42	.62524	.78043	.80115	.2482	.2813	.5994	18
43	.62547	.78025	.80163	.2475	.2816	.5988	17
44	.62570	.78007	.80211	.2467	.2819	.5982	16
45	.62592	.77988	.80258	1.2460	1.2822	1.5976	15
46	.62615	.77970	.80306	.2452	.2825	.5971	14
47	.62638	.77952	.80354	.2445	.2828	.5965	13
48	.62660	.77934	.80402	.2437	.2831	.5959	12
49	.62683	.77915	.80450	.2430	.2834	.5953	11
50	.62706	.77897	.80498	1.2423	1.2837	1.5947	10
51	.62728	.77879	.80546	.2415	.2840	.5942	9
52	.62751	.77861	.80594	.2408	.2843	.5936	8
53	.62774	.77842	.80642	.2400	.2846	.5930	7
54	.62796	.77824	.80690	.2393	.2849	.5924	6
55	.62819	.77806	.80738	1.2386	1.2852	1.5919	5
56	.62841	.77788	.80786	.2378	.2855	.5913	4
57	.62864	.77769	.80834	.2371	.2858	.5907	3
58	.62887	.77751	.80882	.2364	.2861	.5901	2
59	.62909	.77733	.80930	.2356	.2864	.5896	1
60	.62932	.77715	.80978	1.2349	1.2867	1.5890	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.62932	.77715	.80978	1.2349	1.2867	1.5890	60
1	.62955	.77696	.81026	.2342	.2871	.5884	59
2	.62977	.77678	.81075	.2334	.2874	.5879	58
3	.63000	.77660	.81123	.2327	.2877	.5873	57
4	.63022	.77641	.81171	.2320	.2880	.5867	56
5	.63045	.77623	.81219	1.2312	1.2883	1.5862	55
6	.63067	.77605	.81268	.2305	.2886	.5856	54
7	.63090	.77586	.81316	.2297	.2889	.5850	53
8	.63113	.77568	.81364	.2290	.2892	.5845	52
9	.63135	.77549	.81413	.2283	.2895	.5839	51
10	.63158	.77531	.81461	1.2276	1.2898	1.5833	50
11	.63180	.77513	.81509	.2268	.2901	.5828	49
12	.63203	.77494	.81558	.2261	.2904	.5822	48
13	.63225	.77476	.81606	.2254	.2907	.5816	47
14	.63248	.77458	.81655	.2247	.2910	.5811	46
15	.63270	.77439	.81703	1.2239	1.2913	1.5805	45
16	.63293	.77421	.81752	.2232	.2916	.5799	44
17	.63315	.77402	.81800	.2225	.2919	.5794	43
18	.63338	.77384	.81849	.2218	.2922	.5788	42
19	.63360	.77365	.81898	.2210	.2926	.5783	41
20	.63383	.77347	.81946	1.2203	1.2929	1.5777	40
21	.63405	.77329	.81995	.2196	.2932	.5771	39
22	.63428	.77310	.82043	.2189	.2935	.5766	38
23	.63450	.77292	.82092	.2181	.2938	.5760	37
24	.63473	.77273	.82141	.2174	.2941	.5755	36
25	.63495	.77255	.82190	1.2167	1.2944	1.5749	35
26	.63518	.77236	.82238	.2160	.2947	.5743	34
27	.63540	.77218	.82287	.2152	.2950	.5738	33
28	.63563	.77199	.82336	.2145	.2953	.5732	32
29	.63585	.77181	.82385	.2138	.2956	.5727	31
30	.63608	.77162	.82434	1.2131	1.2960	1.5721	30
31	.63630	.77144	.82482	.2124	.2963	.5716	29
32	.63653	.77125	.82531	.2117	.2966	.5710	28
33	.63675	.77107	.82580	.2109	.2969	.5705	27
34	.63697	.77088	.82629	.2102	.2972	.5699	26
35	.63720	.77070	.82678	1.2095	1.2975	1.5694	25
36	.63742	.77051	.82727	.2088	.2978	.5688	24
37	.63765	.77033	.82776	.2081	.2981	.5683	23
38	.63787	.77014	.82825	.2074	.2985	.5677	22
39	.63810	.76996	.82874	.2066	.2988	.5672	21
40	.63832	.76977	.82923	1.2059	1.2991	1.5666	20
41	.63854	.76958	.82972	.2052	.2994	.5661	19
42	.63877	.76940	.83022	.2045	.2997	.5655	18
43	.63899	.76921	.83071	.2038	.3000	.5650	17
44	.63921	.76903	.83120	.2031	.3003	.5644	16
45	.63944	.76884	.83169	1.2024	1.3006	1.5639	15
46	.63966	.76865	.83218	.2016	.3010	.5633	14
47	.63989	.76847	.83267	.2009	.3013	.5628	13
48	.64011	.76828	.83317	.2002	.3016	.5622	12
49	.64033	.76810	.83366	.1995	.3019	.5617	11
50	.64056	.76791	.83415	1.1988	1.3022	1.5611	10
51	.64078	.76772	.83465	.1981	.3025	.5606	9
52	.64100	.76754	.83514	.1974	.3029	.5600	8
53	.64123	.76735	.83563	.1967	.3032	.5595	7
54	.64145	.76716	.83613	.1960	.3035	.5590	6
55	.64167	.76698	.83662	1.1953	1.3038	1.5584	5
56	.64189	.76679	.83712	.1946	.3041	.5579	4
57	.64212	.76660	.83761	.1939	.3044	.5573	3
58	.64234	.76642	.83811	.1932	.3048	.5568	2
59	.64256	.76623	.83860	.1924	.3051	.5563	1
60	.64279	.76604	.83910	1.1917	1.3054	1.5557	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.64279	.76604	.83910	1.1917	1.3054	1.5557	60
1	.64301	.76586	.83959	.1910	.3057	.5552	59
2	.64323	.76567	.84009	.1903	.3060	.5546	58
3	.64345	.76548	.84059	.1896	.3064	.5541	57
4	.64368	.76530	.84108	.1889	.3067	.5536	56
5	.64390	.76511	.84158	1.1882	1.3070	1.5530	55
6	.64412	.76492	.84208	.1875	.3073	.5525	54
7	.64435	.76473	.84257	.1868	.3076	.5520	53
8	.64457	.76455	.84307	.1861	.3080	.5514	52
9	.64479	.76436	.84357	.1854	.3083	.5509	51
10	.64501	.76417	.84407	1.1847	1.3086	1.5503	50
11	.64523	.76398	.84457	.1840	.3089	.5498	49
12	.64546	.76380	.84506	.1833	.3092	.5493	48
13	.64568	.76361	.84556	.1826	.3096	.5487	47
14	.64590	.76342	.84606	.1819	.3099	.5482	46
15	.64612	.76323	.84656	1.1812	1.3102	1.5477	45
16	.64635	.76304	.84706	.1805	.3105	.5471	44
17	.64657	.76286	.84756	.1798	.3109	.5466	43
18	.64679	.76267	.84806	.1791	.3112	.5461	42
19	.64701	.76248	.84856	.1785	.3115	.5456	41
20	.64723	.76229	.84906	1.1778	1.3118	1.5450	40
21	.64745	.76210	.84956	.1771	.3121	.5445	39
22	.64768	.76191	.85006	.1764	.3125	.5440	38
23	.64790	.76173	.85056	.1757	.3128	.5434	37
24	.64812	.76154	.85107	.1750	.3131	.5429	36
25	.64834	.76135	.85157	1.1743	1.3134	1.5424	35
26	.64856	.76116	.85207	.1736	.3138	.5419	34
27	.64878	.76097	.85257	.1729	.3141	.5413	33
28	.64900	.76078	.85307	.1722	.3144	.5408	32
29	.64923	.76059	.85358	.1715	.3148	.5403	31
30	.64945	.76041	.85408	1.1708	1.3151	1.5398	30
31	.64967	.76022	.85458	.1702	.3154	.5392	29
32	.64989	.76003	.85509	.1695	.3157	.5387	28
33	.65011	.75984	.85559	.1688	.3161	.5382	27
34	.65033	.75965	.85609	.1681	.3164	.5377	26
35	.65055	.75946	.85660	1.1674	1.3167	1.5371	25
36	.65077	.75927	.85710	.1667	.3170	.5366	24
37	.65100	.75908	.85761	.1660	.3174	.5361	23
38	.65121	.75889	.85811	.1653	.3177	.5356	22
39	.65144	.75870	.85862	.1647	.3180	.5351	21
40	.65166	.75851	.85912	1.1640	1.3184	1.5345	20
41	.65188	.75832	.85963	.1633	.3187	.5340	19
42	.65210	.75813	.86013	.1626	.3190	.5335	18
43	.65232	.75794	.86064	.1619	.3193	.5330	17
44	.65254	.75775	.86115	.1612	.3197	.5325	16
45	.65276	.75756	.86165	1.1605	1.3200	1.5319	15
46	.65298	.75737	.86216	.1599	.3203	.5314	14
47	.65320	.75718	.86267	.1592	.3207	.5309	13
48	.65342	.75700	.86318	.1585	.3210	.5304	12
49	.65364	.75680	.86368	.1578	.3213	.5299	11
50	.65386	.75661	.86419	1.1571	1.3217	1.5294	10
51	.65408	.75642	.86470	.1565	.3220	.5289	9
52	.65430	.75623	.86521	.1558	.3223	.5283	8
53	.65452	.75604	.86572	.1551	.3227	.5278	7
54	.65474	.75585	.86623	.1544	.3230	.5273	6
55	.65496	.75566	.86674	1.1537	1.3233	1.5268	5
56	.65518	.75547	.86725	.1531	.3237	.5263	4
57	.65540	.75528	.86775	.1524	.3240	.5258	3
58	.65562	.75509	.86826	.1517	.3243	.5253	2
59	.65584	.75490	.86878	.1510	.3247	.5248	1
60	.65606	.75471	.86929	1.1504	1.3250	1.5242	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.65606	.75471	.86929	1.1504	1.3250	1.5242	60
1	.65628	.75452	.86980	.1497	.3253	.5237	59
2	.65650	.75433	.87031	.1490	.3257	.5232	58
3	.65672	.75414	.87082	.1483	.3260	.5227	57
4	.65694	.75394	.87133	.1477	.3263	.5222	56
5	.65716	.75375	.87184	1.1470	1.3267	1.5217	55
6	.65737	.75356	.87235	.1463	.3270	.5212	54
7	.65759	.75337	.87287	.1456	.3274	.5207	53
8	.65781	.75318	.87338	.1450	.3277	.5202	52
9	.65803	.75299	.87389	.1443	.3280	.5197	51
10	.65825	.75280	.87441	1.1436	1.3284	1.5192	50
11	.65847	.75261	.87492	.1430	.3287	.5187	49
12	.65869	.75241	.87543	.1423	.3290	.5182	48
13	.65891	.75222	.87595	.1416	.3294	.5177	47
14	.65913	.75203	.87646	.1409	.3297	.5171	46
15	.65934	.75184	.87698	1.1403	1.3301	1.5166	45
16	.65956	.75165	.87749	.1396	.3304	.5161	44
17	.65978	.75146	.87801	.1389	.3307	.5156	43
18	.66000	.75126	.87852	.1383	.3311	.5151	42
19	.66022	.75107	.87904	.1376	.3314	.5146	41
20	.66044	.75088	.87955	1.1369	1.3318	1.5141	40
21	.66066	.75069	.88007	.1363	.3321	.5136	39
22	.66087	.75049	.88058	.1356	.3324	.5131	38
23	.66109	.75030	.88110	.1349	.3328	.5126	37
24	.66131	.75011	.88162	.1343	.3331	.5121	36
25	.66153	.74992	.88213	1.1336	1.3335	1.5116	35
26	.66175	.74973	.88265	.1329	.3338	.5111	34
27	.66197	.74953	.88317	.1323	.3342	.5106	33
28	.66218	.74934	.88369	.1316	.3345	.5101	32
29	.66240	.74915	.88421	.1309	.3348	.5096	31
30	.66262	.74895	.88472	1.1303	1.3352	1.5092	30
31	.66284	.74876	.88524	.1296	.3355	.5087	29
32	.66305	.74857	.88576	.1290	.3359	.5082	28
33	.66327	.74838	.88628	.1283	.3362	.5077	27
34	.66349	.74818	.88680	.1276	.3366	.5072	26
35	.66371	.74799	.88732	1.1270	1.3369	1.5067	25
36	.66393	.74780	.88784	.1263	.3372	.5062	24
37	.66414	.74760	.88836	.1257	.3376	.5057	23
38	.66436	.74741	.88888	.1250	.3379	.5052	22
39	.66458	.74722	.88940	.1243	.3383	.5047	21
40	.66479	.74702	.88992	1.1237	1.3386	1.5042	20
41	.66501	.74683	.89044	.1230	.3390	.5037	19
42	.66523	.74664	.89097	.1224	.3393	.5032	18
43	.66545	.74644	.89149	.1217	.3397	.5027	17
44	.66566	.74625	.89201	.1211	.3400	.5022	16
45	.66588	.74606	.89253	1.1204	1.3404	1.5018	15
46	.66610	.74586	.89306	.1197	.3407	.5013	14
47	.66631	.74567	.89358	.1191	.3411	.5008	13
48	.66653	.74548	.89410	.1184	.3414	.5003	12
49	.66675	.74528	.89463	.1178	.3418	.4998	11
50	.66697	.74509	.89515	1.1171	1.3421	1.4993	10
51	.66718	.74489	.89567	.1165	.3425	.4988	9
52	.66740	.74470	.89620	.1158	.3428	.4983	8
53	.66762	.74450	.89672	.1152	.3432	.4979	7
54	.66783	.74431	.89725	.1145	.3435	.4974	6
55	.66805	.74412	.89777	1.1139	1.3439	1.4969	5
56	.66826	.74392	.89830	.1132	.3442	.4964	4
57	.66848	.74373	.89882	.1126	.3446	.4959	3
58	.66870	.74353	.89935	.1119	.3449	.4954	2
59	.66891	.74334	.89988	.1113	.3453	.4949	1
60	.66913	.74314	.90040	1.1106	1.3456	1.4945	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.66913	.74314	.90040	1.1106	1.3456	1.4945	60
1	.66935	.74295	.90093	.1100	.3460	.4940	59
2	.66956	.74275	.90146	.1093	.3463	.4935	58
3	.66978	.74256	.90198	.1086	.3467	.4930	57
4	.66999	.74236	.90251	.1080	.3470	.4925	56
5	.67021	.74217	.90304	1.1074	1.3474	1.4921	55
6	.67043	.74197	.90357	.1067	.3477	.4916	54
7	.67064	.74178	.90410	.1061	.3481	.4911	53
8	.67086	.74158	.90463	.1054	.3485	.4906	52
9	.67107	.74139	.90515	.1048	.3488	.4901	51
10	.67129	.74119	.90568	1.1041	1.3492	1.4897	50
11	.67150	.74100	.90621	.1035	.3495	.4892	49
12	.67172	.74080	.90674	.1028	.3499	.4887	48
13	.67194	.74061	.90727	.1022	.3502	.4882	47
14	.67215	.74041	.90780	.1015	.3506	.4877	46
15	.67237	.74022	.90834	1.1009	1.3509	1.4873	45
16	.67258	.74002	.90887	.1003	.3513	.4868	44
17	.67280	.73983	.90940	.0996	.3517	.4863	43
18	.67301	.73963	.90993	.0990	.3520	.4858	42
19	.67323	.73943	.91046	.0983	.3524	.4854	41
20	.67344	.73924	.91099	1.0977	1.3527	1.4849	40
21	.67366	.73904	.91153	.0971	.3531	.4844	39
22	.67387	.73885	.91206	.0964	.3534	.4839	38
23	.67409	.73865	.91259	.0958	.3538	.4835	37
24	.67430	.73845	.91312	.0951	.3542	.4830	36
25	.67452	.73826	.91366	1.0945	1.3545	1.4825	35
26	.67473	.73806	.91419	.0939	.3549	.4821	34
27	.67495	.73787	.91473	.0932	.3552	.4816	33
28	.67516	.73767	.91526	.0926	.3556	.4811	32
29	.67537	.73747	.91580	.0919	.3560	.4806	31
30	.67559	.73728	.91633	1.0913	1.3563	1.4802	30
31	.67580	.73708	.91687	.0907	.3567	.4797	29
32	.67602	.73688	.91740	.0900	.3571	.4792	28
33	.67623	.73669	.91794	.0894	.3574	.4788	27
34	.67645	.73649	.91847	.0888	.3578	.4783	26
35	.67666	.73629	.91901	1.0881	1.3581	1.4778	25
36	.67688	.73610	.91955	.0875	.3585	.4774	24
37	.67709	.73590	.92008	.0868	.3589	.4769	23
38	.67730	.73570	.92062	.0862	.3592	.4764	22
39	.67752	.73551	.92116	.0856	.3596	.4760	21
40	.67773	.73531	.92170	1.0849	1.3600	1.4755	20
41	.67794	.73511	.92223	.0843	.3603	.4750	19
42	.67816	.73491	.92277	.0837	.3607	.4746	18
43	.67837	.73472	.92331	.0830	.3611	.4741	17
44	.67859	.73452	.92385	.0824	.3614	.4736	16
45	.67880	.73432	.92439	1.0818	1.3618	1.4732	15
46	.67901	.73412	.92493	.0812	.3622	.4727	14
47	.67923	.73393	.92547	.0805	.3625	.4723	13
48	.67944	.73373	.92601	.0799	.3629	.4718	12
49	.67965	.73353	.92655	.0793	.3633	.4713	11
50	.67987	.73333	.92709	1.0786	1.3636	1.4709	10
51	.68008	.73314	.92763	.0780	.3640	.4704	9
52	.68029	.73294	.92817	.0774	.3644	.4699	8
53	.68051	.73274	.92871	.0767	.3647	.4695	7
54	.68072	.73254	.92926	.0761	.3651	.4690	6
55	.68093	.73234	.92980	1.0755	1.3655	1.4686	5
56	.68115	.73215	.93034	.0749	.3658	.4681	4
57	.68136	.73195	.93088	.0742	.3662	.4676	3
58	.68157	.73175	.93143	.0736	.3666	.4672	2
59	.68178	.73155	.93197	.0730	.3669	.4667	1
60	.68200	.73135	.93251	1.0724	1.3673	1.4663	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

43°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	M
0	.68200	.73135	.93251	1.0724	1.3673	1.4663	60
1	.68221	.73115	.93306	.0717	.3677	.4658	59
2	.68242	.73096	.93360	.0711	.3681	.4654	58
3	.68264	.73076	.93415	.0705	.3684	.4649	57
4	.68285	.73056	.93469	.0699	.3688	.4644	56
5	.68306	.73036	.93524	1.0692	1.3692	1.4640	55
6	.68327	.73016	.93578	.0686	.3695	.4635	54
7	.68349	.72996	.93633	.0680	.3699	.4631	53
8	.68370	.72976	.93687	.0674	.3703	.4626	52
9	.68391	.72956	.93742	.0667	.3707	.4622	51
10	.68412	.72937	.93797	1.0661	1.3710	1.4617	50
11	.68433	.72917	.93851	.0655	.3714	.4613	49
12	.68455	.72897	.93906	.0649	.3718	.4608	48
13	.68476	.72877	.93961	.0643	.3722	.4604	47
14	.68497	.72857	.94016	.0636	.3725	.4599	46
15	.68518	.72837	.94071	1.0630	1.3729	1.4595	45
16	.68539	.72817	.94125	.0624	.3733	.4590	44
17	.68561	.72797	.94180	.0618	.3737	.4586	43
18	.68582	.72777	.94235	.0612	.3740	.4581	42
19	.68603	.72757	.94290	.0605	.3744	.4577	41
20	.68624	.72737	.94345	1.0599	1.3748	1.4572	40
21	.68645	.72717	.94400	.0593	.3752	.4568	39
22	.68666	.72697	.94455	.0587	.3756	.4563	38
23	.68688	.72677	.94510	.0581	.3759	.4559	37
24	.68709	.72657	.94565	.0575	.3763	.4554	36
25	.68730	.72637	.94620	1.0568	1.3767	1.4550	35
26	.68751	.72617	.94675	.0562	.3771	.4545	34
27	.68772	.72597	.94731	.0556	.3774	.4541	33
28	.68793	.72577	.94786	.0550	.3778	.4536	32
29	.68814	.72557	.94841	.0544	.3782	.4532	31
30	.68835	.72537	.94896	1.0538	1.3786	1.4527	30
31	.68856	.72517	.94952	.0532	.3790	.4523	29
32	.68878	.72497	.95007	.0525	.3794	.4518	28
33	.68899	.72477	.95062	.0519	.3797	.4514	27
34	.68920	.72457	.95118	.0513	.3801	.4510	26
35	.68941	.72437	.95173	1.0507	1.3805	1.4505	25
36	.68962	.72417	.95229	.0501	.3809	.4501	24
37	.68983	.72397	.95284	.0495	.3813	.4496	23
38	.69004	.72377	.95340	.0489	.3816	.4492	22
39	.69025	.72357	.95395	.0483	.3820	.4487	21
40	.69046	.72337	.95451	1.0476	1.3824	1.4483	20
41	.69067	.72317	.95506	.0470	.3828	.4479	19
42	.69088	.72297	.95562	.0464	.3832	.4474	18
43	.69109	.72277	.95618	.0458	.3836	.4470	17
44	.69130	.72256	.95673	.0452	.3839	.4465	16
45	.69151	.72236	.95729	1.0446	1.3843	1.4461	15
46	.69172	.72216	.95785	.0440	.3847	.4457	14
47	.69193	.72196	.95841	.0434	.3851	.4452	13
48	.69214	.72176	.95896	.0428	.3855	.4448	12
49	.69235	.72156	.95952	.0422	.3859	.4443	11
50	.69256	.72136	.96008	1.0416	1.3863	1.4439	10
51	.69277	.72115	.96064	.0410	.3867	.4435	9
52	.69298	.72095	.96120	.0404	.3870	.4430	8
53	.69319	.72075	.96176	.0397	.3874	.4426	7
54	.69340	.72055	.96232	.0391	.3878	.4422	6
55	.69361	.72035	.96288	1.0385	1.3882	1.4417	5
56	.69382	.72015	.96344	.0379	.3886	.4413	4
57	.69403	.71994	.96400	.0373	.3890	.4408	3
58	.69424	.71974	.96456	.0367	.3894	.4404	2
59	.69445	.71954	.96513	.0361	.3898	.4400	1
60	.69466	.71934	.96569	1.0355	1.3902	1.4395	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

46°

M	Sine	Cosine	Tan	Cotan.	Secant	Cosec.	M
0	.69466	.71934	.96569	1.0355	1.3902	1.4395	60
1	.69487	.71914	.96625	.0349	.3905	.4391	59
2	.69508	.71893	.96681	.0343	.3909	.4387	58
3	.69528	.71873	.96738	.0337	.3913	.4382	57
4	.69549	.71853	.96794	.0331	.3917	.4378	56
5	.69570	.71833	.96850	1.0325	1.3921	1.4374	55
6	.69591	.71813	.96907	.0319	.3925	.4370	54
7	.69612	.71792	.96963	.0313	.3929	.4365	53
8	.69633	.71772	.97020	.0307	.3933	.4361	52
9	.69654	.71752	.97076	.0301	.3937	.4357	51
10	.69675	.71732	.97133	1.0295	1.3941	1.4352	50
11	.69696	.71711	.97189	.0289	.3945	.4348	49
12	.69716	.71691	.97246	.0283	.3949	.4344	48
13	.69737	.71671	.97302	.0277	.3953	.4339	47
14	.69758	.71650	.97359	.0271	.3957	.4335	46
15	.69779	.71630	.97416	1.0265	1.3960	1.4331	45
16	.69800	.71610	.97472	.0259	.3964	.4327	44
17	.69821	.71589	.97529	.0253	.3968	.4322	43
18	.69841	.71569	.97586	.0247	.3972	.4318	42
19	.69862	.71549	.97643	.0241	.3976	.4314	41
20	.69883	.71529	.97700	1.0235	1.3980	1.4310	40
21	.69904	.71508	.97756	.0229	.3984	.4305	39
22	.69925	.71488	.97813	.0223	.3988	.4301	38
23	.69945	.71468	.97870	.0218	.3992	.4297	37
24	.69966	.71447	.97927	.0212	.3996	.4292	36
25	.69987	.71427	.97984	1.0206	1.4000	1.4288	35
26	.70008	.71406	.98041	.0200	.4004	.4284	34
27	.70029	.71386	.98098	.0194	.4008	.4280	33
28	.70049	.71366	.98155	.0188	.4012	.4276	32
29	.70070	.71345	.98212	.0182	.4016	.4271	31
30	.70091	.71325	.98270	1.0176	1.4020	1.4267	30
31	.70112	.71305	.98327	.0170	.4024	.4263	29
32	.70132	.71284	.98384	.0164	.4028	.4259	28
33	.70153	.71264	.98441	.0158	.4032	.4254	27
34	.70174	.71243	.98499	.0152	.4036	.4250	26
35	.70194	.71223	.98556	1.0146	1.4040	1.4246	25
36	.70215	.71203	.98613	.0141	.4044	.4242	24
37	.70236	.71182	.98671	.0135	.4048	.4238	23
38	.70257	.71162	.98728	.0129	.4052	.4233	22
39	.70277	.71141	.98786	.0123	.4056	.4229	21
40	.70298	.71121	.98843	1.0117	1.4060	1.4225	20
41	.70319	.71100	.98901	.0111	.4065	.4221	19
42	.70339	.71080	.98958	.0105	.4069	.4217	18
43	.70360	.71059	.99016	.0099	.4073	.4212	17
44	.70381	.71039	.99073	.0093	.4077	.4208	16
45	.70401	.71018	.99131	1.0088	1.4081	1.4204	15
46	.70422	.70998	.99189	.0082	.4085	.4200	14
47	.70443	.70977	.99246	.0076	.4089	.4196	13
48	.70463	.70957	.993 4	.0070	.4093	.4192	12
49	.70484	.70936	.99362	.0064	.4097	.4188	11
50	.70505	.70916	.99420	1.0058	1.4101	1.4183	10
51	.70525	.70895	.99478	.0052	.4105	.4179	9
52	.70546	.70875	.99536	.0047	.4109	.4175	8
53	.70566	.70854	.99593	.0041	.4113	.4171	7
54	.70587	.70834	.99651	.0035	.4117	.4167	6
55	.70608	.70813	.99709	1.0029	1.4122	1.4163	5
56	.70628	.70793	.99767	.0023	.4126	.4159	4
57	.70649	.70772	.99826	.0017	.4130	.4154	3
58	.70669	.70752	.99884	.0012	.4134	.4150	2
59	.70690	.70731	.99942	.0006	.4138	.4146	1
60	.70711	.70711	1.0000	1.0000	1.4142	1.4142	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	M

FACTOR TABLE

The following factor table consists of all numbers that can be made up of two factors not exceeding 100.

Prime numbers are not listed.

Four change gears with teeth between the limits of 20 and 100 will form enough different combinations so that ratios close enough for all practical purposes may be obtained.

To find the prime factors of a number, take the factors given in the table and find the prime factors of them.

EXAMPLE: $1197 = (21 \times 57) = (3 \times 7)\ (3 \times 19) = 3 \times 3 \times 7 \times 19$.

No.	Factors	No.	Factors	No.	Factors	No.	Factors	No.	Factors
20	4–5	99	9–11	176	11–16	259	7–37	350	14–25
21	3–7	100	4–25	177	3–59	260	13–20	351	13–27
22	2–11	102	6–17	178	2–89	261	9–29	352	11–32
24	4–6	104	8–13	180	12–15	264	11–24	354	6–59
25	5–5	105	7–15	182	13–14	265	5–53	355	5–71
26	2–13	106	2–53	183	3–61	266	14–19	356	4–89
27	3–9	108	4–27	184	8–23	267	3–89	357	17–21
28	2–14	110	10–11	185	5–37	268	4–67	360	15–24
30	3–10	111	3–37	186	6–31	270	10–27	361	19–19
32	4–8	112	7–16	187	11–17	272	16–17	363	11–33
33	3–11	114	6–19	188	4–47	273	13–21	364	13–28
34	2–17	115	5–23	189	7–27	275	11–25	365	5–73
35	5–7	116	4–29	190	10–19	276	12–23	366	6–61
36	4–9	117	9–13	192	12–16	279	9–31	368	16–23
38	2–19	118	2–59	194	2–97	280	8–35	369	9–41
39	3–13	119	7–17	195	13–15	282	6–47	370	10–37
40	5–8	120	8–15	196	14–14	284	4–71	371	7–53
42	6–7	121	11–11	198	11–18	285	15–19	372	12–31
44	4–11	122	2–61	200	8–25	286	13–22	374	17–22
45	5–9	123	3–41	201	3–67	287	7–41	375	15–25
46	2–23	124	4–31	203	7–29	288	9–32	376	8–47
48	6–8	125	5–25	204	12–17	289	17–17	377	13–29
49	7–7	126	7–18	205	5–41	290	10–29	378	14–27
50	5–10	128	8–16	207	9–23	291	3–97	380	19–20
51	3–17	129	3–43	208	13–16	292	4–73	384	12–32
52	4–13	130	10–13	209	11–19	294	6–49	385	11–35
54	6–9	132	11–12	210	6–35	295	5–59	387	9–43
55	5–11	133	7–19	212	4–53	296	8–37	388	4–97
56	7–8	134	2–67	213	3–71	297	11–27	390	15–26
57	3–19	135	5–27	215	5–43	299	13–23	391	17–23
58	2–29	136	8–17	216	8–27	300	12–25	392	8–49
60	4–15	138	6–23	217	7–31	301	7–43	395	5–79
62	2–31	140	7–20	219	3–73	304	16–19	396	12–33
63	7–9	141	3–47	220	11–20	305	5–61	399	19–21
64	8–8	142	2–71	221	13–17	306	17–18	400	10–40
65	5–13	143	11–13	222	6–37	308	11–28	402	6–67
66	6–11	144	9–16	224	7–32	310	10–31	403	13–31
68	4–17	145	5–29	225	9–25	312	13–24	405	15–27
69	3–23	146	2–73	228	12–19	315	9–35	406	14–29
70	7–10	147	3–49	230	10–23	316	4–79	407	11–37
72	8–9	148	4–37	231	11–21	318	6–53	408	17–24
74	2–37	150	6–25	232	8–29	319	11–29	410	10–41
75	5–15	152	8–19	234	13–18	320	5–64	413	7–59
76	4–19	153	9–17	235	5–47	322	14–23	414	18–23
77	7–11	154	11–14	236	4–59	323	17–19	415	5–83
78	6–13	155	5–31	237	3–79	324	12–27	416	13–32
80	5–16	156	12–13	238	14–17	325	13–25	418	19–22
81	9–9	158	2–79	240	16–15	328	8–41	420	12–35
82	2–41	159	3–53	242	11–22	329	7–47	423	9–47
84	7–12	160	5–32	243	9–27	330	6–55	425	17–25
85	5–17	161	7–23	244	4–61	332	4–83	426	6–71
86	2–43	162	6–27	245	5–49	333	9–37	427	7–61
87	3–29	164	4–41	246	6–41	335	5–67	429	13–33
88	8–11	165	11–15	247	13–19	336	16–21	430	10–43
90	9–10	166	2–83	248	8–31	338	13–26	432	16–27
91	7–13	168	8–21	249	3–83	340	17–20	434	14–31
92	4–23	169	13–13	250	10–25	341	11–31	435	15–29
93	3–31	170	10–17	252	14–18	342	18–19	437	19–23
94	2–47	171	9–19	253	11–23	343	7–49	438	6–73
95	5–19	172	4–43	255	15–17	344	8–43	440	11–40
96	3–32	174	6–29	256	16–16	345	15–23	441	9–49
98	7–14	175	7–25	258	6–43	348	12–29	442	17–26

No.	Factors	No.	Factors	No.	Factors	No.	Factors	No.	Factors
444	12–37	546	13–42	649	11–59	756	27–28	870	29–30
445	5–89	549	9–61	650	25–26	759	23–33	871	13–67
448	14–32	550	22–25	651	21–31	760	19–40	873	9–97
450	18–25	551	19–29	656	16–41	765	17–45	874	23–38
451	11–41	552	23–24	657	9–73	767	13–59	875	25–35
455	13–35	553	7–79	658	14–47	768	12–64	876	12–73
456	19–24	555	15–37	660	12–55	770	22–35	880	20–44
459	17–27	558	18–31	663	17–39	774	18–43	882	18–49
460	20–23	559	13–43	664	8–83	775	25–31	884	26–34
462	11–42	560	16–35	665	19–35	776	8–97	885	15–59
464	16–29	561	17–33	666	18–37	777	21–37	888	24–37
465	15–31	564	12–47	667	23–29	779	19–41	890	10–89
468	13–36	567	21–27	670	10–67	780	20–39	891	27–33
469	7–67	568	8–71	671	11–61	781	11–71	893	19–47
470	10–47	570	19–30	672	21–32	782	23–34	896	28–32
472	8–59	572	13–44	675	25–27	783	27–29	897	23–39
473	11–43	574	14–41	676	26–26	784	16–49	899	29–31
474	6–79	575	23–25	679	7–97	790	10–79	900	25–36
475	19–25	576	9–64	680	17–40	792	24–33	901	17–53
476	17–28	578	17–34	682	22–31	793	13–61	902	22–41
477	9–53	580	20–29	684	18–38	795	15–53	903	21–43
480	15–32	581	7–83	686	14–49	798	21–38	910	26–35
481	13–37	582	6–97	688	16–43	799	17–47	912	24–38
483	21–23	583	11–53	689	13–53	800	25–32	913	11–83
484	22–22	584	8–73	690	23–30	801	9–89	915	15–61
485	5–97	585	13–45	693	21–33	803	11–73	918	27–34
486	18–27	588	12–49	696	24–29	804	12–67	920	23–40
488	8–61	589	19–31	697	17–41	805	23–35	923	13–71
490	10–49	590	10–59	700	25–28	806	26–31	924	28–33
492	12–41	592	16–37	702	26–27	810	27–30	925	25–37
493	17–29	594	11–54	703	19–37	812	28–29	928	29–32
494	19–26	595	17–35	704	11–64	814	22–37	930	30–31
495	15–33	598	23–26	705	15–47	816	24–34	931	19–49
496	16–31	600	24–25	708	12–59	817	19–43	935	17–55
497	7–71	602	14–43	710	10–71	819	21–39	936	24–39
498	6–83	603	9–67	711	9–79	820	20–41	938	14–67
500	20–25	605	11–55	712	8–89	825	25–33	940	20–47
504	21–24	608	19–32	713	23–31	826	14–59	943	23–41
506	22–23	609	21–29	714	21–34	828	23–36	944	16–59
507	13–39	610	10–61	715	13–55	830	10–83	945	27–35
510	17–30	611	13–47	720	20–36	832	13–64	946	22–43
511	7–73	612	17–36	722	19–38	833	17–49	948	12–79
512	16–32	615	15–41	725	25–29	836	22–38	949	13–73
513	19–27	616	22–28	726	22–33	837	27–31	950	19–50
516	12–43	620	20–31	728	13–56	840	24–35	952	28–34
517	11–47	621	23–27	729	27–27	841	29–29	954	18–53
518	14–37	623	7–89	730	10–73	845	13–65	957	29–33
520	13–40	624	24–26	731	17–43	846	18–47	960	30–32
522	18–29	625	25–25	732	12–61	847	11–77	961	31–31
525	21–25	627	19–33	735	21–35	848	16–53	962	26–37
527	17–31	629	17–37	736	23–32	850	25–34	966	23–42
528	16–33	630	18–35	737	11–67	851	23–37	968	22–44
529	23–23	632	8–79	738	18–41	852	12–71	969	19–51
530	10–53	636	12–53	740	20–37	854	14–61	970	10–97
531	9–59	637	13–49	741	19–39	855	19–45	972	27–36
532	19–28	638	22–29	742	14–53	858	22–39	975	25–39
533	13–41	639	9–71	744	24–31	860	20–43	976	16–61
534	6–89	640	10–64	747	9–83	861	21–41	979	11–89
536	8–67	644	23–28	748	44–17	864	27–32	980	20–49
539	11–49	645	15–43	750	25–30	867	17–51	984	24–41
540	20–27	646	19–34	752	16–47	868	28–31	986	29–34
544	17–32	648	24–27	754	26–29	869	11–79	987	21–47

No.	Factors	No.	Factors	No.	Factors	No.	Factors	No.	Factors
988	26–38	1105	17–65	1230	30–41	1358	14–97	1485	27–55
989	23–43	1106	14–79	1232	28–44	1360	20–68	1488	24–62
990	22–45	1107	27–41	1235	19–65	1363	29–47	1491	21–71
992	31–32	1110	30–37	1239	21–59	1364	31–44	1494	18–83
994	14–71	1113	21–53	1240	40–31	1365	35–39	1495	23–65
996	12–83	1116	31–36	1241	17–73	1368	24–57	1496	34–44
999	27–37	1118	26–43	1242	23–54	1369	37–37	1500	25–60
1000	25–40	1120	28–40	1245	15–83	1372	28–49	1501	19–79
1001	13–77	1121	19–59	1246	14–89	1375	25–55	1504	32–47
1003	17–59	1122	33–34	1247	29–43	1376	32–43	1505	35–43
1005	15–67	1125	25–45	1248	32–39	1377	27–51	1508	26–58
1007	19–53	1127	23–49	1250	25–50	1378	26–53	1512	28–54
1008	21–48	1128	24–47	1254	33–38	1380	23–60	1513	17–89
1012	22–46	1131	29–39	1258	34–37	1386	21–66	1517	37–41
1014	26–39	1134	21–54	1260	28–45	1387	19–73	1518	33–46
1015	29–35	1136	16–71	1261	13–97	1392	29–48	1519	31–49
1020	20–51	1139	17–67	1264	16–79	1394	34–41	1520	38–40
1022	14–73	1140	20–57	1265	23–55	1395	31–45	1521	39–39
1023	31–33	1144	26–44	1269	27–47	1400	35–40	1525	25–61
1024	32–32	1147	31–37	1271	31–41	1403	23–61	1530	30–51
1025	25–41	1148	28–41	1272	24–53	1404	36–39	1533	21–73
1026	27–38	1150	25–46	1273	19–67	1406	37–38	1534	26–59
1027	13–79	1152	32–36	1274	26–49	1407	21–67	1536	32–48
1029	21–49	1155	33–35	1275	25–51	1408	32–44	1537	29–53
1032	24–43	1156	34–34	1276	29–44	1410	30–47	1539	27–57
1034	22–47	1157	13–89	1278	18–71	1411	17–83	1540	28–55
1035	23–45	1159	19–61	1280	16–80	1416	24–59	1541	23–67
1036	28–37	1160	20–58	1281	21–61	1419	33–43	1547	17–91
1037	17–61	1161	27–43	1287	33–39	1420	20–71	1548	36–43
1040	26–40	1162	14–83	1288	28–46	1421	29–49	1550	31–50
1044	29–36	1164	12–97	1290	30–43	1422	18–79	1551	33–47
1045	19–55	1166	22–53	1292	34–38	1424	16–89	1552	16–97
1050	25–42	1168	16–73	1295	35–37	1425	25–57	1554	37–42
1053	27–39	1170	30–39	1296	16–81	1426	31–46	1558	38–41
1054	31–34	1173	23–51	1298	22–59	1428	34–42	1560	24–65
1056	24–44	1175	25–47	1300	25–52	1430	26–55	1562	22–71
1058	23–46	1176	24–49	1302	21–62	1431	27–53	1564	34–46
1060	20–53	1178	31–38	1305	29–45	1435	35–41	1566	29–54
1062	18–59	1180	20–59	1309	17–77	1440	32–45	1568	28–56
1064	28–38	1183	13–91	1311	23–57	1443	37–39	1575	25–63
1065	15–71	1184	32–37	1312	32–41	1444	38–38	1577	19–83
1066	26–41	1185	15–79	1314	18–73	1445	17–85	1580	20–79
1067	11–97	1188	27–44	1316	28–47	1449	23–63	1581	31–51
1068	12–89	1189	29–41	1320	24–55	1450	29–50	1584	33–48
1071	21–51	1190	34–35	1323	27–49	1452	33–44	1586	26–61
1072	16–67	1196	23–52	1325	25–53	1455	15–97	1587	23–69
1073	29–37	1197	21–57	1326	26–51	1456	26–56	1590	30–53
1075	25–43	1200	25–48	1328	16–83	1457	31–47	1591	37–43
1078	22–49	1204	28–43	1330	35–38	1458	18–81	1593	27–59
1079	13–83	1206	18–67	1332	36–37	1460	20–73	1595	29–55
1080	24–45	1207	17–71	1333	31–43	1462	34–43	1596	38–42
1081	23–47	1209	31–39	1334	29–46	1463	19–77	1598	34–47
1083	19–57	1210	22–55	1335	15–89	1464	24–61	1599	39–41
1085	31–35	1215	27–45	1340	20–67	1470	30–49	1600	40–40
1088	32–34	1216	32–38	1342	22–61	1472	32–46	1602	18–89
1089	33–33	1218	29–42	1343	17–79	1474	22–67	1606	22–73
1092	28–39	1219	23–53	1344	21–64	1475	25–59	1608	24–67
1095	15–73	1220	20–61	1349	19–71	1476	36–41	1610	35–46
1098	18–61	1221	33–37	1350	25–54	1479	29–51	1612	26–62
1100	20–55	1222	26–47	1352	26–52	1480	37–40	1615	19–85
1102	29–38	1224	24–51	1353	33–41	1482	38–39	1617	33–49
1104	23–48	1225	35–35	1357	23–59	1484	28–53	1620	30–54

No.	Factors	No.	Factors	No.	Factors	No.	Factors	No.	Factors
1624	29–56	1755	39–45	1896	24–79	2046	33–62	2190	30–73
1625	25–65	1760	32–55	1898	26–73	2047	23–89	2193	43–51
1628	37–44	1763	41–43	1900	25–76	2048	32–64	2196	36–61
1632	32–51	1764	36–49	1904	28–68	2050	41–50	2200	44–50
1633	23–71	1767	31–57	1908	36–53	2052	38–54	2201	31–71
1634	38–43	1768	34–52	1909	23–83	2054	26–79	2204	38–58
1638	26–63	1769	29–61	1911	39–49	2058	42–49	2205	45–49
1640	40–41	1770	30–59	1914	33–58	2059	29–71	2208	32–69
1643	31–53	1771	23–77	1917	27–71	2064	43–48	2209	47–47
1645	35–47	1775	25–71	1920	30–64	2065	35–59	2210	34–65
1647	27–61	1776	37–48	1922	31–62	2067	39–53	2211	33–67
1649	17–97	1780	20–89	1924	37–52	2068	44–47	2212	28–79
1650	30–55	1782	33–54	1925	35–55	2070	45–46	2214	41–54
1652	28–59	1785	35–51	1927	41–47	2072	37–56	2220	37–60
1653	29–57	1786	38–47	1932	42–46	2074	34–61	2223	39–57
1656	36–46	1792	32–56	1935	43–45	2075	25–83	2225	25–89
1659	21–79	1794	39–46	1936	44–44	2077	31–67	2226	42–53
1660	20–83	1798	31–58	1938	38–51	2079	33–63	2231	23–97
1664	32–52	1800	36–50	1940	20–97	2080	32–65	2232	31–72
1665	37–45	1802	34–53	1943	29–67	2088	36–58	2233	29–77
1666	34–49	1804	41–44	1944	36–54	2090	38–55	2236	43–52
1672	38–44	1805	19–95	1947	33–59	2091	41–51	2240	40–56
1674	31–54	1806	42–43	1950	39–50	2093	23–91	2241	27–83
1675	25–67	1809	27–67	1952	32–61	2100	42–50	2242	38–59
1677	39–43	1813	37–49	1953	31–63	2106	39–54	2244	44–51
1679	23–73	1815	33–55	1955	23–85	2107	43–49	2250	45–50
1680	30–56	1817	23–79	1958	22–89	2108	34–62	2254	46–49
1681	41–41	1820	35–52	1960	40–49	2109	37–57	2255	41–55
1682	29–58	1824	32–57	1961	37–53	2112	33–64	2256	47–48
1683	33–51	1825	25–73	1968	41–48	2115	45–47	2257	37–61
1690	26–65	1826	22–83	1971	27–73	2116	46–46	2262	39–58
1691	19–89	1827	29–63	1972	34–58	2117	29–73	2263	31–73
1692	36–47	1829	31–59	1974	42–47	2120	40–53	2268	28–81
1694	22–77	1830	30–61	1975	25–79	2124	36–59	2272	32–71
1696	32–53	1833	39–47	1976	38–52	2125	25–85	2275	35–65
1700	20–85	1836	36–51	1978	43–46	2128	38–56	2277	33–69
1701	27–63	1840	40–46	1980	36–55	2130	30–71	2278	34–67
1702	37–46	1843	19–97	1984	31–64	2132	41–52	2279	43–53
1704	24–71	1845	41–45	1988	28–71	2133	27–79	2280	40–57
1705	31–55	1846	26–71	1989	39–51	2134	22–97	2288	44–52
1708	28–61	1848	42–44	1992	24–83	2135	35–61	2291	29–79
1710	38–45	1849	43–43	1995	35–57	2136	24–89	2294	37–62
1711	29–59	1850	37–50	1998	37–54	2139	31–69	2295	45–51
1715	35–49	1855	35–53	2000	50–40	2142	42–51	2296	41–56
1716	39–44	1856	32–58	2001	29–69	2144	32–67	2300	46–50
1720	40–43	1860	31–60	2002	26–77	2145	39–55	2301	39–59
1722	41–42	1862	38–49	2006	34–59	2146	37–58	2303	49–47
1725	25–69	1863	27–69	2009	41–49	2150	43–50	2304	36–64
1728	32–54	1869	21–89	2010	30–67	2156	44–49	2310	30–77
1729	19–91	1870	34–55	2013	33–61	2158	26–83	2312	34–68
1734	34–51	1872	39–48	2014	38–53	2160	40–54	2314	26–89
1736	31–56	1875	25–75	2015	31–65	2162	46–47	2318	38–61
1738	22–79	1876	28–67	2016	36–56	2166	38–57	2320	29–80
1739	37–47	1880	40–47	2021	43–47	2170	35–62	2322	43–54
1740	30–58	1881	33–57	2024	44–46	2173	41–53	2324	28–83
1742	26–67	1885	29–65	2025	45–45	2175	25–87	2325	31–75
1743	21–83	1886	41–46	2028	39–52	2176	34–64	2328	24–97
1746	18–97	1887	37–51	2030	35–58	2178	33–66	2331	37–63
1748	38–46	1888	32–59	2035	37–55	2183	37–59	2332	44–53
1749	33–53	1890	35–54	2037	21–97	2184	39–56	2336	32–73
1750	25–70	1891	31–61	2040	34–60	2185	23–95	2337	41–57
1752	24–73	1892	43–44	2044	28–73	2187	27–81	2340	45–52

No.	Factors	No.	Factors	No.	Factors	No.	Factors	No.	Factors
2343	33–71	2494	43–58	2655	45–59	2816	44–64	2982	42–71
2345	35–67	2496	39–64	2656	32–83	2820	47–60	2988	36–83
2346	46–51	2499	49–51	2660	38–70	2821	31–91	2989	49–61
2349	29–81	2500	50–50	2664	36–74	2822	34–83	2990	46–65
2350	47–50	2501	41–61	2665	41–65	2829	41–69	2992	44–68
2352	48–49	2508	44–57	2666	43–62	2832	48–59	2993	41–73
2356	38–62	2511	31–81	2668	46–58	2835	45–63	2997	37–81
2360	40–59	2516	34–74	2670	30–89	2838	43–66	3000	50–60
2365	43–55	2520	36–70	2673	27–99	2840	40–71	3002	38–79
2366	26–91	2522	26–97	2679	47–57	2842	49–58	3003	39–77
2368	37–64	2523	29–87	2680	40–67	2844	36–79	3007	31–97
2370	30–79	2528	32–79	2684	44–61	2847	39–73	3008	47–64
2375	25–95	2530	46–55	2686	34–79	2848	32–89	3009	51–59
2376	44–54	2535	39–65	2688	42–64	2849	37–77	3010	43–70
2378	41–58	2537	43–59	2691	39–69	2850	50–57	3015	45–67
2379	39–61	2538	47–54	2695	49–55	2852	46–62	3016	52–58
2380	34–70	2541	33–77	2697	31–87	2856	51–56	3021	53–57
2385	45–53	2542	41–62	2698	38–71	2860	52–55	3024	48–63
2387	31–77	2544	48–53	2700	50–54	2862	53–54	3025	55–55
2392	46–52	2546	38–67	2701	37–73	2867	47–61	3026	34–89
2394	38–63	2548	49–52	2703	51–53	2870	41–70	3034	41–74
2397	47–51	2550	50–51	2704	52–52	2871	33–87	3036	46–66
2400	48–50	2552	44–58	2706	33–82	2880	45–64	3038	49–62
2401	49–49	2553	37–69	2709	43–63	2881	43–67	3040	40–76
2403	27–89	2555	35–73	2714	46–59	2883	31–93	3042	39–78
2405	37–65	2556	36–71	2716	28–97	2886	39–74	3045	35–87
2407	29–83	2560	40–64	2720	40–68	2888	38–76	3050	50–61
2408	43–56	2562	42–61	2726	47–58	2890	34–85	3053	43–71
2409	33–73	2565	45–57	2728	44–62	2891	49–59	3055	47–65
2412	36–67	2573	31–83	2730	42–65	2898	46–63	3060	51–60
2414	34–71	2574	39–66	2736	48–57	2900	50–58	3066	42–73
2415	35–69	2576	46–56	2738	37–74	2904	44–66	3068	52–59
2418	39–62	2580	43–60	2739	33–83	2905	35–83	3069	33–93
2419	41–59	2581	29–89	2744	49–56	2907	51–57	3071	37–83
2420	44–55	2583	41–63	2745	45–61	2910	30–97	3072	48–64
2425	25–97	2584	38–68	2747	41–67	2911	41–71	3074	53–58
2430	45–54	2585	47–55	2750	50–55	2912	52–56	3075	41–75
2432	38–64	2590	37–70	2752	43–64	2914	47–62	3078	54–57
2436	42–58	2592	48–54	2754	51–54	2915	53–55	3080	55–56
2438	46–53	2596	44–59	2755	29–95	2916	54–54	3081	39–79
2440	40–61	2597	49–53	2756	52–53	2920	40–73	3082	46–67
2442	33–74	2600	50–52	2759	31–89	2923	37–79	3087	49–63
2444	47–52	2601	51–51	2760	46–60	2924	43–68	3094	34–91
2448	48–51	2604	42–62	2765	35–79	2925	45–65	3096	43–72
2449	31–79	2607	33–79	2769	39–71	2926	38–77	3100	50–62
2450	49–50	2610	45–58	2772	36–77	2928	48–61	3102	33–94
2451	43–57	2613	39–67	2773	47–59	2937	33–89	3104	32–97
2457	39–63	2618	34–77	2774	38–73	2940	49–60	3105	45–69
2460	41–60	2619	27–97	2775	37–75	2944	46–64	3108	42–74
2464	32–77	2622	46–57	2784	48–58	2945	31–95	3111	51–61
2465	29–85	2623	43–61	2788	41–68	2948	44–67	3115	35–89
2470	38–65	2624	41–64	2790	45–62	2950	50–59	3116	38–82
2475	45–55	2625	35–75	2793	49–57	2952	36–82	3120	48–65
2478	42–59	2627	37–71	2795	43–65	2958	51–58	3124	44–71
2479	37–67	2628	36–73	2800	50–56	2960	40–74	3127	53–59
2480	40–62	2632	47–56	2805	51–55	2961	47–63	3128	46–68
2482	34–73	2635	31–85	2806	46–61	2964	52–57	3132	54–58
2484	36–69	2639	29–91	2808	52–54	2967	43–69	3135	55–57
2485	35–71	2640	48–55	2809	53–53	2968	53–56	3136	49–64
2490	30–83	2646	49–54	2812	38–74	2970	54–55	3139	43–73
2491	47–53	2650	50–53	2813	29–97	2975	35–85	3145	37–85
2492	28–89	2652	39–68	2814	42–67	2976	48–62	3149	47–67

No.	Factors	No.	Factors	No.	Factors	No.	Factors	No.	Factors
5088	53–96	5313	69–77	5561	67–83	5810	70–83	6076	62–98
5092	67–76	5320	70–76	5568	64–87	5820	60–97	6080	76–80
5096	52–98	5325	71–75	5576	68–82	5822	71–82	6083	77–79
5100	51–100	5328	72–74	5580	62–90	5828	62–94	6084	78–78
5103	63–81	5329	73–73	5586	57–98	5829	67–87	6090	70–87
5104	58–88	5330	65–82	5589	69–81	5832	72–81	6097	67–91
5106	69–74	5332	62–86	5590	65–86	5840	73–80	6100	61–100
5110	70–73	5335	55–97	5600	56–100	5841	59–99	6106	71–86
5112	71–72	5336	58–92	5605	59–95	5846	74–79	6110	65–94
5115	55–93	5340	60–89	5607	63–89	5848	68–86	6111	63–97
5120	64–80	5346	66–81	5609	71–79	5850	65–90	6120	72–85
5124	61–84	5355	63–85	5610	66–85	5852	76–77	6132	73–84
5130	57–90	5358	57–94	5612	61–92	5856	61–96	6138	66–93
5133	59–87	5360	67–80	5616	72–78	5859	63–93	6141	69–89
5135	65–79	5368	61–88	5621	73–77	5865	69–85	6142	74–83
5141	53–97	5369	59–91	5624	74–76	5874	66–89	6144	64–96
5146	62–83	5372	68–79	5625	75–75	5880	70–84	6150	75–82
5148	66–78	5376	64–84	5626	58–97	5888	64–92	6156	76–81
5152	56–92	5382	69–78	5628	67–84	5890	62–95	6160	77–80
5159	67–77	5390	70–77	5632	64–88	5893	71–83	6162	78–79
5160	60–86	5394	62–87	5640	60–94	5896	67–88	6164	67–92
5162	58–89	5395	65–83	5642	62–91	5900	59–100	6174	63–98
5166	63–82	5396	76–71	5643	57–99	5904	72–82	6175	65–95
5168	68–76	5400	54–100	5644	68–83	5913	73–81	6177	71–87
5170	55–94	5402	73–74	5655	65–87	5915	65–91	6188	68–91
5175	69–75	5412	66–82	5658	69–82	5916	68–87	6192	72–86
5180	70–74	5415	57–95	5664	59–96	5917	61–97	6200	62–100
5183	71–73	5418	63–86	5670	70–81	5920	74–80	6204	66–94
5184	64–81	5427	67–81	5673	61–93	5922	63–94	6205	73–85
5185	61–85	5428	59–92	5676	66–86	5925	75–79	6208	64–97
5192	59–88	5429	61–89	5680	71–80	5928	76–78	6210	69–90
5194	53–98	5432	56–97	5684	58–98	5929	77–77	6216	74–84
5200	52–100	5440	64–85	5688	72–79	5934	69–86	6225	75–83
5208	62–84	5445	55–99	5694	73–78	5940	60–99	6230	70–89
5214	66–79	5451	69–79	5695	67–85	5950	70–85	6231	67–93
5220	60–87	5456	62–88	5696	64–89	5952	64–93	6232	76–82
5225	55–95	5460	65–84	5698	74–77	5963	67–89	6237	77–81
5226	67–78	5467	71–77	5700	57–100	5964	71–84	6240	65–96
5229	63–83	5472	57–96	5704	62–92	5976	72–83	6241	79–79
5236	68–77	5475	73–75	5712	68–84	5978	61–98	6248	71–88
5238	54–97	5476	74–74	5720	65–88	5980	65–92	6256	68–92
5244	57–92	5478	66–83	5723	59–97	5984	68–88	6264	72–87
5246	61–86	5481	63–87	5727	69–83	5985	63–95	6270	66–95
5247	53–99	5487	59–93	5733	63–91	5986	73–82	6272	64–98
5248	82–64	5488	56–98	5734	61–94	5994	74–81	6278	73–86
5250	70–75	5490	61–90	5740	70–82	6000	60–100	6279	69–91
5251	59–89	5494	67–82	5742	66–87	6003	69–87	6290	74–85
5254	71–74	5500	55–100	5751	71–81	6004	76–79	6298	67–94
5256	72–73	5504	64–86	5760	64–90	6006	77–78	6300	63–100
5264	56–94	5508	68–81	5762	67–86	6014	62–97	6305	65–97
5265	65–81	5510	58–95	5766	62–93	6016	64–94	6308	76–83
5270	62–85	5518	62–89	5767	73–79	6020	70–86	6314	77–82
5278	58–91	5520	69–80	5772	74–78	6030	67–90	6318	78–81
5280	66–80	5525	65–85	5775	75–77	6035	71–85	6319	71–89
5292	63–84	5529	57–97	5776	76–76	6039	61–99	6320	79–80
5293	67–79	5530	70–79	5780	68–85	6045	65–93	6324	68–93
5300	53–100	5538	71–78	5782	59–98	6048	63–96	6336	64–99
5301	57–93	5544	66–84	5785	65–89	6052	68–89	6348	69–92
5304	68–78	5546	59–94	5795	61–95	6059	73–83	6351	73–87
5307	61–87	5548	73–76	5796	69–84	6068	74–82	6364	74–86
5310	59–90	5550	74–75	5800	58–100	6072	69–88	6365	67–95
5312	64–83	5551	61–91	5808	66–88	6075	75–81	6370	65–98

No.	Factors	No.	Factors	No.	Factors	No.	Factors	No.	Factors
6375	75–85	6693	69–97	7030	74–95	7410	78–95	7872	82–96
6384	76–84	6696	72–93	7031	79–89	7425	75–99	7885	83–95
6390	71–90	6699	77–87	7040	80–88	7426	79–94	7896	84–94
6391	77–83	6700	67–100	7047	81–87	7440	80–93	7900	79–100
6392	68–94	6708	78–86	7050	75–94	7448	76–98	7905	85–93
6396	78–82	6715	79–85	7052	82–86	7452	81–92	7912	86–92
6399	81–79	6716	73–92	7055	83–85	7462	82–91	7917	87–91
6400	64–100	6720	70–96	7056	72–98	7469	77–97	7920	88–90
6402	66–97	6723	81–83	7068	76–93	7470	83–90	7921	89–89
6408	72–89	6724	82–82	7081	73–97	7476	84–89	7938	81–98
6417	69–93	6732	68–99	7084	77–92	7480	85–88	7954	82–97
6424	73–88	6734	74–91	7098	78–91	7482	86–87	7968	83–96
6432	67–96	6745	71–95	7100	71–100	7488	78–96	7980	84–95
6435	65–99	6750	75–90	7104	74–96	7500	75–100	7990	85–94
6438	74–87	6762	69–98	7110	79–90	7505	79–95	7998	86–93
6440	70–92	6764	76–89	7120	80–89	7520	80–94	8000	80–100
6450	75–86	6768	72–94	7125	75–95	7524	76–99	8004	87–92
6460	76–85	6776	77–88	7128	81–88	7533	81–93	8008	88–91
6461	71–91	6786	78–87	7134	82–87	7544	82–92	8010	89–90
6468	77–84	6789	73–93	7138	83–86	7546	77–98	8019	81–99
6474	78–83	6790	70–97	7140	84–85	7553	83–91	8036	82–98
6478	79–82	6794	79–86	7144	76–94	7560	84–90	8051	83–97
6480	72–90	6800	68–100	7154	73–98	7565	85–89	8064	84–96
6486	69–94	6804	81–84	7161	77–93	7566	78–97	8075	85–95
6497	73–89	6806	82–83	7176	78–92	7568	86–88	8084	86–94
6499	67–97	6808	74–92	7178	74–97	7569	87–87	8091	87–93
6500	65–100	6816	71–96	7189	79–91	7584	79–96	8096	88–92
6510	70–93	6825	75–91	7200	72–100	7600	80–95	8099	89–91
6512	74–88	6831	69–99	7209	81–89	7614	81–94	8100	81–100
6525	75–87	6840	76–90	7216	82–88	7623	77–99	8118	82–99
6528	68–96	6853	77–89	7220	76–95	7626	82–93	8134	83–98
6532	71–92	6860	70–98	7221	83–87	7636	83–92	8148	84–97
6534	66–99	6862	73–94	7224	84–86	7644	78–98	8160	85–96
6536	76–86	6864	78–88	7225	85–85	7650	85–90	8170	86–95
6545	77–85	6873	79–87	7227	73–99	7654	86–89	8178	87–94
6552	78–84	6880	80–86	7238	77–94	7656	87–88	8184	88–93
6555	69–95	6882	74–93	7252	74–98	7663	79–97	8188	89–92
6557	79–83	6885	81–85	7254	78–93	7680	80–96	8190	90–91
6560	80–82	6887	71–97	7268	79–92	7695	81–95	8200	82–100
6561	81–81	6888	82–84	7275	75–97	7700	77–100	8217	83–99
6566	67–98	6889	83–83	7280	80–91	7708	82–94	8232	84–98
6570	73–90	6900	69–100	7290	81–90	7719	83–93	8245	85–97
6580	70–94	6912	72–96	7296	76–96	7722	78–99	8256	86–96
6586	74–89	6916	76–91	7298	82–89	7728	84–92	8265	87–95
6596	68–97	6930	77–90	7300	73–100	7735	85–91	8272	88–94
6600	66–100	6935	73–95	7304	83–88	7740	86–90	8277	89–93
6603	71–93	6942	78–89	7308	84–87	7742	79–98	8280	90–92
6612	76–87	6952	79–88	7310	85–86	7743	87–89	8281	91–91
6622	77–86	6956	74–94	7315	77–95	7744	88–88	8300	83–100
6624	69–96	6958	71–98	7326	74–99	7760	80–97	8316	84–99
6630	78–85	6960	80–87	7332	78–94	7776	81–96	8330	85–98
6633	67–99	6966	81–86	7347	79–93	7790	82–95	8342	86–97
6636	79–84	6970	82–85	7350	75–98	7800	78–100	8352	87–96
6640	80–83	6972	83–84	7360	80–92	7802	83–94	8360	88–95
6642	81–82	6975	75–93	7371	81–91	7812	84–93	8366	89–94
6643	73–91	6984	72–97	7372	76–97	7820	85–92	8370	90–93
6650	70–95	6992	76–92	7380	82–90	7821	79–99	8372	91–92
6660	74–90	7000	70–100	7387	83–89	7826	86–91	8400	84–100
6664	68–98	7007	77–91	7392	84–88	7830	87–90	8415	85–99
6674	71–94	7008	73–96	7395	85–87	7832	88–89	8428	86–98
6675	75–89	7020	78–90	7396	86–86	7840	80–98	8439	87–97
6688	76–88	7029	71–99	7400	74–100	7857	81–97	8448	88–96

No.	Factors	No.	Factors	No.	Factors	No.	Factors	No.	Factors
8455	89–95	8640	90–96	8832	92–96	9100	91–100	9405	95–99
8460	90–94	8645	91–95	8835	93–95	9108	92–99	9408	96–98
8463	91–93	8648	92–94	8836	94–94	9114	93–98	9409	97–97
8464	92–92	8649	93–93	8900	89–100	9118	94–97	9500	95–100
8500	85–100	8700	87–100	8910	90–99	9120	95–96	9504	96–99
8514	86–99	8712	88–99	8918	91–98	9200	92–100	9506	97–98
8526	87–98	8722	89–98	8924	92–97	9207	93–99	9600	96–100
8536	88–97	8730	90–97	8928	93–96	9212	94–98	9603	97–99
8544	89–96	8736	91–96	8930	94–95	9215	95–97	9604	98–98
8550	90–95	8740	92–95	9000	90–100	9216	96–96	9700	97–100
8554	91–94	8742	93–94	9009	91–99	9300	93–100	9702	98–99
8556	92–93	8800	88–100	9016	92–98	9306	94–99	9800	98–100
8600	86–100	8811	89–99	9021	93–97	9310	95–98	9801	99–99
8613	87–99	8820	90–98	9024	94–96	9312	96–97	9900	99–100
8624	88–98	8827	91–97	9025	95–95	9400	94–100	10000	100–100
8633	89–97								

DECIMAL EQUIVALENTS

$\frac{1}{64}$ .015625	$\frac{11}{32}$ —.34375	$\frac{43}{64}$ .671875
$\frac{1}{32}$ —.03125	$\frac{23}{64}$ .359375	$\frac{11}{16}$ ——.6875
$\frac{3}{64}$ .046875	$\frac{3}{8}$ ——.375	$\frac{45}{64}$ .703125
$\frac{1}{16}$ ——.0625	$\frac{25}{64}$ .390625	$\frac{23}{32}$ —.71875
$\frac{5}{64}$ .078125	$\frac{13}{32}$ —.40625	$\frac{47}{64}$ .734375
$\frac{3}{32}$ —.09375	$\frac{27}{64}$ .421875	$\frac{3}{4}$ ——.750
$\frac{7}{64}$ .109375	$\frac{7}{16}$ ——.4375	$\frac{49}{64}$ .765625
$\frac{1}{8}$ ——.125	$\frac{29}{64}$ .453125	$\frac{25}{32}$ —.78125
$\frac{9}{64}$ .140625	$\frac{15}{32}$ —.46875	$\frac{51}{64}$ .796875
$\frac{5}{32}$ —.15625	$\frac{31}{64}$ .484375	$\frac{13}{16}$ ——.8125
$\frac{11}{64}$ .171875	$\frac{1}{2}$ ——.500	$\frac{53}{64}$ .828125
$\frac{3}{16}$ ——.1875	$\frac{33}{64}$ .515625	$\frac{27}{32}$ —.84375
$\frac{13}{64}$ .203125	$\frac{17}{32}$ —.53125	$\frac{55}{64}$ .859375
$\frac{7}{32}$ —.21875	$\frac{35}{64}$ .546875	$\frac{7}{8}$ ——.875
$\frac{15}{64}$ .234375	$\frac{9}{16}$ ——.5625	$\frac{57}{64}$ .890625
$\frac{1}{4}$ ——.250	$\frac{37}{64}$ .578125	$\frac{29}{32}$ —.90625
$\frac{17}{64}$ .265625	$\frac{19}{32}$ —.59375	$\frac{59}{64}$ .921875
$\frac{9}{32}$ —.28125	$\frac{39}{64}$ .609375	$\frac{15}{16}$ ——.9375
$\frac{19}{64}$ .296875	$\frac{5}{8}$ ——.625	$\frac{61}{64}$ .953125
$\frac{5}{16}$ ——.3125	$\frac{41}{64}$ .640625	$\frac{31}{32}$ —.96875
$\frac{21}{64}$ .328125	$\frac{21}{32}$ —.65625	$\frac{63}{64}$ .984375

INDEX

Addendum, 180, 188, 194

Addendum angle (bevel gears), 187, 188

Addition, of angles, 107; of decimals, 22; of fractions, 9

Algebraic problems solved by similar figures, 118

Aliquot part, 31

Altitude of a triangle, 103, 165

Angles, 79, 106

Angular addendum (bevel gears), 188

Angular indexing (dividing head), 199

Approximation of a decimal, 207

Approximation of a fraction, 206

Arc, definition of, 84

Area, of a circle, 90; of a cone, 91; of a cylinder, 91; of a frustum of a cone, 91; of a parallelogram, 90; of a rectangle, 89; of a sphere, 92; of a square, 89; of a trapezoid, 90; of a triangle, 89

Bevel gears, dimensions of, 187, 188, 189; shafts forming an acute angle, 191; shafts forming a right angle, 187

Bisector of an angle, 86

Brown and Sharp tapers, 124

Calculations of change gears for hobbing machines, 250

Cancellation, 13

Center distance of worm and worm wheel, 194

Change gears for automatic gear hobbing, 250, 251, 252, 254; for backing off, 236, 237; for the lathe, 217, 218, 220, 223; for spiral gear hobbing, 250, 251, 252, 254; for spiral milling, 226, 228

Checking bevel gear teeth, 188

Checking gear teeth, with a square gauge, 184; with plugs, 181, 182, 183; with verniers, 180, 184, 188, 195

Checking holes bored at an angle, 178

Checking rack teeth, with plugs, 182; with verniers, 180

Checking worm threads, 195

Checking worm wheel teeth, 195

Chord, definition of, 84

Circle, area of, 90; definition of, 82

Circular pitch, 179

Circumscribed circle, 84, 86

Circumference of circle, 82

Clearance of bevel gear teeth, 188; of spur gear teeth, 180

Closest set of approximate factors for compound gears, 209

Common fractions, 6

Complex fractions, 16

Compound gearing, 47; of the lathe, 220

Compound gear ratios for a lathe, 220

Compound gears, and lead screw, 56; and rack, 50; for given revolutions, 50; teeth required, 50, 215

Compound proportion, 63

Compounded ratios, 62

Cone, area of, 91; volume of, 97

Continued fractions, 204; compound gear ratios, 209; simple gear ratios, 206
Convergents(continued fractions), 204
Corresponding angles, 80
Cosecant, definition of, 131; use of, 137
Cosine, definition of, 130; use of, 135
Cotangent, definition of, 130; use of, 136
Cutters for gear teeth, 182
Cutting speeds and feeds, 74
Cylinder, area of, 91; volume of, 97

Decimal degree, 108
Decimal equivalents, table of, 316
Decimal fractions, 22
Decimals, 22
Dedendum, 180, 188
Dedendum angle (bevel gears), 187, 188
Degree, definition of, 106
Denominator, 6
Dials, 66
Diameter of a circle, 82
Diametral pitch, 179
Diametral pitch hobs, 230
Differential indexing (dividing head), 199
Dimensions, for checking bevel gear teeth, 188; for checking rack teeth, 180, 182; for checking spur gear teeth, 180, 181, 182, 183, 184; for checking worm threads, 195; for checking worm wheel teeth, 195; for standard tapers, 124
Dividing an angle, 108
Dividing head, 198
Division, of decimals, 23; of fractions, 18
Divisions on a vernier, 111
Double-threaded screw, definition of, 54

Enlarging figures, 116
Excess of 9's, 28
Exponent, definition of, 3
Expressions, involving complex fractions, 18, 20; involving multiplication and division, 4; involving grouping symbols, 36, 37; involving radical signs, 39

Face angle (bevel gears), 187, 188
Factors for compound gearing, 47, 48
Factors of a number, 1
Factor table, 305
Feed gears for hobbing machine, 251, 254
Feed, of the hob, 243; to be used on a hob, 254
Feed spirals, 243
Feet per minute, 74, 75
Formula, definition of, 38; for differential indexing, 201; for inscribed circle in a right triangle, 87; for intersections of chords, 87; involving radical sign, 38
Fraction, definition of, 6
Fractional expressions, 14
Frustum of cone, area of, 91
Functions of angles, 130

Gage problems, 175
Gashes of a hob, 230
Gear and lead screw, 54
Gear box of a lathe, 222
Gearing, of the lathe, 55, 217; of the milling machine, 226
Gear ratios, 42, 43; for backing off, 236, 237; for a lathe, 55, 217
Gears, dimensions of, 179, 180; factors for compounding, 47; teeth required, 42, 43; teeth required for a lathe, 55, 217

Gear-tooth verniers, 112
Graduated dials, 66
Grouping symbols, 36

Hand, of a hob, 230; of spiral gear, 243
Helix angle, of a hob, 231; of a worm, 194
Hexagon, definition of, 84
Hob, definition of, 230
Hobbing machine, 242, 250
Hobbing machine change gears, 250
Hobbing spiral gears, 242
Horizontal line, definition of, 79
Hypotenuse, definition of, 126

Idler gears, definition of, 43
Imaginary simple gears, 47, 50
Improper fraction, definition of, 6
Index gears for hobbing machine, 250, 252
Indexing (dividing head), 198, 199
Infinity, definition of, 212
Inscribed circle, 84, 86
Involute gears, 179

Jarno tapers, 124

Lathe, gearing of, 55, 217
Lathe gears, 55, 217
Lead, definition of, 54; of diametral-pitch hobs, 230, 232; of hob gash, 232; of spiral gears, 240
Least common denominator, 9
Linear pitch, 194
Linear pitch hobs, 230
Lower-end fraction (continued fractions), 210
Lowest terms of fractions, 7

Maximum speed of crank slides, 76
Micrometer, 55
Milling-machine gears, 226
Minute, definition of, 106
Mixed number, 6
Morse tapers, 124
Multiplying an angle, 107
Multiplication of decimals, 23; of fractions, 12; indicated by a dot, 38; sign of, omitted, 38, 39

Numerator, definition of, 6

Oblique triangle, definition of, 80
Odd leads, 222
Opposite angles, 79
Outside diameter, of bevel gears, 187, 188; of hobs, 230; of spiral gears, 241; of spur gears, 180

Parallelepiped, volume of, 96
Parallel lines, definition of, 79
Parallelogram, area of, 90
Percentage, 33
Peripheral speed, 74
Perpendicular line, definition of, 79
Perpendiculars of a triangle, 149
Pitch angle of a bevel gear, 187, 188
Pitch cone radius (bevel gear), 187, 188
Pitch diameter, of a bevel gear, 187, 188; of a spiral gear, 240; of a spur gear, 180; of a worm, 194; of a worm wheel, 194
Pitch of gears, 179
Point of tangency, definition of, 82
Polygon, definition of, 85
Practical problems in trigonometry, 167
Pressure angle of a gear, 179
Prime factors, 1
Prime numbers, 1
Product of ratios, 62
Proof, of addition, 28; of division, 29; of multiplication, 29; of

square root, 71; of subtraction, 29; of $Y=(A^2-C^2)\div 2B$, 104
Proper fractions, 6
Proportion, 60; rule, 61; rule for tapers, 123
Protractors, 113
Pyramid, volume of, 96
Pythagorean theorem, 100

Rack, and compound gears, 50, 51; and simple gears, 44
Rack teeth, dimensions of, 179, 180
Radical sign, use of, 3, 38, 39, 70
Radius of circle, 82
Ratio, 59; of arc to radius, 106; of similar figures, 116
Reciprocal, of an aliquot part, 31; of a number, 17; of a fraction, 17; of a ratio, 59
Rectangle, area of, 89
Reducing figures, 116
Reduction, of complex fractions, 16; of decimals, 24; of decimal degree, 108; of fractions, 7
Relation of altitude line and midpoint of base, 103
Removal of grouping symbols, 36, 37
Revolutions per minute, 74
Root angle (bevel gears), 187, 188
Rule, for addition of fractions, 10; for area of a triangle, 38; for compound gearing, 47; for compound proportion, 64; for determining divisions on verniers, 111; for hypotenuse of a right triangle, 100; for proportion, 61; for side of a right triangle, 101; for subtraction of fractions, 10; for tapers, 123

Secant, definition of, 130; use of, 137
Second, definition of, 106
Sector, definition of, 84
Segment, definition of, 84
Short cuts in division, 31; in multiplication, 31
Side adjoining, definition of, 126
Side opposite, definition of, 126
Signs of operation, 3
Similar figures, 116
Similar triangles, trigonometry, 126, 127
Simple gearing, 41; of the lathe, 217
Simple indexing (dividing head), 198
Simple proportion, 60
Sine bar problems, 173
Sine, definition of, 130; use of, 134
Single-threaded screw, 54
Solution, of feed gears for hobbing machine, 251, 254; of index gears for hobbing machine, 250, 252; of oblique triangles, 150, 156, 160; of right triangles, 134, 139, 142
Sphere, area of, 92; volume of, 98
Spiral-fluted hobs, 230
Spiral gears, 239
Spiral intersections, 235, 245
Spur gear, 41, 179
Spur gear and rack, dimensions of, 179, 180
Square, area of, 89; definition of, 84
Square root, 70; of a fraction, 72
Standard tapers, 124
Subtraction, of angles, 107; of decimals, 22; of fractions, 10

Table, of cutters for gear teeth, 182; of decimal equivalents, 316; of factors, 305; of trigonometric functions, 257
Tangent, of a circle, 82; definition of, 130; use of, 136
Tapers, 121

Teeth required in compound gears, 50, 215
Theory of the intersecting points of spirals, 235, 245
Threads per inch, 54
Throat diameter of worm wheels, 194
Time required for drilling, 75
Tooth angle of spiral gears, 239
Trapezoid, area of, 90
Triangle, area of, 38, 89; definition of, 80
Trigonometry, 126
Trigonometric functions, 130; table of, 25
Triple-threaded screw, 54

Uniting fractions, 208
Unit method, 63
Upper-end fraction (continued fractions), 210
Use of gear-tooth verniers, 180, 188, 195
Use of trigonometric table, 131

Vernier, 111
Vernier calipers, 111, 112; for checking gear teeth, 180, 188, 195
Vernier measuring instruments, 111
Vernier protractors, 113
Vertex, 83
Vertical line, definition of, 79
Vinculum, 70
Volume, of cone, 97; of cylinder, 97; of parallelepiped, 96; of pyramid, 96, 97; of sphere, 98

Worm, dimensions of, 194
Worm gearing, 193
Worm wheel, dimensions of, 194

SOLUTIONS AND ANSWERS

EXERCISE 1

1. 5, 7. **2.** 2, 29. **3.** 2, 2, 2, 3, 5. **4.** 3, 41.

5. 3, 3, 41. **6.** 2, 2, 3, 5, 7. **7.** 37. **8.** 29, 29.

9. 27, 59. **10.** 39, 41. **11.** 83, 99. **12.** 81, 97.

13. 99, 99.

EXERCISE 2

1. 3. **2.** $2-125-8+200-24=2+200-125-8-4=202-157=45$. **3.** $2+8\times3-7\times9+100\div2=2+24-63+50=13$.

4. 525. **5.** 25. **6.** 2. **7.** 8. **8.** 4.

EXERCISE 3

1. $\frac{5}{7}$. **2.** $\frac{1}{2}$. **3.** $2\frac{3}{22}$. **4.** $15\frac{26}{27}$. **5.** $\frac{78}{29}$.

6. $\frac{327}{4}$. **7.** $\frac{18}{22}\frac{15}{35}=\frac{6}{7}\frac{05}{45}=\frac{1}{1}\frac{21}{49}$. **8.** $\frac{15}{20}=\frac{45}{60}$. 45. *Ans.*

9. $\frac{36}{63}=\frac{12}{21}=\frac{48}{84}$. 84. *Ans.*

10. $\frac{21}{25}\frac{60}{20}=\frac{216}{252}=\frac{54}{63}=\frac{6}{7}=\frac{42}{49}$. 49. *Ans.*

EXERCISE 4

1. $4\left|\frac{12, 5, 4}{3, 5, 1}\right.$. L.C.M. $=4\times3\times5\times1=60$.

2. $\frac{3}{8}+\frac{5}{6}+\frac{7}{9}+\frac{1}{4}=\frac{27}{72}+\frac{60}{72}+\frac{56}{72}+\frac{18}{72}=\frac{161}{72}=2\frac{17}{72}$.

3. $\frac{7}{8}+2\frac{1}{2}+\frac{6}{5}+\frac{3}{2}+17=\frac{35}{40}+2\frac{20}{40}+\frac{48}{40}+\frac{60}{40}+17=19\frac{163}{40}=23\frac{3}{40}$.

4. $\frac{1}{10}+\frac{1}{9}+\frac{1}{8}+\frac{1}{7}=\frac{252}{2520}+\frac{280}{2520}+\frac{315}{2520}+\frac{360}{2520}=\frac{1207}{2520}$.

5. $\frac{5}{12}-\frac{3}{10}=\frac{25}{60}-\frac{18}{60}=\frac{7}{60}$.

6. $\frac{1}{8}-\frac{1}{9}=\frac{9}{72}-\frac{8}{72}=\frac{1}{72}$.

7. $\frac{3}{4}+\frac{3}{5}=\frac{15}{20}+\frac{12}{20}=\frac{27}{20}$.

$\frac{11}{35}+\frac{1}{12}=\frac{132}{420}+\frac{35}{420}=\frac{167}{420}$.

$\frac{27}{20}-\frac{167}{420}=\frac{567}{420}-\frac{167}{420}=\frac{400}{420}=\frac{20}{21}$.

8.

9073	16247	1
7174	9073	1
1899	7174	3
1477	5697	1
422	1477	3
422	1266	2
0	211	

$\frac{9073\div211}{16247\div211}=\frac{43}{77}$.

9. $7\frac{17}{128}$.

10. $\frac{3}{8}+\frac{49}{64}+\frac{7}{16}+\frac{11}{32}=\frac{24}{64}+\frac{49}{64}+\frac{28}{64}+\frac{22}{64}=\frac{123}{64}=1\frac{59}{64}$.

11. $\frac{11}{7}+\frac{2}{7}=\frac{13}{7}=1\frac{6}{7}$.

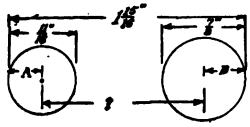


FIG. 4S

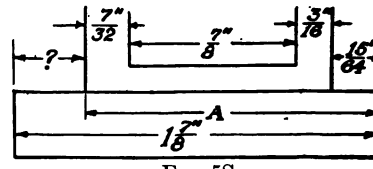


FIG. 5S

12. $A=1\frac{1}{16}''\div 2=\frac{11}{32}''$.
$B=\frac{7}{8}''\div 2=\frac{7}{16}''$.
$?=1\frac{15}{16}''-\frac{7}{16}''-\frac{11}{32}''=1\frac{5}{32}''$.

13. $A=\frac{7}{32}''+\frac{7}{8}''+\frac{3}{16}''+\frac{15}{64}''=\frac{14}{64}''+\frac{56}{64}''+\frac{12}{64}''+\frac{15}{64}''=\frac{97}{64}''=1\frac{33}{64}''$.
$?=1\frac{7}{8}''-1\frac{33}{64}''=1\frac{56}{64}''-1\frac{33}{64}''=\frac{23}{64}''$.

EXERCISE 5

1. $\frac{\overset{3}{\cancel{6}}\times\cancel{8}\times\overset{2}{\cancel{9}}\times\cancel{10}}{\cancel{8}\times\cancel{3}\times\cancel{5}}=36$.

2. $\frac{\cancel{3}\times\cancel{3}\times\cancel{4}}{\cancel{48}\times\cancel{72}\times\cancel{8}}=\frac{4}{3}=1\frac{1}{3}$.
$\cancel{32}\times\cancel{24}\times\cancel{27}$
$\cancel{2}\quad\cancel{9}$
3

3. $6\frac{6}{65}$.

4. $\frac{3}{1280}$.

5. 7.

6. 36.

7. 1.

8. $\frac{7\times 8-3+8}{8+3\times 9-5}=\frac{56-3+8}{8+27-5}=\frac{61}{30}=2\frac{1}{30}$.

9. $\frac{7\times 11-102+7\times 4}{8+7\times 3-17}=\frac{77-102+28}{8+21-17}=\frac{3}{12}=\frac{1}{4}$.

10. $6\frac{2}{3}$.

11. $5\frac{1}{4}\frac{8}{8}$.

12. $\frac{7\times 4+6-9\div 3}{8\div 2+4}=\frac{28+6-3}{4+4}=\frac{31}{8}=3\frac{7}{8}$.

13. $1773\frac{1}{8}$.

14. $88\frac{7}{9}$.

15. $7\times\frac{43}{64}''+2\times\frac{1}{2}''=\frac{301}{64}''+1''=5\frac{45}{64}''$.

16. $\frac{9}{10}$ of a foot $=\frac{9}{10}\times 12=10\frac{4}{5}''$. If $\frac{5}{8}$ is turned, $\frac{1}{8}$ is unfinished. $\frac{1}{8}$ of $10\frac{4}{5}''=\frac{1}{8}\times\frac{54}{5}''=\frac{9}{5}''=1\frac{4}{5}''$. $1\frac{4}{5}$ inches. *Ans.*

EXERCISE 6

1. $\frac{\frac{7}{8}\times 8}{\frac{3}{4}\times 8}=\frac{7}{6}=1\frac{1}{6}$.

2. $\frac{\frac{5}{12}\times 12}{9\times 12}=\frac{5}{108}$.

3. $\frac{15\frac{3}{4}}{16\frac{2}{3}}=\frac{\frac{63}{4}\times 12}{\frac{50}{3}\times 12}=\frac{189}{200}$.

4. $18\frac{2}{3}$.

EXERCISE 7

1. $\frac{1}{72}$.

2. 6.

3. $1\frac{15}{17}$.

4. $\frac{2}{567}$.

5. $\frac{\frac{7}{12}}{\frac{2}{5}}=1\frac{11}{24}$.

6. $54\div\frac{6}{7}=54\times\frac{7}{6}=63$.

7. $\frac{7}{9}\div\frac{14}{6}=\frac{7}{9}\times\frac{6}{14}=\frac{1}{3}$.

8. $34\frac{2}{5}\div 46\frac{2}{7}=\frac{172}{5}\div\frac{324}{7}=\frac{172}{5}\times\frac{7}{324}=\frac{301}{405}$.

9. $\frac{5\frac{7}{16}''-(6\times\frac{1}{8}'')}{7}=\frac{5\frac{7}{16}''-\frac{3}{4}''}{7}=\frac{4\frac{11}{16}''}{7}=4\frac{11}{16}''\times\frac{1}{7}=\frac{75}{16}''\times\frac{1}{7}=\frac{75}{112}''$.

EXERCISE 8

1. ~~$24\times\frac{35}{24}=35$~~. $\frac{35}{24}$. *Ans.*

2. $\frac{4}{25}$.

3. $\frac{5}{6}\times\frac{\frac{9}{281}}{\frac{5}{6}}=\frac{9}{281}$. $\frac{\frac{9}{281}}{\frac{5}{6}}$. *Ans.*

4. $\frac{11\frac{1}{2}}{8\frac{2}{3}}$.

5. $\frac{\frac{17}{4}}{\frac{3}{8}}$.

6. $\frac{\frac{4}{5}}{7}$.

7. $\frac{6}{\frac{5}{7}}$

8. $\frac{2\times\frac{7}{6}}{6\times\frac{7}{6}}=\frac{2\frac{1}{3}}{7}$. $2\frac{1}{3}$. *Ans.*

9. $\frac{\frac{3}{4}}{\frac{5}{6}}=\frac{9}{10}$; $\frac{11}{\frac{10}{5}}=\frac{66}{25}$; then $\frac{\frac{3}{4}}{\frac{5}{6}}\times\frac{11}{\frac{10}{5}}=\frac{9}{10}\times\frac{66}{25}=\frac{297}{125}$.

10. $\frac{78\frac{1}{3}}{\frac{9}{12}}=\frac{940}{9}$; $\frac{2\frac{8}{9}}{3}=\frac{26}{27}$; then $\frac{78\frac{1}{3}}{\frac{9}{12}}\div\frac{2\frac{8}{9}}{3}=\frac{940}{9}\div\frac{26}{27}=\frac{940}{9}\times\frac{27}{26}=\frac{1410}{13}=108\frac{6}{13}$.

11. $\frac{47\frac{2}{3}\times\frac{6\frac{2}{3}}{\frac{9}{14}}}{\frac{9}{14}\times\frac{6\frac{2}{3}}{\frac{9}{14}}}=\frac{?}{6\frac{2}{3}}$; $47\frac{2}{3}\times\frac{6\frac{2}{3}}{\frac{9}{14}}=\frac{143}{3}\times\frac{20}{3}\times\frac{14}{9}=\frac{40040}{81}=494\frac{26}{81}$; $494\frac{26}{81}$. *Ans.*

EXERCISE 9

1. 338.9158.

2. 4000.0004.

3. $4.675+.4+.045+.87+18.11=24.10$.

4. 12.080.

5. 11.78509. **6.** .0782. **7.** 34.202. **8.** .999″.

9. 1.872″ + .7165″ − 1.031″ = 1.5575″.

10. .9025″ − .502″ + .801″ − .502″ = .6995″.

11. $\frac{1.125''}{2} - .183'' + \frac{.904''}{2} = .8315''$.

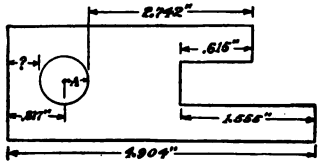


FIG. 13S

12. A = 4.904″ − .817″ − 1.555″ + .615″ − 2.742″ = .405″.

? = .817″ − .405″ = .412″.

13. 2.5845″. **14.** $\frac{5.503'' - (4 \times .241'')}{5} = .9078''$. **15.** 1.439″.

16. 20.187. **17.** .0057. **18.** .0894. **19.** 59.31.

20. 16.677. **21.** $\frac{6.6}{6.7} + \frac{87}{100} - .934 = .985 + .87 - .934 = .921$.

22. $\frac{9.06 \times \frac{13 \cdot 2}{9 \cdot 06}}{.054 \times \frac{13 \cdot 2}{9 \cdot 06}} = \frac{13.2}{.0786}$.

23. .624″ *Ans.*

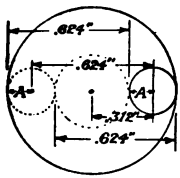


FIG. 17S

24. $\frac{1}{2} \times 1.125'' - \frac{4}{8} \times 1.125'' = .0625''$.

25. .5384. **26.** $.3875 = \frac{3875 \div 125}{10000 \div 125} = \frac{31}{80}$.

27. $\frac{18.6}{47.43} = \frac{1860 \div 93}{4743 \div 93} = \frac{20}{51}$. **28.** .5456.

29. $\frac{4.444}{8.674}$ or .51233.

30. $\frac{17\frac{2}{3}}{23\frac{5}{7}} = \frac{371}{498} = .74497$. **31.** 2.2043. **32.** 1.6667.

33. $\frac{14 \times .8}{17.4 \div 6} = \frac{?}{18}$

$\frac{11.2}{2.9} = \frac{?}{18}$

$\frac{11.2 \times \frac{18}{2 \cdot 9}}{2.9 \times \frac{18}{2 \cdot 9}} = \frac{69.51}{18}$.

69.51 *Ans.*

34. 2.5548″.

EXERCISE 10

1.
54326=2
64832=5
434608 10=1
1738432
3476864
3522063232=1

2.
62541=0
54515=2
312705 0
938115
2814345
3409422615=0

3.
68952=3
48896=8
551616 24=6
3309696
6619392
3371476992=6

4.
712653=6
560749=4
4988571 24=6
34919997
39908568
399619457097=6

5.
70425=0
998=8
70425000 0
140850
70284150=0

6.
40365=0
499=4
20182500 0
40365
20142135=0

7. $7241 \div 3125 = \frac{7241}{3125} = \frac{.07241}{.03125} = \frac{.07241}{\frac{1}{32}} = .07241 \times 32$

.07241=5
32=5
14482 25=7
21723
2.31712=7

8. $4\overline{)965200}$
241300

EXERCISE 11

1. 1.36=1%, then 17=12½%. **2.** 258¾ lb. of copper.

3. .4%. **4.** 44. **5.** 2883.9 lb. **6.** ¼ of 1½%=⅜%; ⅜% of 2200 lb.$=\frac{66.00 \text{ lb.}}{8}=8.25$ lb.

7. \$.0095=1%; \$2.25−\$0.95=\$1.30=Increase in price; \$1.30÷.0095=136.8%.

8. 2500 parts in 24 hours equals 104⅙ parts in 1 hour.
1500 " " 10 " " 150 " " 1 "

This makes an increase of 45⅚ parts in 1 hour.

If 104⅙ equals 100%, 45⅚ equals 44%. 44% increase. *Ans.*

9. $12\frac{1}{2}\%$ of 96 is 12; 12 is $33\frac{1}{3}\%$ of 36. **10.** .0096″.

11. A = .750375″. B = .749625″.

EXERCISE 12

1. $42-(9+6)+2=42-15+2=44-15=29$. **2.** 8.

3. 46. **4.** 107. **5.** $(16-2)(17+9)=14\times26=364$. **6.** 4.

7. $5\left(\frac{15-6}{2(4+3)}\right)-\frac{34-2}{16}=5\left(\frac{9}{14}\right)-\frac{32}{16}=\frac{45}{14}-2=1\frac{3}{14}$.

8. $2+\{14-(6+2)\}\{(6+2)(18+2)-90\}-6\div2=2+\{14-8\}\{(8)(20)-90\}-3=2+6(160-90)-3=2+6\times70-3=2+420-3=419$.

EXERCISE 13

1. $\frac{3(2+B)+ABC-3}{CD-(BC-A)+3B}=\frac{3(2+2)+6\times2\times8-3}{8\times8-(2\times8-6)+3\times2}=\frac{12+96-3}{64-(16-6)+6}=\frac{105}{60}=\frac{7}{4}=1\frac{3}{4}$.

2. 10825″. **3.** 282.744 cubic inches. **4.** .054125″.

5. 1.1686″. **6.** 4. **7.** $1\frac{7}{16}$″. **8.** 76.9692 cubic inches.

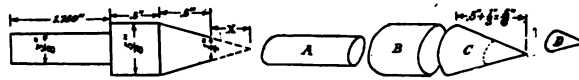


FIG. 29S

9. $X=\frac{\frac{1}{4}\times.5}{\frac{5}{8}-\frac{1}{4}}=\frac{1}{3}$.

$A=.7854(\frac{3}{8})^2(1.750)=.1932$ cubic inch.

$B=.7854(\frac{5}{8})^2(.5)=.1534$ cubic inch.

$C=1.0472(\frac{5}{16})^2(\frac{5}{8})=.0852$ cubic inch.

$D=1.0472(\frac{1}{8})^2(\frac{1}{3})=.0054$ cubic inch.

Volume = A+B+C−D = .1932+.1534+.0852−.0054 = .4264 cubic inch.

10. 2.298″. **11.** 2.348″. **12.** .100.

EXERCISE 14

1. $26\times\frac{4}{7}=14\frac{6}{7}$ teeth. **2.** $7\times\frac{70}{81}=6\frac{4}{81}$ revolutions.

3. $\frac{4}{9}\times\frac{81}{70}=\frac{18}{35}$ revolution. **4.** A = 37 teeth; B = 55 teeth.

5. $\dfrac{A=\frac{4}{7}\text{ revolution}}{B=\frac{11}{18}\text{ revolution}}=\dfrac{A=\frac{11}{18}\text{ teeth.}}{B=\frac{4}{7}\text{ teeth.}}=\dfrac{11}{18}\times\dfrac{7}{4}=\dfrac{77}{72}$ teeth in A.
$\frac{77}{72}$ " " B.

6. $\frac{11}{13}=\frac{66}{78}$, A = 78 teeth.

7.

1537	2597	1
1060	1537	1
477	1060	2
424	954	4
53	106	2
	106	
	0	

$$\frac{1537\div 53}{2597\div 53}=\frac{29}{49}.$$

29 teeth and 49 teeth. *Ans.*

8. $\frac{56}{88}=\frac{7}{11}=\frac{35}{55}$, X = 35 teeth. **9.** $3\times 12\times .200''=7.2''$.

EXERCISE 15

1. After factoring ($A=1739=37\times 47$; $B=1541=23\times 67$) and placing in proper compound order the following sets of compound gears are formed.

C = 37; D = 23; E = 47; F = 67.
C = 37; D = 67; E = 47; F = 23.
C = 47; D = 23; E = 37; F = 67.
C = 47; D = 67; E = 37; F = 23.

2. C and E are the factors of A; therefore, if A = 4753 and C = 49, E must equal $\frac{47}{49}\frac{53}{}$ or 97 teeth.

D and F are the factors of B; therefore, if B = 1333 and F = 43, D must equal $\frac{13}{43}\frac{33}{}$ or 31 teeth.

3. After obtaining suitable factors

$$\left(\frac{X}{Y}=\frac{360}{47}=\frac{4\times 90}{1\times 47}=\frac{(25\times 4)\times 90}{(25\times 1)\times 47}=\frac{100\times 90}{25\times 47}\right)$$

and placing in proper compound order, the following sets of compound gears are obtained.

A = 100; B = 47; C = 90; D = 25.
A = 100; B = 25; C = 90; D = 47.
A = 90; B = 47; C = 100; D = 25.
A = 90; B = 25; C = 100; D = 47.

4. After factoring ($1092=28\times39$; $1224=24\times51$) and placing in proper compound order, the following sets of compound gears are formed.

$A=28;\ B=24;\ C=39;\ D=51.$
$A=28;\ B=51;\ C=39;\ D=24.$
$A=39;\ B=24;\ C=28;\ D=51.$
$A=39;\ B=51;\ C=28;\ D=24.$

5. $\frac{36\times42}{88\times C}=\frac{63}{187}=\frac{9\times7}{11\times17}=\frac{(4\times9)\times7}{(4\times11)\times17}=\frac{(4\times9)\times(2\times7)}{(2\times4\times11)\times17}=$
$\frac{(4\times9)\times(3\times2\times7)}{(2\times4\times11)\times(3\times17)}=\frac{36\times42}{88\times51}$. $C=51$ teeth. *Ans.*

6. After obtaining suitable factors
$\left(\frac{X}{Y}=\frac{71}{403}=\frac{1\times71}{13\times31}=\frac{(3\times1)\times71}{13\times(3\times31)}=\frac{(7\times3\times1)\times71}{(7\times13)\times(3\times31)}=\frac{21\times71}{91\times93}\right)$
and placing in proper compound order, the following sets of compound gears are obtained.

$A=21;\ B=91;\ C=71;\ D=93.$
$A=21;\ B=93;\ C=71;\ D=91.$
$A=71;\ B=91;\ C=21;\ D=93.$
$A=71;\ B=93;\ C=21;\ D=91.$

7. $\frac{A}{B}=\frac{35\times64}{25\times40}=\frac{(35\div5)\times(64\div8)}{(25\div5)\times(40\div8)}=\frac{7\times8}{5\times5}=\frac{56}{25}$, $A=56$ teeth; $B=25$ teeth. *Ans.*

8. $\frac{6.455}{1}\times\frac{22}{17}\times\frac{25}{26}=8.0322$ revolutions.

9. If A′ makes 203 revolutions while D′ makes 121 revolutions, A′ must have 121 teeth and D′ 203 teeth. After obtaining suitable factors
$\left(\frac{A'}{D'}=\frac{121}{203}=\frac{11\times11}{7\times29}=\frac{(3\times11)\times(2\times11)}{(3\times7)\times(2\times29)}=\frac{33\times22}{21\times58}\right)$

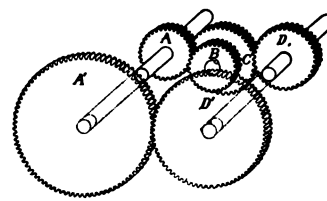


FIG. 60S

and placing in proper compound order, the following sets of compound gears are obtained.

A=33; B=21; C=22; D=58.
A=33; B=58; C=22; D=21.
A=22; B=21; C=33; D=58.
A=22; B=58; C=33; D=21.

10. 8 inches is equal to 40 teeth in rack. When rack moves 40 teeth, X will make 40÷20 or 2 revolutions. When A makes 4 revolutions, X must make 2 revolutions. The number of revolutions inverted equals the number of gear teeth. Thus, A=2 teeth and X=4 teeth. A must be multiplied by 15 to equal 30, therefore X must be multiplied by the same number. X=4×15 or 60 teeth.

EXERCISE 16

1. $4\frac{2}{3}\times\frac{1}{7}''=\frac{2}{3}''$.

2. $8.5\times\frac{1}{7}''=1.21428''$.

3. $1.703\div\frac{1}{7}''=11.921''$.

4. While A makes $5\frac{1}{5}$ revolutions, the screw will make $5\frac{1}{5}\times\frac{36}{45}$ or $\frac{104}{25}$ revolution. One revolution of screw moves slide $\frac{1}{5}''$; then $\frac{104}{25}$ revolution will move it $\frac{104}{25}\times\frac{1}{5}''=\frac{104}{125}''=.832''$.

5. Screw must make $4.362\div\frac{1}{5}$ or 21.81 revolutions to move slide 4.362″. While screw makes 21.81 revolutions, A will make $21.81\times\frac{45}{36}$ or 27.2625 revolutions.

6. While A makes 1 revolution, B will make $\frac{76}{81}$ or .93827 revolution.

7. While A makes 2 revolutions, B will make $2\times\frac{57}{39}\times\frac{72}{30}$ or 7.0153 revolutions.

8. While B makes 2 revolutions, A will make $2\times\frac{30}{72}\times\frac{39}{57}$ or .57017 revolution.

9. B must make $.355\div\frac{1}{5}$ or 1.775 revolutions to move slide .355″. While A makes 1 revolution, B makes 1.775 revolutions, therefore A must have 1775 teeth and B 1000 teeth or A=71 teeth and B=40 teeth. PROOF: $\frac{71}{40}\times\frac{1}{5}''=.355''$.

10. While crank makes $1\frac{5}{8}$ revolutions, the worm wheel will make $1\frac{5}{8}\times\frac{20}{41}\times\frac{2}{51}$ or .03108 revolution.

11. While A makes 1 revolution, B will make $1\times\frac{40}{1}\times\frac{53}{67}$ or 31.641 revolutions.

EXERCISE 17

1. 1.0993. **2.** 28. **3.** $\frac{13}{24}$. **4.** $\frac{\$11.43}{1}\times\frac{4}{9}=\$5.08$.

5. $23''\times\frac{3.52}{4.56}=17.75''$. **6.** $\frac{17}{64}$. **7.** $100\times\frac{54}{29}=186.2$ revolutions.

8. $72\times\frac{12}{18}=48$ teeth. **9.** $.236''\times\frac{\frac{7}{9}}{\frac{4}{5}}=.236''\times\frac{7}{9}\times\frac{5}{4}=.2294''$.

EXERCISE 18

1. $\$12.50\times\frac{7}{4}\times\frac{8}{5}=\$26.25$. **2.** $7\times\frac{4}{12}\times\frac{40}{26}=3.59$ hours.

3. $45\times\frac{112}{250}\times\frac{7}{3}=47.04$ pounds. **4.** $\frac{2053.9}{1}\times\frac{10}{8}\times\frac{4}{3}\times\frac{3}{2}=$ 51347.5 pounds. **5.** $1000\times\frac{18}{6}\times\frac{18}{8}=6000$ revolutions.

6. $100\times\frac{31}{12}\times\frac{45}{29}=400.8$ revolutions. **7.** $5\times\frac{80}{71}\times\frac{1}{6}''=.93896''$.

8. $50\times\frac{240}{115}=104.34$ revolutions.

9. $6\times\frac{37}{20}\times\frac{25}{31}=8.9516$ revolutions.

EXERCISE 19

1. While A makes 1 revolution, the table is elevated $.250''\times\frac{2}{5}$ or $.100''$. If one revolution elevates it $.100''$, to elevate it $.001''$ will require $.001\div.100$ or .01 of a revolution. Therefore, the collar must have 100 divisions so that each division will cause A to revolve .01 of a revolution. 100 divisions. *Ans.*

2. While A makes 1 revolution, the slide moves $1\times.01\times.1''$ or $.001''$. If 1 revolution of A makes the slide move .001 of an inch, to move it .0001 of an inch would require $.0001\div.001$ or .1 of a revolution. Therefore the collar A must have 10 divisions so that each division will move it .1 of a revolution. 10 divisions. *Ans.*

3. One revolution of the cross-slide screw moves the slide $.250''$. If collar A has 125 divisions, each division will move it $.002''$. When the slide moves $.002''$, the diameter of the shaft is reduced $.004''$. If one division on the collar reduces the shaft diameter $.004''$, to reduce it $.018''$ it must be revolved $.018\div.004$ or 4.5 divisions. $4\frac{1}{2}$ divisions. *Ans.*

4. While A makes 1 revolution, the slide moves $.0375''$. If 1 revolution of A moves the slide $.0375''$, to move it $.0005''$ would require $.0005\div.0375$ or $\frac{1}{75}$ of a revolution. Therefore, A must have 75 divisions so that each division will move it $\frac{1}{75}$ of a revolution. 75 divisions. *Ans.*

5. While A moves 3 notches, the screw makes $\frac{3}{50}\times\frac{52}{26}$ or $\frac{3}{25}$ of a revolution. Since 1 revolution of the screw moves the table $\frac{2}{5}''$, $\frac{3}{25}$ of a revolution will move it $\frac{2}{5}''\times\frac{3}{25}$ or .048″. *Ans.*

6. While the rack moves 1 inch, C makes 1 revolution, and D makes 10 revolutions, because each revolution of D moves the rack .100″. When D makes 10 revolutions, A makes $10\times\frac{15}{60}$ or $2\frac{1}{2}$ revolutions; therefore, while the rack moves 1 inch, C makes 1 revolution, and A makes $2\frac{1}{2}$ revolutions.

If we know the number of revolutions the simple gears (A and C) must make, the numbers of teeth in them are obtained by inverting the number of the revolutions. Thus, if A makes $2\frac{1}{2}$ revolutions while C makes 1 revolution, A must have 1 tooth and B $2\frac{1}{2}$ teeth. B must be multiplied by 24 to obtain 60; therefore, A must also be multiplied by 24 to keep the ratio the same. Then A equals 24×1 or 24. 24 teeth. *Ans.*

EXERCISE 20

1. 213. **2.** 1010. **3.** .750. **4.** 47.138. **5.** .06.

6. 3.00001. **7.** 28.536. **8.** $\sqrt{\frac{32.1}{16.07}}=\sqrt{1.99751}=1.4133.$

EXERCISE 21

1. 175.5 minutes or 2 hours, 55 minutes, 30 seconds.

2. $\frac{45}{\frac{1}{4}}\times\frac{42}{11}$ or 687.2 R.P.M. **3.** 120 revolutions.

4. 21.48 R.P.M.

5. 5342 feet per minute. **6.** 15.71 feet per minute.

7. $\frac{6000}{6}\times\frac{42}{11}=3818$ R.P.M.; $B=10''\times\frac{210}{3818}=2.383''$.

8. When spindle makes 1 revolution, the spindle feeds

$$1\times\tfrac{45}{15}\times\tfrac{3}{8}\times\tfrac{20}{40}\times\tfrac{1}{130}\times12\times.3142''=\frac{8.4834''}{520}=.0163''.$$

To have a cutting speed of 60 feet per minute drill must make $\frac{60}{2}\times\frac{42}{11}$ or 114.5 R.P.M. To drill a hole 5″ deep by using a feed of .0163″, drill must make 306.7 revolutions. For the drill to make 306.7 revolutions at 114.5 R.P.M. it would take 2.67 minutes.

9. To have a cutting speed of 60 feet per minute, a 3.750″ shaft must make $\frac{60}{3.750}\times\frac{42}{11}$ or 61+ revolutions. The cutting is not to

exceed 60 feet per minute; therefore, the lathe would run at 44 R.P.M. on first cut. By using a .030″ feed the lathe would have to make $\frac{10.25}{.030}$ or 341.6 revolutions to finish the first cut on the shaft. At 44 R.P.M. it would take $\frac{341.6}{44}$ or 7.76 minutes to reduce shaft to 3.5″.

The lathe would have to make 65+ R.P.M. to have a cutting speed of 60 feet per minute on a 3.5″ shaft; therefore, on the second cut the lathe would also run 44 R.P.M., and the cutting time would be 7.76 minutes to reduce the shaft from 3.5″ to 3.25″.

The lathe would have to make 70+ R.P.M. to have a cutting speed of 60 feet per minute on a 3.25″ shaft; therefore, on the next cut the lathe would run at 68 R.P.M. By using a .030″ feed the lathe would have to make 208.3 revolutions to reduce both ends of the shaft to 3″. At 68 R.P.M. it would take 3.06 minutes to do this.

The total time on shaft would then be 7.76+7.76+3.06 or 18.58 minutes.

10. Crank must make $\frac{45}{2\times 2}\times\frac{42}{11}$ or 42.95 R.P.M. for tool to have a maximum speed of 45 feet per minute. While crank makes 42.95 R.P.M., A will make $42.95\times\frac{90}{12}$ or 214.75 R.P.M.

EXERCISE 22

1. $?=180°-123°\ 14'=56°\ 46'$.

2. $?=180°-(40°+58°+31°)=51°$.

3. $?=32°-25°+7°=14°$.

4. $90°-37°=53°$.

5. $A=90°-50°=40°$.
$?=70\frac{1}{2}°-40°=30\frac{1}{2}°$.

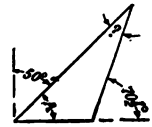


FIG. 115S

6. $?=64°+59°=123°$.

7. $?=\frac{180°-50°}{2}=65°$.

FIG. 118S

8. $A=60°-29°=31°$.
$B=60°$.
$?=180°-(60°+31°)=89°$.

EXERCISE 23

1. $5.674'' \times 3.1416 = 17.825''$.

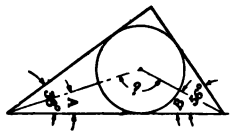


FIG. 124S

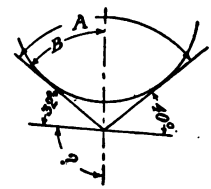


FIG. 125S

2. $A = \frac{38}{2}° = 19°$.
$B = \frac{56}{2}° = 28°$.
$? = 180° - (19° + 28°) = 133°$.

3. $A = 180° - (40° + 32°) = 108°$.
$B = \frac{108}{2}° = 54°$.
$? = 180° - (54° + 32°) = 94°$.

FIG. 126S

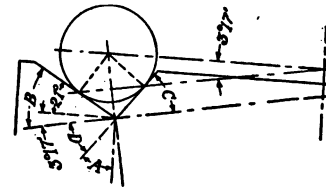


FIG. 127S

4. $A = \frac{360° - 250°}{2} = 55°$.
$? = 64° - 55° = 9°$.

5. $B = 24°\ 17'$. $C = 24°\ 17'$.
$D = 24°\ 17'$, because it is an opposite angle to C.
$A = 90° - (24°\ 17' + 3°\ 17') = 62°\ 26'$.

EXERCISE 24

1. $1.375'' \times 1.4142 = 1.9445''$.

2. $3.5'' \div 1.4142 = 2.4749''$.

3. $1.125'' \times 1.1547 = 1.29903''$.

4. $1.700'' \div 1.1547 = 1.4722''$.

5. $180° - 2(128° - 90°) = 104°$.

6. The sum of the interior angles $= (6-2)180° = 720°$. $A = 720° - (44° + 300° + 42° + 142° + 72°) = 720° - 600° = 120°$.

7. $X = 3.1416'' \div 10 = .31416''$. Circumference of small gear $= 1.2'' \times 3.1416$. Then $(1.2'' \times 3.1416) \div .31416'' = 12$, the number of teeth in the small gear.

EXERCISE 25

1. $A = \frac{5}{8}'' \times \frac{5}{8}'' \div 1\frac{5}{16}'' = .2976''$.

$? = 1\frac{5}{16}'' + .2976'' = 1.6101''$.

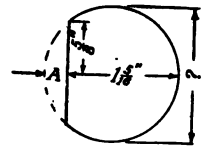


FIG. 148S

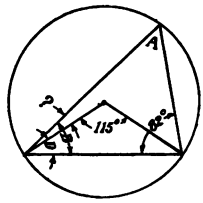


FIG. 149S

2. $D = \frac{180° - 115°}{2} = 32\frac{1}{2}°$.

$A = 1\frac{1}{2}5 = 57\frac{1}{2}°$.

$B = 180° - (57\frac{1}{2}° + 82°) = 40\frac{1}{2}°$.

$C = 40\frac{1}{2}° - 32\frac{1}{2}° = 8°$.

3. $? = 16'' + 12'' - 8'' = 20''$. **4.** $? = 2\,(180° - 80° - 26°) = 148°$.

EXERCISE 26

1. $10'' \times 5.4'' = 54$ square inches. **2.** 45.36 square inches.

3. 10.455 square inches. **4.** 34.16 square inches.

5. $\frac{3.1'' + 9.2''}{2} \times 4.1'' = 25.215$ square inches.

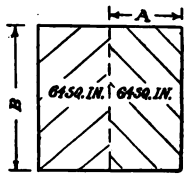


FIG. 169S

6. 103.869 square inches.

7. $\pi R^2 = 12.76$ square inches.

$R^2 = \frac{12.76}{\pi} = 4.06162$.

$R = \sqrt{4.06162} = 2.015''$.

Diameter $= 2 \times 2.015'' = 4.030''$.

8. $B = \sqrt{128} = 11.313''$.

9. $X = \frac{160}{5} = 32$ square inches.

$Y = 2 \times 32 = 64$ square inches.

$A = \sqrt{64} = 8''$.

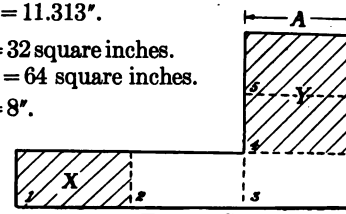


FIG. 170S

10. Area of a 14″ square = 196 square inches. Area of a 14″ circle = 153.9384 square inches. The difference = 42.0616 square inches, and the area of A = 42.0616 ÷ 4 = 10.515 square inches.

11. $4 \times 6'' \times 6'' \times 3.1416 = 452.3904$ square inches.

EXERCISE 27

1. $7'' \times 5'' \times 7.3'' = 255.5$ cubic inches. **2.** 126.9632 cubic inches. **3.** $3.1'' \times 3.1'' \times 3.1416 \times 7.1'' = 214.354$ cubic inches.

4. 103.90842 cubic inches. **5.** $\frac{1}{2}(4'' \times 7.1'')\ (\frac{1}{3})\ (5.2'') = 24.613$ cubic inches. **6.** $4'' \times 4'' \times 3.1416 \times \frac{1}{3} \times 6.2'' = 103.882$ cubic inches.

7. $\frac{2''+4''}{2} \times 6'' \times \frac{1}{3} \times 4.2'' = 25.2$ cubic inches.

8. $\frac{4}{3} \times 7''^3 \times 3.1416 = 1436.7584$ cubic inches.

EXERCISE 28

1. $C = \sqrt{10.11^2 + 6.72^2} = 12.139$. **2.** $C = \sqrt{11^2 + 14^2} = 17.804$.

3. $A = \sqrt{7.2^2 - 6.4^2} = 3.2984$. **4.** $B = \sqrt{8.04^2 - .312^2} = 8.0339$.

5. $A = \sqrt{10.7^2 - 5.661^2} = 9.0798$.

6. $? = 2\sqrt{2^2 - (\frac{7}{8})^2} = 2 \times \sqrt{3.234375} = 3.5968''$.

7. $? = (1\frac{3}{8} + 1\frac{1}{4}) - \sqrt{(1\frac{3}{8})^2 - 1} = 2.625'' - .9437'' = 1.6813''$.

8. $? = \sqrt{1^2 - .5^2} = .86603''$.

9. $? = .375 + .250 + \sqrt{(4.5 - .500 - .250)^2 + (3.750 - .500 - .375)^2}$
$= .625 + \sqrt{22.3281} = .625'' + 4.7252'' = 5.3502''$.

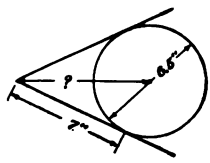


FIG. 193S

10. $? = \sqrt{3.25^2 + 7^2}$
$? = \sqrt{10.5625 + 49}$
$? = \sqrt{59.5625}$
$? = 7.7177''$.

11. $2.600 + 1.62 - \sqrt{2.6^2 + 1.62^2} =$
$4.22 - \sqrt{6.76 + 2.6244} =$
$4.22 - \sqrt{9.3844} =$
$4.22'' - 3.0634'' = 1.1566''$. Diameter.

12. $B=\sqrt{.8^2-.250^2}=$
$\sqrt{.64-.0625}=$
$\sqrt{.5775}=$
$.7599''.$
$?=.8''+.7599''-.250''=$
$1.3099''.$

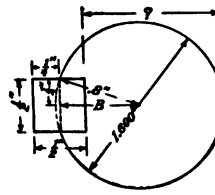


FIG. 195S

13. $A=4''-.9''=3.1''.$
$B=4''+\frac{7}{8}''=4\frac{7}{8}''.$
$C=\sqrt{A^2+B^2}.$
$C=\sqrt{3.1^2+4.875^2}=$
$\sqrt{9.61+23.7656}=$
$\sqrt{33.3756}=5.7771''.$
$?=5.7771''+.9''+.875''=$
$7.5521''.$

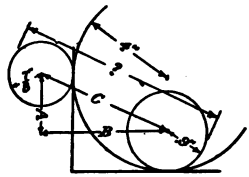


FIG. 196S

14. $?=\sqrt{6^2+2.7^2}=6.5795''.$

15. $A=\sqrt{7^2+7^2}=9.8994''.$
$B=9.8994''-7''=2.8994''.$
$C=2B=5.7988''.$
$D=1.4142''\times 2.8994''=4.1003''.$
$?=2\times 4.1003''-5.7988''=$
$2.4018''.$

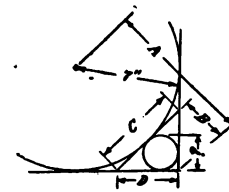


FIG. 198S

EXERCISE 29

1. $?=\dfrac{8.4^2-5.2^2}{2\times 10.5}=2.0723.$

2. $?=\dfrac{12.6^2-9.3^2}{2\times 6}=6.0225.$

3. $Y=\dfrac{7.21^2-5.9^2}{2\times 8.2}=1.0472''.$
$X=4.1''-1.0472''=$
$3.0528''.$
$?=\sqrt{5.9^2-3.0528^2}=$
$5.0488''.$

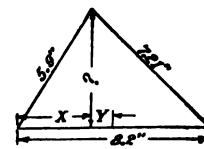


FIG. 204S

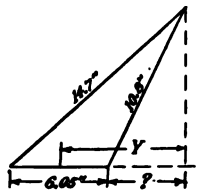


FIG. 205S

4. $Y=\dfrac{14.7^2-10.8^2}{2\times 6.05}=8.219''$.

$?=8.219''-3.025''=5.194''$.

EXERCISE 30

1. 44°. **2.** $\dfrac{180°}{8}=22\frac{1}{2}°$. **3.** 17° 56′ 42″. **4.** 79° 22′ 45″.

5. $26°\,37'\,23''=26°\,37\frac{23}{60}'=\left(26\dfrac{37\frac{23}{60}}{60}\right)°=26.623°$

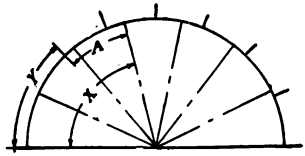


FIG. 216S

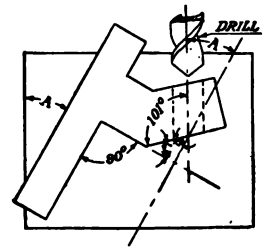


FIG. 217S

6. $X=\frac{3}{7}\times 180°=77\frac{1}{7}°$.
$Y=\frac{2}{5}\times 180°=45°$.
$A=X-Y=77\frac{1}{7}°-45°=32\frac{1}{7}°$.
$A=32°\,8'\,43''$.

7. $B=90°-41°\,7'=48°\,53'$.
$A=180°-(101°+48°\,53')=30°\,7'$.

8. $168°=\frac{168}{360}$ or $\frac{7}{15}$ of a revolution; $172°=\frac{172}{360}$ or $\frac{43}{90}$ of a revolution. By inverting the revolutions and calling them gear teeth, $A=\frac{43}{90}$ of a tooth and $B=\frac{7}{15}$ of a tooth. By multiplying both by 90, A = 43 teeth and B = 42 teeth.

9. $123°\times\frac{48}{37}=159.567°=159°\,34'\,1''$.

10. Circumference $=3.1416\times 6.35''=19.949''$. $\dfrac{3.271}{19.949}$ = part of the whole circle that θ equals. $\theta=360°\times\dfrac{3.271}{19.949}=59.028°$ or 59° 1′ 41″.

11. Circumference $= 1.865'' \times \frac{360°}{62°14'} = 1.865'' \times \frac{360}{62\frac{7}{30}} = 10.788''$.

Diameter $= \frac{10.788}{3.1416} = 3.4339''$.

EXERCISE 31

1. $A = 1.317'' - .892'' = .425''$.

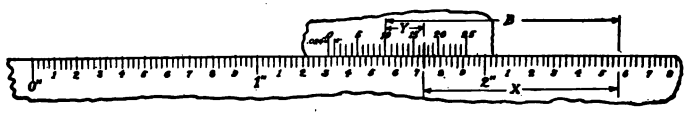


FIG. 229S

2. $X = 2.575'' - 1.725'' = .850''$. $Y = 7 \times .024'' = .168''$. $B = .850'' + .168'' = 1.018''$.

3. $C = 2.545'' - .892'' = 1.653''$.

4. $D = (1.300'' - .150'') - (14 \times .024'') = .814''$. **5** $A = 1.014''$.

6. $B = (1.350'' - .475'') - (.024'' \times 7) = .707''$. **7.** $A = .284''$.

8. $B = .289''$.

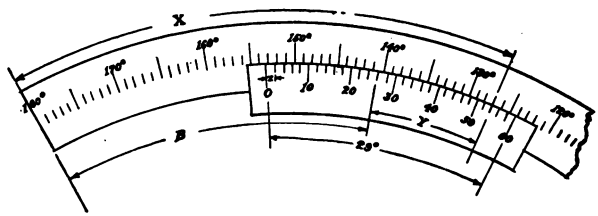


FIG. 232S (a)

9. $A = 26° 54'$.

10. $X = 53°$. $Z = \frac{29}{30}°$.
$Y = 14 \times \frac{29}{30}° = 14 \times 58' = 13° 32'$.
$B = 53° - 13° 32' = 39° 28'$.

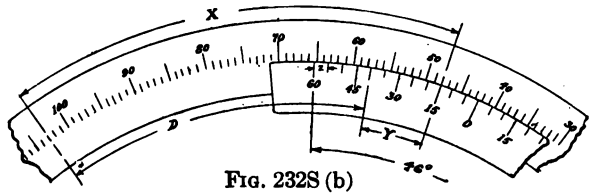


FIG. 232S (b)

11. C = 42° 15′.

12. X = 55°.
Z = $\frac{4\frac{6}{4}}{4}$° = $1\frac{1\frac{1}{2}}{2}$° = 1° 55′.
Y = 5 × 1° 55′ = 9° 35′.
D = 55° − 9° 35′ = 45° 25′.

EXERCISE 32

1. $? = .601'' \times \frac{1.764}{.913} = 1.1611''$. **2.** $? = 3.1416 \times \frac{.687}{1} = 2.1583$.

3. $A = \sqrt{.75^2 + .5^2} = .90138''$.

$? = .90138'' \times \frac{2}{.750} = 2.4036''$.

FIG. 242S

4. $? = .875 \times \frac{1.1}{2.31} = .416666$.

5. $? = .4 \times \frac{1.33}{.871} = .61079''$. **6.** $? = 1'' \times \frac{1}{.8571} = 1.1667''$.

7. $? = 8.1'' \times \frac{4}{9} = 3.6''$.

8. $A = \sqrt{2.41^2 + 1.32^2} = 2.7478$.

$? = 1.32'' \times \frac{1.71}{2.7478} = .82145''$.

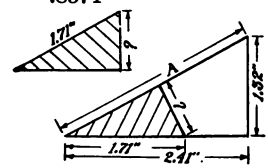


FIG. 247S

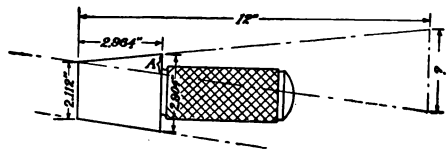


FIG. 248S

9. $A = 2.904'' - 2.112'' = .792''$.

$? = .792'' \times \frac{12}{2.964} = 3.2064''$.

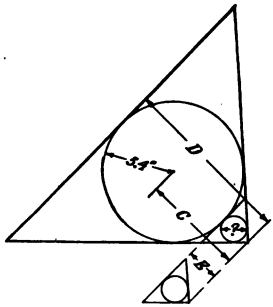


FIG. 249S

10. $C = 1.4142 \times 5.4'' = 7.6366''$.

$D = 7.6368'' + 5.4'' = 13.0366''$.

$E = 7.6368'' - 5.4'' = 2.2366''$.

$? = 10.8'' \times \frac{2.2366}{13.0366} = 1.8529''$.

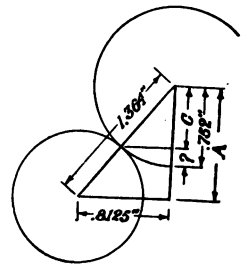


FIG. 250S

11. $A = \sqrt{1.364^2 - .8125^2} = 1.0956''$.

$C = 1.0956'' \times \frac{.752}{1.364} = .60402''$.

$? = .752'' - .60402'' = .14798''$.

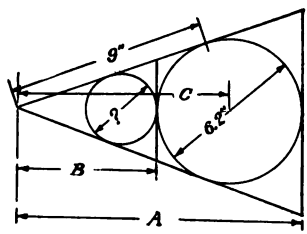


FIG. 251S

12. $C = \sqrt{9^2 + 3.1^2} = 9.5189''$.

$A = 9.5189'' + 3.1'' = 12.6189''$.

$B = 9.5189'' - 3.1'' = 6.4189''$.

$? = 6.2'' \times \frac{6.4189}{12.6189} = 3.1537''$.

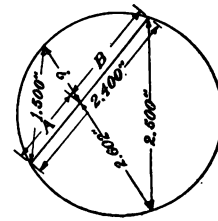


FIG. 252S

13. $A = 2.602'' \times \frac{1.5}{2.5} = 1.5612''$.

$B = 2.400'' - 1.5612'' = .8388''$.

$? = .8388'' \times \frac{1.5}{2.5} = .50328''$.

EXERCISE 33

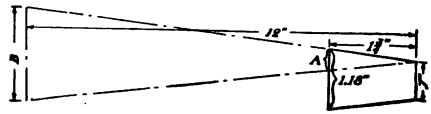


FIG. 258S

1. $A = 1.18'' - .7'' = .48''$.

$B = .48'' \times \frac{12}{1\frac{3}{4}} = \frac{23.04''}{7} = 3.2914'' =$ taper per foot.

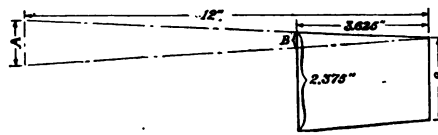


FIG. 259S

2. $A = .4375''$ or taper per foot.

$B = .4375'' \times \frac{3.625}{12} = \frac{1.5859''}{12} = .1321''$.

$? = 2.375'' - .1321'' = 2.2429''$.

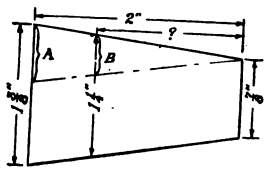


FIG. 260S

3. $A = 1\frac{3}{8}'' - \frac{7}{8}'' = \frac{4}{8}'' = \frac{1}{2}''$.

$B = 1\frac{1}{4}'' - \frac{7}{8}'' = \frac{3}{8}''$.

$? = 2 \times \frac{\frac{3}{8}}{\frac{1}{2}} = 1.5''$.

4. $A = 3\frac{3}{8}'' - 1\frac{3}{4}'' = 1\frac{5}{8}''$.

$B = 1\frac{5}{8}'' \times \frac{1\frac{7}{8}}{4\frac{1}{4}} = .7169''$.

$? = 1.750'' + .7169'' = 2.4669''$.

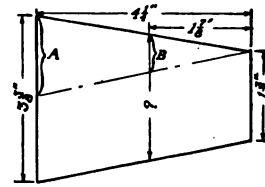


FIG. 261S

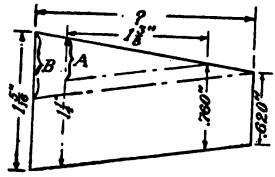


FIG. 262S

5. A = 1.250″ − .760″ = .490″.
B = 1.3125″ − .620″ = .6925″.
$? = 1\frac{3}{8}'' \times \frac{.6925}{.490} = 1.9432''$.

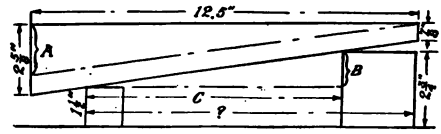


FIG. 263S

6. $A = 2\frac{5}{8}'' - \frac{7}{8}'' = 1\frac{3}{4}''$. $B = 2\frac{3}{4}'' - 1\frac{1}{2}'' = 1\frac{1}{4}''$.
$C = 12.5'' \times \frac{1\frac{1}{4}}{1\frac{3}{4}} = 8.9285''$.
$? = 8.9285'' + 2.750'' = 11.6785''$.

7. $A = 1'' - \frac{3}{8}'' = \frac{5}{8}''$.
$B = 5\frac{7}{8}'' \times \frac{.009}{\frac{5}{8}} = .0846''$.

If one revolution of screw moves gib $\frac{1}{12}$ of an inch, to move it .0846″ would require $.0846 \div \frac{1}{12}$ or 1.0152 revolutions.

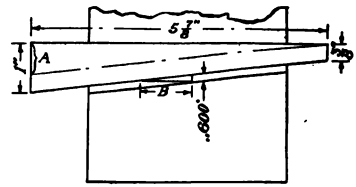
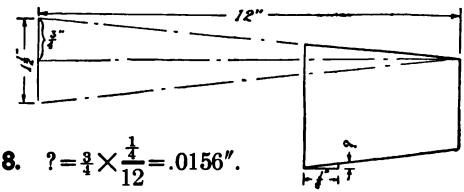


FIG. 264S



8. $? = \frac{3}{4} \times \frac{\frac{1}{4}}{12} = .0156''$.

FIG. 265S

EXERCISE 34

1. A = 10 × .25882 = 2.5882.
2. B = 10 × .96592 = 9.6592.
3. A = 8 × 1.0075 = 8.06.
4. B = 8 × .12278 = .98224.
5. A = 2.175 × 1.7434 = 3.7919.
6. B = 2.175 × 1.4281 = 3.1061.
7. A = 4.6947 ÷ 10 = .46947.
8. B = 8.8295 ÷ 10 = .88295.
9. A = 2.3877 ÷ 2.025 = 1.179.
10. B = 1.2654 ÷ 2.025 = .62488.
11. A, E, I. **12.** B, D, H. **13.** C, F, G.

EXERCISE 35

1. Sine. **2.** Cosine. **3.** Tangent. **4.** Secant.
5. Cotangent. **6.** Cosecant. **7.** A = secant 23° = 1.0864.
8. B = tangent 23° = .42447. **9.** A = sine 29° = .48481.
10. B = cosine 29° = .87462. **11.** A = cosecant 20° 3′ = 2.9168.
12. B = cotangent 20° 3′ = 2.7400. **13.** A = secant 46° = 1.4395.
14. B = tangent 46° = 1.0355. **15.** A = sine 59° = .85717.
16. B = cosine 59° = .51504. **17.** A = cosecant 16° = 3.6279.
18. B = cotangent 16° = 3.4874.

EXERCISE 36

1. A = 4.3 (secant 16°) = 4.3 × 1.0403 = 4.47329.
2. B = 4.3 (tangent 16°) = 4.3 (.28674) = 1.23298.
3. C = 4.3 (sine 16°) = 4.3 (.27564) = 1.18525.
4. A = 1.760 (cotangent 31°) = 1.760 (1.6643) = 2.929168.
5. B = 1.760 (cosecant 31°) = 1.760 (1.9416) = 3.4172.
6. A = 1.01 (cosine 42° 11′) = 1.01 (.74100) = 7.4841.

EXERCISE 37

1. 2.3428 = cosecant 25° 16′. **2.** 2.7450 = tangent 69° 59′.
3. 3.1379 = secant 71° 25′. **4.** 2.4525 = cotangent 22° 11′.
5. 1.1699 = secant 31° 16′. **6.** .76754 = sine 50° 8′.
7. .52646 = cosine 58° 14′. **8.** 3.0595 = tangent 71° 54′.

EXERCISE 38

1. Cotangent $A = \frac{2.170}{\frac{3}{4}} = 2.89333$, A = 19° 3′ 59″; or tangent $A = \frac{\frac{3}{4}}{2.170} = .345622$, A = 19° 3′ 59″. **2.** Secant $A = \frac{12.021}{10.5} = 1.14485$, A = 29° 8′ 8″; or cosine $A = \frac{10.5}{12.021} = .87347$, A = 29° 8′ 8″.

3. A = .375″ (cotangent $17\frac{1}{2}°$) = .375″(3.1716) = 1.18935″.

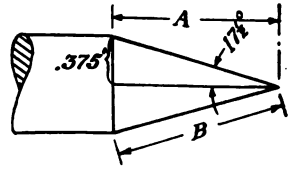


FIG. 314S

4. B = .375″(cosecant $17\frac{1}{2}°$) = .375″(3.3255) = 1.24707″.

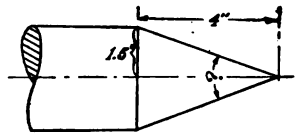


FIG. 315S

5. Tangent of $\frac{1}{2}$ the angle $= \frac{1.5}{4} =$ .375; $\frac{1}{2}$ angle $= 20° 33' 21''$; or cotangent of $\frac{1}{2}$ the angle $= \frac{4}{1.5} = 2.6667$; $\frac{1}{2}$ angle $=$ $20° 33' 21''$.
$? = 2(20° 33' 21'') = \mathbf{41° 6' 42''}$.

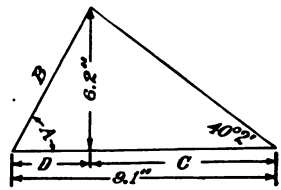


FIG. 316S

6. $C = 6.2''(\text{cotangent } 40° 2') = 6.2''(1.1903) = 7.37986''$.
$D = 9.1'' - 7.37986'' = 1.72014''$.

$\text{Cotangent A} = \frac{1.72014}{6.2} = .27744.$ $\quad A = 74° 29' 38''$.

7. $B = 6.2''(\text{cosecant } 74° 29' 38'') = 6.2''(1.03778) = 6.4343''$; or

$B = \sqrt{D^2 + 6.2^2} = \sqrt{1.72014^2 + 6.2^2} = \sqrt{2.9589 + 38.44} = \sqrt{41.3989} = 6.4343''$.

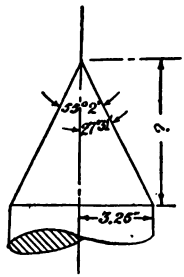


FIG. 317S

8. $? = 3.25''$ (cotangent $27° 31'$) $= 3.25''(1.9196) =$ $6.2387''$.

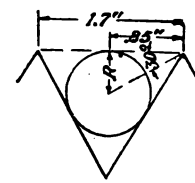


FIG. 318S

9. $R = .850''(\text{tangent } 30°) =$ $.850''(.57735) =$ $.49074''$.
Diameter $= 2(.49074'') =$ $.98148''$.

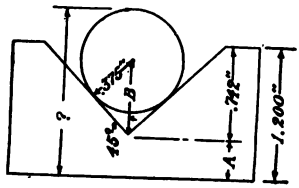


FIG. 319S

10. A = 1.200″ − .742″ = .458″.
B = .375″(cosecant 45°) = .375″(1.4142) = .53033″.
? = .458″ + .53033″ + .375″ = 1.3633″.

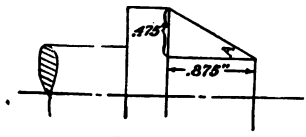


FIG. 320S

11. Cotangent $A = \frac{.875}{.475} =$ 1.8421.
A = 28° 29′ 45″.
? = 2(28° 29′ 45″) = 56° 59′ 30″.

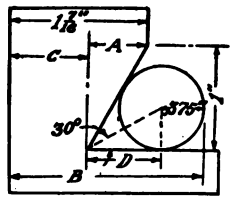


FIG. 321S

12. A = 1″(tangent 30°) = .57735″.

13. C = 1.4375″ − .57735″ = .86015″
D = .375″ (cotangent 30°) = .375″ × 1.7321 = .64954″.
B = .86015″ + .64954″ + .375″ = 1.8847″.

15. C = 1.125″.
A = 1.125″ − .250″ = .875″.
D = 1.125″(sine 60°) = 1.125″(.86603) = .97428″.

16. B = .97428″ + .1875″ − .125″ = 1.03678″.

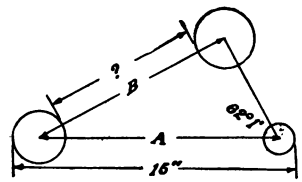


FIG. 322S

14. A = 15″ − .750″ − .4375″ = 13.8125″.
B = A(sine 62° 1′) = 13.8125″(.88308) = 12.1975″.
? = 12.1975″ − .750″ − .9375″ = 10.510″.

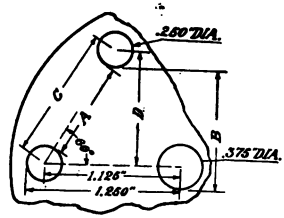


FIG. 323S

EXERCISE 39

1. Tangent A = 1.75 ÷ 3.45 = .50725. A = 26° 53′ 47″.
   Cotangent A = 3.45 ÷ 1.75 = 1.9714. A = 26° 53′ 47″.
2. X = 3.45(secant 26° 53′ 47″) = 3.45 × 1.1213 = 3.8685.
   X = 1.75(cosecant 26° 53′ 47″) = 1.75 × 2.2106 = 3.8685
   $X = \sqrt{3.45^2 + 1.75^2} = 3.8685$.
3. Sine A = 2.67 ÷ 4.006 = .66650. A = 41° 47′ 52″.
   Cosecant A = 4.006 ÷ 2.67 = 1.50037. A = 41° 47′ 52″.
4. X = 4.006(cosine 41° 47′ 52″) = 4.006 × .74551 = 2.9864.
   X = 2.67(cotangent 41° 47′ 52″) = 2.67 × 1.1185 = 2.9864.
   $X = \sqrt{4.006^2 - 2.67^2} = 2.9864$.
5. Cosine A = (5.62″ − 1″) ÷ (6.01″ − 1″) = .92215.
   A = 22° 45′ 27″.
6. X = 1″ + (5.62″ − 1″)(tangent 22° 45′ 27″) = 1″ + 4.62″ × .41949
   = 2.938″, or $X = 1'' + \sqrt{(6.01-1)^2 - (5.62-1)^2} = 2.938''$.
7. Tangent A = (1.1″ + 2″) ÷ (8.52″ − 2.50″) = .51495.
   A = 27° 14′ 47″.
8. X = (8.52″ − 2.50″)(secant 27° 14′ 47″) − 1.5″ =
   6.02″ × 1.1248 − 1.5″ = 5.2713″, or
   $X = \sqrt{(8.52-2.50)^2 + (1.1+2)^2} - 1.5'' = 5.2713''$.

EXERCISE 40

FIG. 332S

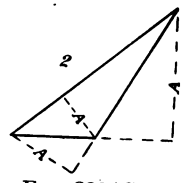


FIG. 332AS

FIG. 332BS

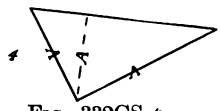


FIG. 332CS

FIG. 332DS

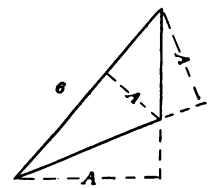


FIG. 332ES

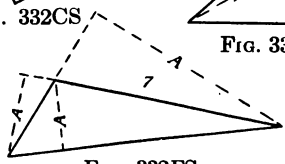


FIG. 332FS

EXERCISE 41

1. $X=7+3=10$. **2.** $X=9-3.5=5.5$. **3.** $X=180°-84°=96°$.

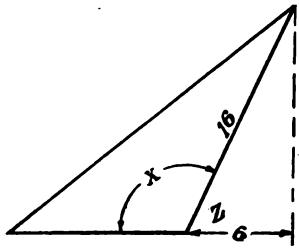


FIG. 339S

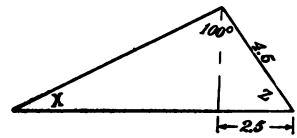


FIG. 340S

4. Cosine $Z=6\div16=.375$.
$Z=67°\ 58'\ 32''$.
$X=180°-67°\ 58'\ 32''=112°\ 1'\ 28''$.

5. Cosine $Z=2.5\div4.5=.55555$.
$Z=56°\ 15'\ 4''$.
$X=180°-(100°+56°\ 15'\ 4'')=23°\ 44'\ 56''$.

6. $X=\dfrac{4^2-3^2}{2\times6}=\dfrac{7}{12}=.58333$.

7. $X=8+\dfrac{14^2-12^2}{2\times16}=8+\dfrac{52}{32}=9.625$.

8. $X=\dfrac{5.5^2-3^2}{2\times3.5}=\dfrac{21.25}{7}=3.0357$.

9. $X=1.25+\dfrac{5.25^2-3.5^2}{2\times2.5}=1.25+\dfrac{15.3125}{5}=4.3125$.

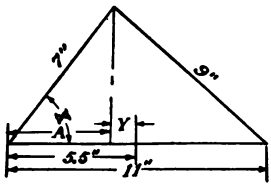


FIG. 345S

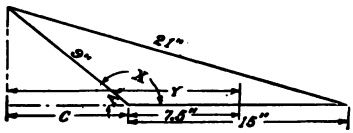


FIG. 346S

10. $Y=\dfrac{9^2-7^2}{2\times11}=\dfrac{81-49}{22}=\dfrac{32}{22}=1.4545''$.
$A=5.5''-1.4545''=4.0455''$.
Cosine $X=\dfrac{4.0455}{7}=.57793$.
$X=54°\ 41'$.

11. $Y=\dfrac{21^2-9^2}{2\times15}=\dfrac{441-81}{30}=12''$.
$C=12''-7.5''=4.5''$.
Cosine $A=\dfrac{4.5}{9}=.5$. $A=60°$.
$X=180°-60°=120°$.

EXERCISE 42

1. X = 5.5(sine 30°) = 5.5 × .50000 = 2.75.

2. X = 21 − 14.5 = 6.5.

3. X = 8.5(cosine 27°) = 8.5 × .89101 = 7.5735.

4. X = 7.2(cosine 42°) = 7.2 × .74314 = 5.3506.

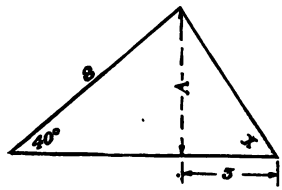


FIG. 356S

5. A = 8(sine 40°) = 8 × .64279 = 5.14232.
Tangent X = 5.14232 ÷ 3 = 1.714141.
X = 59° 44′ 27″.

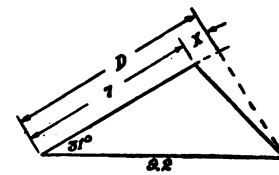


FIG. 357S

6. D = 9.2(cosine 31°) = 9.2 × .85717 = 7.8859.
X = 7.8859 − 7 = .8859.

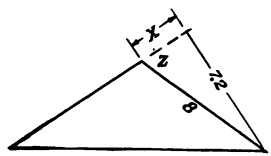


FIG. 358S

7. Sine Z = 7.2 ÷ 8 = .90000.
Z = 64° 9′ 30″.
X = 8(cosine 64° 9′ 30″) = 8 × .43588 = 3.487, or
X = $\sqrt{8^2 - 7.2^2}$ = 3.487.

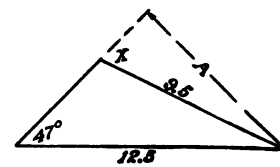


FIG. 359S

8. A = 12.5(sine 47°) = 12.5 × .73135 = 9.1419.
Sine X = 9.1419 ÷ 9.5 = .96230.
X = 74° 13′.

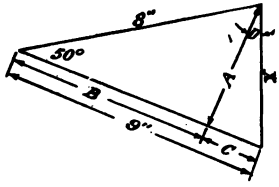


FIG. 360S

9. $A = 8''(\text{sine } 50°) = 8''(.76604) = 6.12832''$.

$B = 8''(\text{cosine } 50°) = 8''(.64279) = 5.14232''$.

$C = 9'' - 5.14232'' = 3.85768''$.

$\text{Cotangent } D = \dfrac{6.12832}{3.85768} = 1.58860$.

$D = 32° 11' 24''$.

$X = 6.12832''(\text{secant } 32° 11' 24'') = 6.12832''(1.1816) = 7.2412''$.

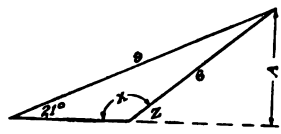


FIG. 361S

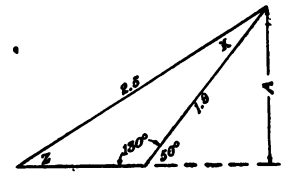


FIG. 362S

10. $A = 9(\text{sine } 21°) = 9 \times .35837 = 3.22533$.

$\text{Sine } Z = 3.2253 \div 6 = .53755$.

$Z = 32° 31' 3''$.

$X = 180° - 32° 31' 3'' = 147° 28' 57''$.

11. $A = 1.9(\text{sine } 50°) = 1.9 \times .76604 = 1.45547$.

$\text{Sine } Z = 1.45547 \div 2.5 = .58219$.

$Z = 35° 36' 17''$.

$X = 180° - (130° + 35° 36' 17'') = 14° 23' 43''$.

EXERCISE 43

1. $X = 2.426(\text{sine } 30°) = 2.426 \times .50000 = 1.213$.

2. $X = 8(\text{sine } 69°) = 8 \times .93358 = 7.4686$.

3. $X = 7.5(\text{tangent } 24°) = 7.5 \times .44523 = 3.3392$.

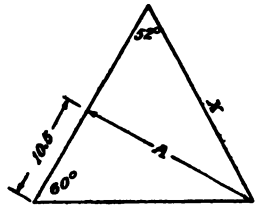


FIG. 375S

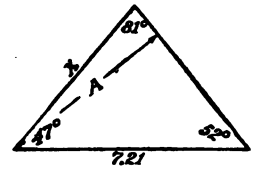


FIG. 376S

4. A = 10.5 (tangent 60°) = 10.5 × 1.7320 = 18.186.
X = 18.186 (cosecant 52°) = 18.186 × 1.2690 = 23.078.

5. A = 7.21 (sine 52°) = 7.21 × .78801 = 5.6815.
X = 5.6815 (cosecant 81°) = 5.6815 × 1.0125 = 5.7525.

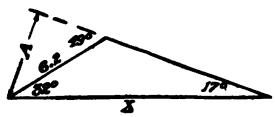


FIG. 377S

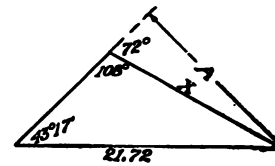


FIG. 378S

6. A = 6.2 (sine 49°) = 6.2 × .75471 = 4.6792.
X = 4.6792 (cosecant 17°) = 4.6792 × 3.4203 = 16.004.

7. A = 21.75 (sine 43° 17′) = 21.75 × .68561 = 14.8914.
X = 14.8914 (cosecant 72°) = 14.8914 × 1.0515 = 15.6583.

8. B = 18° + 31° = 49°
A = 6 (sine 49°) = 6 (.75471) = 4.52826.
X = 4.52826 (cosecant 18°) = 4.52826 (3.2361) = 14.6539.

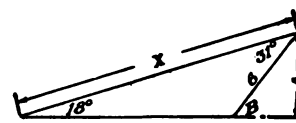


FIG. 379S

EXERCISE 44

1. $X=\dfrac{9}{\text{cotangent } 48°\ 2'+\text{cotangent } 51°}=\dfrac{9}{.89935+.80978}=5.2658.$

2. $X=\dfrac{4.72}{\text{cotangent } 52°+\text{cotangent } 62°}=3.5948.$

3. $X=\dfrac{6}{\text{cotangent } 21°-\text{cotangent } 50°}=\dfrac{6}{2.6051-.83910}=3.3975.$

4. $Y=\dfrac{10.2^2-6.6^2}{2(11.6)}=2.6069''.$

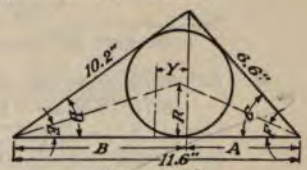


FIG. 385S

$A=\dfrac{11.6''}{2}-2.6069''=3.1931''.$

$B=\dfrac{11.6''}{2}+2.6069''=8.4069''.$

Cosine $C=\dfrac{3.1931}{6.6}=.48380$; $C=61°\ 3'\ 58''.$

Cosine $D=\dfrac{8.4069}{10.2}=.82420$; $D=34°\ 29'\ 34''.$

$E=\frac{1}{2}(34°\ 29'\ 34'')=17°\ 14'\ 47''.$

$F=\frac{1}{2}(61°\ 3'\ 58'')=30°\ 31'\ 59''.$

$R=\dfrac{11.6}{\text{cotangent } 17°\ 14'\ 47''+\text{cotangent } 30°\ 31'\ 59''}=\dfrac{11.6}{3.2229+1.6954}=\dfrac{11.6}{4.9183}=2.3585''.$

Diameter $=2(2.3585'')=4.717''.$

5. $A=8''(\text{sine } 59°)=8''(.85717)=6.85736''.$

Sine $B=\dfrac{6.85736}{7.6}=.90228$; $B=64°\ 27'\ 35''.$

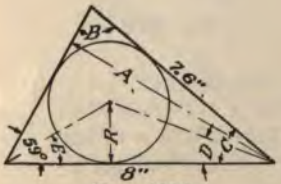


FIG. 386S

$C=180°-(59°+64°\ 27'\ 35'')=56°\ 32'\ 25''.$

$D=28°\ 16'\ 12''.$ $E=29°\ 30'.$

$R=\dfrac{8''}{\text{cot. } 29°\ 30'+\text{cot. } 28°\ 16'\ 12''}=\dfrac{8''}{1.7675+1.8595}=2.2056''.$

Diameter $=2(2.2056'')=4.4102''.$

EXERCISE 45

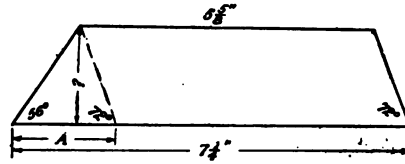


FIG. 387S

1.

$A = 7\frac{1}{4}'' - 5\frac{5}{8}'' = 1\frac{5}{8}''.$

$$? = \frac{1.625''}{\text{cotangent } 56^\circ + \text{cotangent } 72^\circ} = 1.6259''.$$

2. $A = .4375''(\text{cosecant } 14^\circ\ 30') =$
$.4375''(3.9939) = 1.74733''.$
$C = \frac{1}{2}(1\frac{3}{32}'') = .54687''.$
$D = .54687''(\text{cotangent } 14^\circ\ 30'') =$
$.54687''(3.8667) = 2.11458''.$
$E = 1.74733'' + .4375'' = 2.18483''.$
$? = 2.1848'' - 2.11458'' = .07022''.$

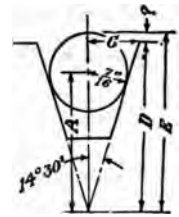


FIG. 388S

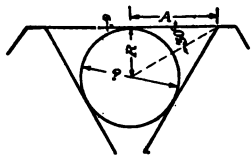


FIG. 389S

3.

$$A = \frac{.250'' - .03125''}{2} = .109375''.$$

$R = .1094''(\text{tangent } 30^\circ) =$
$.1094''(.57735) = .06316''.$
$? = 2(.06316'') = .12632''.$

4. $C = .28125''(\text{cotangent } 22^\circ\ 30') =$
$.28125''\ (2.4142) = .67899''.$
$A = 2.500'' - 2(.67899'') +$
$.5625'' = 1.7045''.$

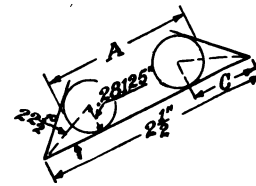


FIG. 390S (a)

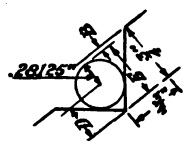


FIG. 390S (b)

5. $D = .28125''(\text{cosecant } 45^\circ) =$
$.28125''(1.4142) = .39774''.$
$E = .39774'' + .28125'' = .67899''.$
$B = .750'' - .67899'' = .07101''.$

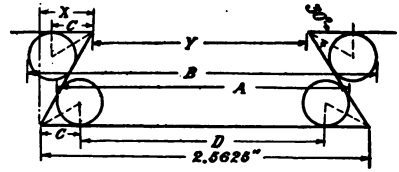


FIG. 391S

6. C = .1875″(cotangent 30°) = .1875″(1.732) = .3247″.
D = 2.5625″ − 2(.3247″) = 1.9131″.
A = 1.9131″ + .375″ = 2.2881″.

7. X = .750(tangent 30°) = .750″(.57735) = .4330″.
Y = 2.5625″ − 2(.433″) = 1.6965″.
B = 1.6965″ + 2(.3247″) + .375″ = 2.7209″.

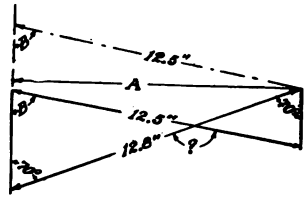


FIG. 392S

8. A = 12.8″(sine 70°) = 12.8″ × .93969 = 12.028″.
$\text{Sine B} = \dfrac{12.028}{12.5} = .96224.$
B = 74° 12′ 15″. ? = 70° + 74° 12′ 15″ = 144° 12′ 15″.

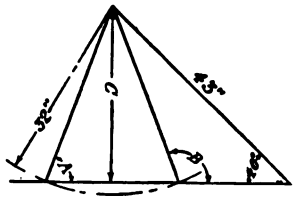


FIG. 393S

9. C = 43″(sine 40°) = 43″(.64279) = 27.63997″.
$\text{Sine A} = \dfrac{27.63997}{32} = .86374.$
A = 59° 44′ 21″.

10. B = 180° − 59° 44′ 21″ = 120° 15′ 39″.

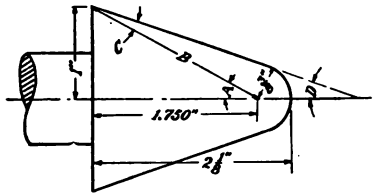


FIG. 394S

11. Cotangent $A=\frac{1.750}{1}=1.750.$ $A=29°\ 44'\ 41''.$

$B=1''(\text{cosecant } 29°\ 44'\ 41'')=2.0156''.$

Cosecant $C=\frac{2.0156}{.375}=5.3749.$ $C=10°\ 43'\ 20''.$

$D=29°\ 44'\ 41''-10°\ 43'\ 20''=19°\ 1'\ 21''.$

$?=2(19°\ 1'\ 21'')=38°\ 2'\ 42''.$

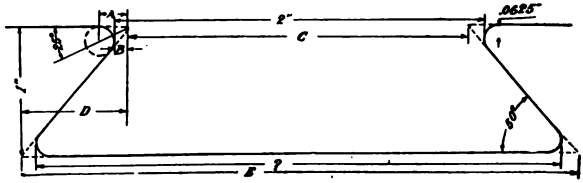


FIG. 395S

12. $A=.0625''(\text{cotangent } 25°)=.0625''(2.1445)=.13403''.$

$B=.13403''-.0625''=.07153''.$

$C=2''-2(.07153'')=1.857''.$

$D=1''(\text{cotangent } 50°)=.83910''.$

$E=1.857''+2(.83910'')=3.5352''.$

$?=3.5352''-2(.07153'')=3.3921''.$

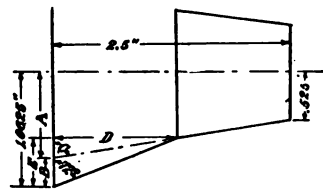


FIG. 396S

13. $A=\frac{1}{2}\left(2.25''\times\frac{2.5}{12}\right)+.525''=.75937''.$

$B = 1.0625'' - .75937'' = .30313''$.

Cotangent $C = \frac{\frac{1}{2}(2.25)}{12} = .09375$; $C = 84° 38' 39''$.

$$D = \frac{.30313''}{\text{cotangent } 72° - \text{cotangent } 84° 38' 39''} =$$

$$\frac{.30313''}{.32492 - .09375} = \frac{.30313''}{.23117} = 1.3113''.$$

$E = 1.3113''(\text{cotangent } 72°) = 1.3113''(.32492) = .42607''$.

$? = 2.125'' - 2(.42607'') = 1.27286''$.

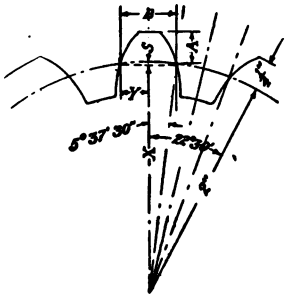


FIG. 397S

14. $X = 2''(\text{cosine } 5° 37' 30'') = 2''(.99518) = 1.99036''$.
$S = 2'' - 1.99036'' = .00964''$.
$A = .250'' + .00964'' = .2596''$.

15. $Y = 2''(\text{sine } 5° 37' 30'') = 2''(.09801) = .19602''$.
$B = 2Y = 2(.19602'') = .39204''$.

16. Cotangent $E = \frac{14.5}{13} = 1.11538$.
$E = 41° 52' 40''$.
$F = 90° - 41° 52' 40'' = 48° 7' 20''$.
$A = 13''(\text{cosecant } 41° 52' 40'') = 13''(1.49804) = 19.47452''$.

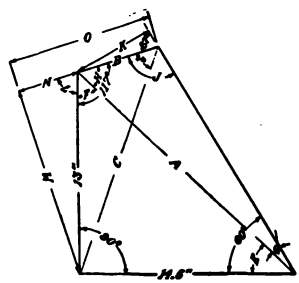


FIG. 398S

17. $G = 60° - 41° 52' 40'' = 18° 7' 20''$.
$H = 117° - 48° 7' 20'' = 68° 52' 40''$.
$J = 360° - (60° + 117° + 90°) = 93°$.
$K = 19.4745''(\text{sine } 18° 7' 20'') = 19.4745'' (.311043) = 6.05741''$.
$B = 6.05741''(\text{cosecant } 87°) = 6.05741'' (1.0014) = 6.06589''$.

18. L = 180° − 117° = 63°.
M = 13″(sine 63°) = 13″(.89101) = 11.58313″.
N = 13″(cosine 63°) = 13″(.45399) = 5.90187″.
O = 6.06589″ + 5.90187″ = 11.96776″.

$$\text{Cotangent P} = \frac{11.96776}{11.58313} = 1.03321.$$

P = 44° 3′ 11″.
C = 11.58313″(cosecant 44° 3′ 11″).
C = 11.58313″(1.4381) = 16.656″.

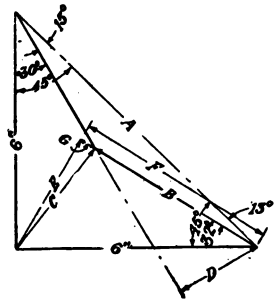


FIG. 399S

19. A = 6″(secant 45°) =
6″(1.4142) = 8.4852″.

20. D = 8.4852″(sine 15°) =
8.4852″(.25882) =
2.19614″.
B = 2.19614″(cosecant 28°) =
2.19614″(2.13) = 4.6778″.

21. E = 6″(sine 32°) =
6″(.52992) = 3.17952″.
F = 6″(cosine 32°) =
6″(.84805) = 5.08830″.

G = 5.08830″ − 4.6778″ = .4105″.
C = $\sqrt{3.17952^2 + .4105^2}$ = 3.2059″.

22. D = .125″(sine 20°) =
.125″(.34202) =
.04275″.
A = 1.500″ + .04275″ =
1.54275″.

23. E = .125″(cosine 20°) =
.125″ (.93969) =
.11746″.

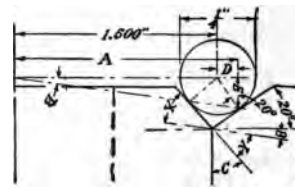


FIG. 400AS

$$\text{Cotangent B} = \frac{1.54275}{.11746} = 13.1342.$$

B = 4° 21′ 14″.

24. X = 20° + 4° 21′ 14″ = 24° 21′ 14″.
Y = 24° 21′ 14″.
C = 90° − 24° 21′ 14″ − 4° 21′ 14″ = 61° 17′ 32″.

EXERCISE 46

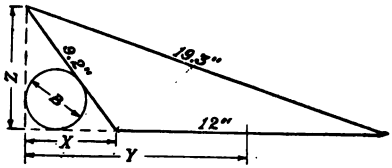


FIG. 401S

1. $Y=\dfrac{19.3^2-9.2^2}{2(12)}=11.9937''.$

   $A=6''+11.9937''=17.9937''.$

2. $X=11.9937''-6''=5.9937''.$

   $Z=\sqrt{9.2^2-5.9937^2}=6.9796''.$

   $B=6.9796''+5.9937''-9.2''=3.7733''.$

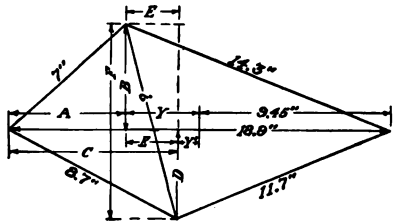


FIG. 402S

3. $Y=\dfrac{14.3^2-7^2}{2(18.9)}=4.1135''.$

   $Y'=\dfrac{11.7^2-8.7^2}{2(18.9)}=1.6190''.$

   $A=9.45''-4.1135''=5.3365''.$

   $B=\sqrt{7^2-5.3365^2}=4.5301''.$

   $C=9.45''-1.619''=7.8310''.$

   $D=\sqrt{8.7^2-7.8310^2}=3.7902''.$

   $E=Y-Y'=4.1135''-1.619''=2.4945''.$

   $F=B+D=4.5301''+3.7902''=8.3203''.$

   $?=\sqrt{8.3203^2+2.4945^2}=8.6862''.$

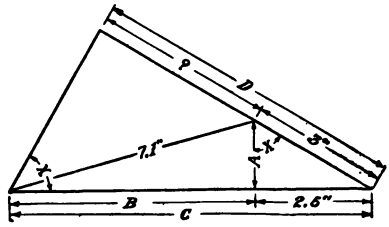


FIG. 403S

4. $A=\sqrt{3^2-2.5^2}=1.6583''$.

$B=\sqrt{7.1^2-1.6583^2}=6.9036''$.

$C=2.5''+6.9036''=9.4036''$.

$D=9.4036''\times\frac{2.5}{3}=7.8363''$.

$?=7.8363''-3''=4.8363''$.

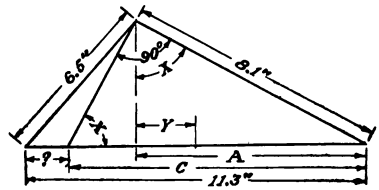


FIG. 404S

5. $Y=\frac{8.1^2-6.5^2}{2(11.3)}=1.0336''$.

$A=\frac{1}{2}(11.3'')+1.0336''=6.6836''$.

$C=8.1''\times\frac{8.1}{6.6836}=9.8165''$.

$?=11.3''-9.8165''=1.4835''$.

EXERCISE 47

1. $X = 1.5''(\text{cosine } 30°) = 1.5''(.86603) = 1.29904''$.
$R = \frac{1}{2}(1.1875'') = .59375''$.
$E = .59375''(\text{cosecant } 45°) =$
$.59375''(1.4142) = .83968''$.
$C = 30° + 45° = 75°$.
$D = .83968''(\text{sine } 75°) =$
$.83968''(.96592) =$
$.811064''$
$A = .59375'' + .811064'' -$
$1.29904'' = .10577''$.

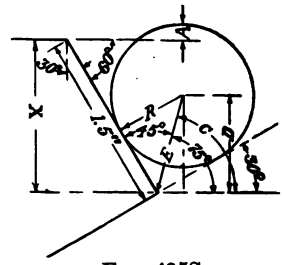


FIG. 405S

2. $B = 2(5)(\text{sine } 15°) =$
$10(.25882) = 2.5882''$.

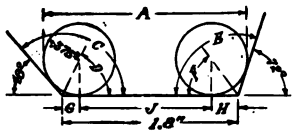


FIG. 406S

3. $C = 180° - 48° = 132°$.
$D = \frac{1}{2}(132°) = 66°$.
$E = 180° - 72° = 108°$.
$F = \frac{1}{2}(108°) = 54°$.
$G = .375''(\text{cotangent } 66°) =$
$.375''(.44523) = .16696''$.
$H = .375''(\text{cotangent } 54°) = .375''(.72654) = .27245''$.
$J = 1.8'' - (.16696'' + .27245'') = 1.3606''$.
$A = 1.3606'' + .750'' = 2.1106''$.

4. $B = 5''(\text{sine } 48°) = 5''(.74314) = 3.7157''$.

5. $C = \dfrac{180° - 12\frac{1}{2}°}{2} = 83°\ 45'$.
$D = \frac{1}{2}(83°\ 45') = 41°\ 52'\ 30''$.
$R = .750''(\text{tangent } 41°\ 52'\ 30'') =$
$.750''(.89646) = .67234''$.
$A = 2(.67234'') = 1.34468''$.

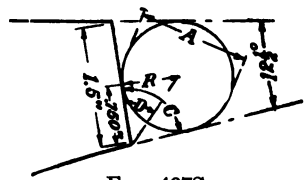


FIG. 407S

6. $B = 10''(\text{sine } 12\frac{1}{2}°) = 10''(.21644) = 2.1644''$.

7. Tangent of $\frac{1}{2}C = \frac{\frac{1}{2}(4.50)}{12} = .1875$.

$\frac{1}{2}C = 10° 37' 10''$. $C = 21° 14' 20''$.

$A = 10''(\text{sine } 21° 14' 20'') = 10''(.36226) = 3.6226''$

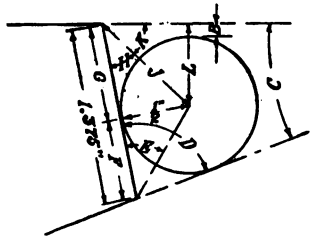


FIG. 408S

8. $D = 90° - 10° 37' 10'' = 79° 22' 50''$.

$E = 39° 41' 25''$.

$F = .5''(\text{cotangent } 39° 41' 25'') = .5''(1.2049) = .60245''$.

$G = 1.375'' - .60245'' = .77255''$.

$\text{Cotangent } H = \frac{.77255}{.500} = 1.5451$.

$H = 32° 54' 42''$.

$J = .5''(\text{cosecant } 32° 54' 42'') = .5''(1.8405) = .92025''$.

$K = 79° 22' 50'' - 32° 54' 42'' = 46° 28' 8''$.

$L = .92025''(\text{sine } 46° 28' 8'') = .92025''(.725) = .66718''$.

$B = .66718'' - .5000'' = .16718''$.

EXERCISE 48

1. $C = 1.18''(\text{secant } 26°) = 1.18''(1.1126) = 1.312868''$.

$D = 90° + 22° - 26° = 86°$.

$E = \frac{1}{2}(86°) = 43°$.

$F = 90° + 26° = 116°$

$G = \frac{1}{2}(116°) = 58°$.

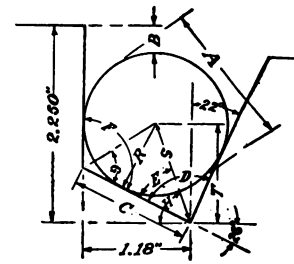


FIG. 409S

$R = \frac{1.312868''}{\text{cotangent } 43° + \text{cotangent } 58°} = \frac{1.312868''}{1.0724 + .62487} = .77351''$.

$H = E + 26° = 69°$.

$A = 2(.77351'') = 1.5470''$.

2. $S = .77351''(\text{cosecant } 43°) = .77351''(1.4663) = 1.1341''$.

$T = 1.1341''(\text{sine } 69°) = 1.1341''(.93358) = 1.05877''$.

$B = 2.250'' - (1.05877'' + .77351'') = .41773''$.

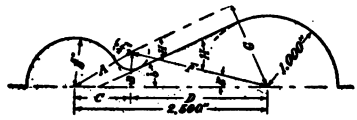


FIG. 410S

3. $A=\frac{5}{8}''+\frac{1}{4}''=.875''$. $B=\frac{5}{16}''+\frac{1}{4}''=.5625''$.

$C=\sqrt{.875^2-.5625^2}=.67023''$. $D=2.5''-.67023''=1.8298''$.

Cotangent $E=\dfrac{1.8298}{.5625}=3.2529$. $E=17°\ 5'\ 18''$.

$F=.5625''(\text{cosecant } 17°\ 5'\ 18'')=.5625''(3.4031)=1.9142''$.

$G=1.000''+.250''=1.25''$.

Cosecant $H=\dfrac{1.9142}{1.25}=1.53136$. $H=40°\ 46'\ 5''$.

$?=H-E=40°\ 46'\ 5''-17°\ 5'\ 18''=23°\ 40'\ 47''$.

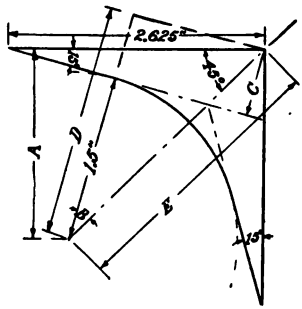


FIG. 411S

4. $C=2.625''(\text{sine } 15°)=2.625''(.25882)=.67940''$.

$D=.67940''+1.5''=2.1794''$. $B=45°-15°=30°$.

$E=2.1794''(\text{secant } 30°)=2.1794''(1.1547)=2.51655''$.

$A=2.51655''(\text{sine } 45°)=2.51655''(.70711)=1.7795''$.

5. $Y=\dfrac{9.5^2-6.000^2}{2(14.500)}=1.8707''.$

$A=\frac{1}{2}(14.5'')-1.8707''=5.3793''.$

$B=\sqrt{6^2-5.3793^2}=2.65767''.$

$\text{Diameter}=6.00''\times\dfrac{9.5}{2.65767}=21.4473''.$

$?=\frac{1}{2}(21.4473'')=10.7236''.$

NOTE:—The two right triangles are similar.

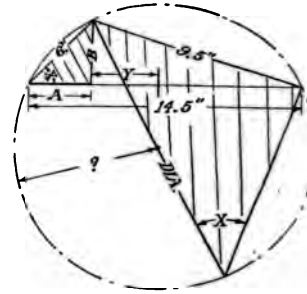


FIG. 412S

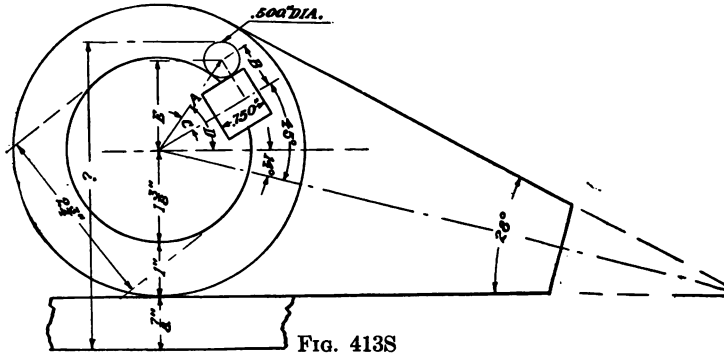


FIG. 413S

6. $A=\frac{1}{2}(2.750''+.500'')=1.625''.$ $B=\frac{1}{2}(.500''+.750'')=.625''.$

$\text{Cosecant } C=\dfrac{1.625}{.625}=2.600.$ $C=22°\ 37'\ 10''.$

$D=22°\ 37'\ 10''+45°\ 14'=53°\ 37'\ 10''.$

$E=A(\text{sine } 53°\ 37'\ 10'')=1.625''(.80510)=1.3083''.$

$?=.250''+1.3083''+1.375''+1.00''+.875''=4.8083''.$

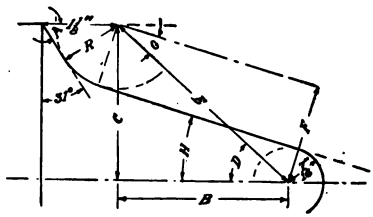


FIG. 414S

7. $A=90°-31°=59°.$

$R=1.125''(\text{sine } 59°)=1.125''(.85717)=.96432''.$

8. $B = 3.25'' - 1.125'' = 2.125''$.
$C = \frac{1}{2}(4.250'') = 2.125''$.
Cotangent $D = \dfrac{2.125}{2.125} = 1$. $D = 45°$.
$E = 2.125''(\text{cosecant } 45°) = 2.125''(1.4142) = 3.0052''$.
$F = .96432'' + .4375'' = 1.40182''$.
Sine $G = \dfrac{1.40182}{3.0052} = .46646$. $G = 27° 48' 17''$.
$H = 45° - 27° 48' 17'' = 17° 11' 43''$.
$? = 2(17° 11' 43'') = 34° 23' 26''$.

EXERCISE 49

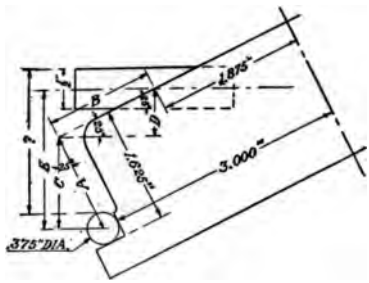


FIG. 415S

1. $A = 1.625'' - \frac{1}{2}(.375'') = 1.4375''$.
$B = 3.000'' + \frac{1}{2}(.375'') - 1.875'' = 1.3125''$.
$C = 1.4375''(\text{cosine } 25°) = 1.4375''(.90631) = 1.30282''$.
$D = 1.3125''(\text{sine } 25°) = 1.3125''(.42262) = .55468''$.
$E = 1.30282'' + .55468'' = 1.8575''$.
$? = 1.8575'' - \frac{1}{2}(.375'') + .250'' = 1.9200''$.

2. $A = 1\frac{1}{4}'' + \frac{1}{2}'' = 1\frac{3}{4}''$.
$B = \frac{1}{4}''$ (tangent 45°) = $.250''$.
$C = 1\frac{3}{4}'' - .250'' = 1.500''$.
$D = 1.500''(\text{sine } 45°) = 1.500''(.70711) = 1.06066''$.
$? = 1.06066'' + .250'' + .200'' = 1.51066''$.

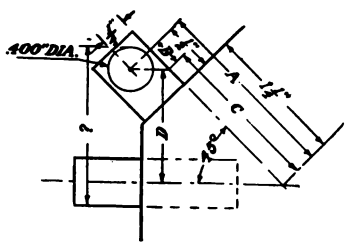


FIG. 416S

EXERCISE 50

1. Outside diameter $= \frac{N+2}{D.P.} = \frac{22+2}{8} = 3''$.

2. Addendum $= \frac{1}{D.P.} = \frac{1}{8} = .125''$.

3. Pitch diameter $= \frac{N}{D.P.} = \frac{20}{7} = 2.85714''$.

4. No. $5\frac{1}{2}$ cutter.

5. Whole depth $= \frac{2.157}{D.P.} = \frac{2.157}{7} = .30814''$.

6. $\frac{1}{2}(\frac{16}{9} + \frac{20}{9}) = 2''$.

7. Clearance $= \frac{.157}{9} = .01744''$.

8. If O.D. $= \frac{N+2}{D.P.}$; then, D.P. $= \frac{N+2}{O.D.} = \frac{21+2}{3.2857} = 7$.

9. Chordal thickness $= \frac{R}{D.P.} = \frac{1.5694}{7} = .2242''$.

10. Depth to set calipers $= \frac{U}{D.P.} = \frac{1.0294}{7} = .14705''$.

11. $Z = S\left[\text{secant}\left(14\frac{1}{2}° + \frac{90°}{N}\right)\right] = .2242''(\text{secant } 18° 47' 9'') =$
$.2242''(1.0562) = .2368''$.

12. $C = \frac{1}{2}P\left(\text{cosine}\frac{90°}{N}\right) = \frac{1}{2} \times 3''(\text{cosine } 4° 17' 9'') = 1.5''(.99721) =$
$1.4958''$; or $C = \frac{1}{2}(3.2857'' - .14705'' = 1.4958.''$
$\frac{Z}{2} = \frac{.2368''}{2} = 1184''$.
$\frac{S}{2} = \frac{.2242}{2} = .1121''$.
$T = C + \frac{Z}{2} + \frac{S}{2}\left[\text{tangent}\left(14\frac{1}{2}° + \frac{90°}{N}\right)\right] =$
$1.4958'' + .1184'' + .1121''(\text{tangent } 18° 47' 9'') =$
$1.4958'' + .1184'' + .1121''(.33949) = 1.6522.''$

13. $A=2\left(\frac{3.1416}{5}\right)-\frac{3.1416}{4}=.4712''$.

14. $B=3\left(\frac{3.1416}{5}\right)-\frac{3.1416}{2}=.3142''$.

15. $C=\frac{1}{2}\times\frac{9}{8}=1.1875''$.
$D=\frac{1}{2}\times\frac{7}{8}=1.0625''$.
$E=\frac{1}{2}\times\frac{9}{8}=.750''$.
$F=C+D=1.1875''+1.0625''=2.25''$.
$G=E+C=.750''+1.1875''=1.9375''$.
$H=2.25''(\text{sine } 65°)=2.0392''$.
$J=2.25''(\text{cosine } 65°)=.95089''$.
$K=1.9375''-.95089''=.98661''$.
$A=\sqrt{2.0392^2-.98661^2}=2.2653''$.

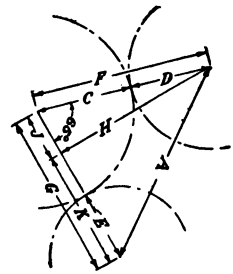


FIG. 428S

16. See Article 225. $A=.1840''$, $B=.1012''$.

EXERCISE 51

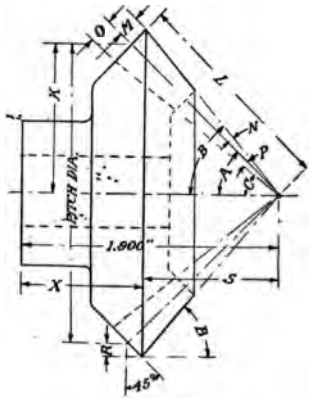


FIG. 432S

1. $A=45°$.

2. Pitch diameter $=\frac{15}{6}=2.5''$.
$K=\frac{1}{2}(2.5'')=1.25''$.
$L=K(\text{cosecant } 45°)=1.25''(1.4142)=1.7677''$.

$M = \frac{1}{D.P.} = \frac{1}{6}''$.

Cotangent $N = \frac{1.7677}{\frac{1}{6}} = 10.6062$. $N = 5° 23' 11''$.

$B = 45° + 5° 23' 11'' = 50° 23' 11''$.

3. $O = \frac{1.157}{D.P.} = \frac{1.157}{6} = .19283''$.

Cotangent $P = \frac{1.7677}{.19283} = 9.1671$. $P = 6° 13' 32''$.

$C = 45° - 6° 13' 32'' = 38° 46' 28''$.

4. $R = \frac{1}{6}''(\text{cosine } 45°) = \frac{1}{6}'' \times .70711 = .1178''$.
Outside diameter = pitch diameter + $2(.1178'') = 2.5'' + .2356'' = 2.7356''$.

5. $S = \frac{1}{2}$(outside diameter)(cotangent $50° 23' 11''$) = $1.3678''(.82767) = 1.1321''$.
$X = 1.900'' - 1.1321'' = .7679''$.

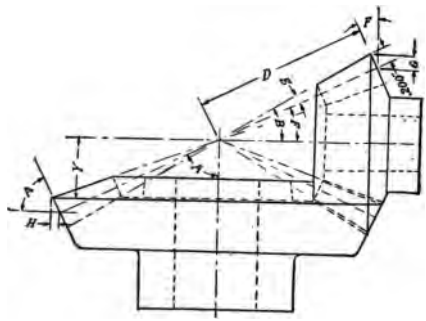


FIG. 433S

6. Pitch diameter of gear $= \frac{28}{5} = 5.6''$.
Pitch diameter of pinion $= \frac{13}{5} = 2.6''$.
Tangent of pitch angle $A = \frac{28}{13} = 2.1538$.
Angle $A = 65° 5' 41''$.

7. $F = 90° - 65° 5' 41'' = 24° 54' 19''$.
$D = \frac{1}{2}$(pitch diameter of pinion)(cosecant $24° 54' 19''$) = $1.3''(2.3746) = 3.087''$.

Cotangent $E=\frac{3.087}{.200}=15.435$. $E=3°\ 42'\ 24''$.

B = face angle of pinion = 24° 54′ 19″ + 3° 42′ 24″ = 28° 36′ 43″.

8. G = .200″(cosine 24° 54′ 19″) = .200″(.90700) = .1814″.
O.D. of pinion = pitch diameter + 2(.1814″) = 2.6″ + .3628″ = 2.9628″.

9. H = .200″(cosine 65° 5′ 41″) = .200″(.42118) = .08429″.
O.D. of gear = pitch diameter + 2(.08429″) = 5.6″ + .1686″ = 5.7686″.

10. $Y=\frac{1}{2}$(O.D. of gear)(cotangent 68° 48′ 5″) = 2.8843″(.38785) = 1.1187″.

EXERCISE 52

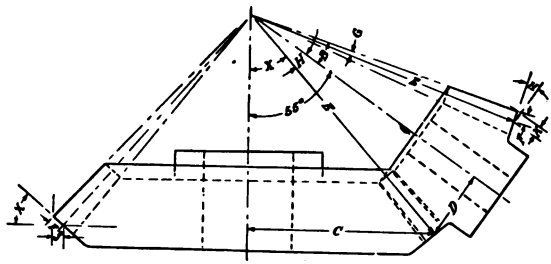


FIG. 435S

1. $C=\frac{1}{2}$(pitch diameter of gear) $=\frac{1}{2}(40)(.5'')\left(\frac{1}{\pi}\right)=3.1831''$.
$D=\frac{1}{2}$(pitch diameter of pinion) $=\frac{1}{2}(15)(.5'')\left(\frac{1}{\pi}\right)=1.1936''$.

$$\text{Cotangent } X=\frac{[3.1831''+1.1936''(\text{secant } 55°)](\text{cotangent } 55°)}{3.1831}$$

$$=\frac{3.6859}{3.1831}=1.1580. \qquad X=40°\ 48'\ 43''.$$

2. E = C(cosecant 40° 48′ 43″) = 3.1831″(1.5300) = 4.8701″.
F = .3183(C.P.) = .3183(.5″) = .15915″.

$$\text{Cotangent } G=\frac{4.8701}{.15915}=30.601. \qquad G=1°\ 52'\ 22''.$$

H = 55° − 40° 48′ 43″ = 14° 11′ 17″.
B = 14° 11′ 17″ + 1° 52′ 22″ = 16° 3′ 39″.

3. J = .15915″(cosine 40° 48′ 43″) = .15915″(.75686) = .12045″.
O.D. of gear = 2(3.1831″ + .12045″) = 6.6071″.

4. K = .15915″(cosine 14° 11′ 17″) = .15915″(.96950) = .1543″.
O.D. of pinion = 2(1.1936″ + .1543″) = 2.6958″.

EXERCISE 53

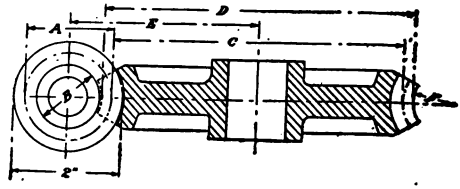


FIG. 441S

1. A = 2 − (.6366)(.5″) = 1.6817″.

2. B = 2 − 2(.6866)(.5″) = 1.3134″.

3. C = (.5″)(45)(.31831) = 7.1619″.

4. D = 7.1619″ + (.6366)(.5″) = 7.4802″.

5. E = ½(7.1619″ + 1.6817″) = 4.4218″.

6. $F = \dfrac{2'' - 1.2732(.5'')}{2} = .6817''$.

7. Let X = pitch.
Then A = 3″ − .6366X.
C = X(40)(.31831) =
12.7324X.
Center distance = ½(A + C) =
½(12.7324X + 3″ − .6366X) =
½(3″ + 12.0958X).
Center distance also = 5.5″;
therefore,
½(3″ + 12.0958X) = 5.5″
3″ + 12.0958X = 11″.
12.0958X = 8″.
$X = \dfrac{8''}{12.0958} = .66139''$.

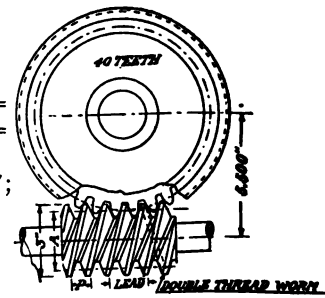


FIG. 442S

8. Lead = 2(.66139″) = 1.32278″.

9. $A = 3'' - .66139''(.6366) = 3'' - .4210'' = 2.5790''$.

EXERCISE 54

1. 40 revolutions.

2. $\frac{3}{7} \times 40 = \frac{120}{7} = 17\frac{1}{7}$ revolutions.

3. $\frac{1}{9} \times 40 = \frac{40}{9} = 4\frac{4}{9}$ revolutions.

4. $\frac{1}{17} \times 40 = 2\frac{6}{17}$ revolutions.

5. A must make $3\frac{1}{13}$ revolutions to cause C to make $\frac{1}{13}$ of a revolution. To take care of the fractional part of a revolution of A, the plate must be divisible by 13; therefore, a plate with 39 holes is used.

6. $\frac{1}{13} = \frac{3}{39}$; therefore, A would be moved 3 holes in the 39 circle for $\frac{1}{13}$ of a revolution of the crank.

7. $\frac{1}{206} \times 40 = \frac{20}{103}$ revolution.

8. If 1 revolution of A causes C to revolve 9°, to cause C to revolve 22°, the crank A must make $22 \div 9$ or $2\frac{4}{9}$ revolutions.

9. $82\frac{1}{2}$ revolutions.

10. If 1 revolution of A causes C to revolve 9°, to cause C to revolve 21° 27′ 21″, the crank A must make $\frac{21^\circ\ 27'\ 21''}{9^\circ}$ or $\frac{77241''}{32400''}$ or $2\frac{4147}{10800}$ revolutions.

11. While C makes $\frac{1}{107}$ of a revolution, A makes $\frac{40}{107}$ of a revolution. But $\frac{40}{107}$ equals approximately $\frac{10}{27}$. If A is revolved $\frac{10}{27}$ of a revolution, it would be off the difference between $\frac{40}{107}$ and $\frac{10}{27}$ or $\frac{10}{27 \times 107}$ of a revolution. Thus, $\frac{40 \times 27}{107 \times 27} - \frac{10 \times 107}{27 \times 107} = \frac{10}{27 \times 107}$. While D makes $\frac{1}{107}$ of a revolution, the plate B and gear E must make $\frac{10}{27 \times 107}$ revolutions. By inverting these revolutions the number of teeth in the gears D and E are obtained. Thus, $D = \frac{10}{27 \times 107}$ of a tooth and $E = \frac{1}{107}$ of a tooth. By multiplying both of these by 107×27 to clear of fractions, D = 10 teeth and E = 27 teeth. A better set of gears would be D = 30 teeth and E = 81 teeth. Move the crank 10 holes in the 27 circle.

12. 123 divisions solved by the formula, $\frac{D}{E}=40-(X)(N)$.

$X=\frac{40}{N}=\frac{40}{123}=$ approximately $\frac{10}{31}$.

$\frac{D}{E}=40-\frac{10}{31}\times 123=40-\frac{1230}{31}=40-39\frac{21}{31}=\frac{10}{31}=\frac{30}{93}$.

Move the crank 10 holes in the 31 circle; D = 30 teeth; E = 93 teeth.

13. Move the crank 3 holes in the 39 circle; D = 28 teeth; E = 26 teeth.

14. Move the crank 1 hole in the 29 circle; D = 27 teeth; E = 87 teeth.

EXERCISE 55

1. $\frac{77}{244}$ *Ans.*

		3	5	1	12
1	0	1	5	6	77
0	1	3	16	19	244
		3	5	1	12

2. $\frac{1}{0}, \frac{0}{1}, \frac{1}{10}, \frac{4}{41}, \frac{21}{215}, \frac{46}{471}$.

EXERCISE 56

1. $\sqrt{2}=1.4142=\frac{14142}{10000}$.

10000	14142	1
8284	10000	2
1716	4142	2
1420	3432	2
296	710	2
236	592	2
	118	

		1	2	2	2	2	2
1	0	1	2	5	12	29	70
0	1	1	3	7	17	41	99
		1	2	2	2	2	2

Ans. $\frac{99}{70}=1.41428$.

.0008 = Variation.

2.

9063	7420	1
7420	6572	4
1643	848	1
848	795	1
795	53	15
795		
0		

		1	4	1	1	15	8
1	0	1	4	5	9	77	
0	1	1	5	6	11	94	
		1	4	1	1	15	8

$\frac{9063}{7420}=1.2214$.

Ans. $\frac{94}{77}=1.2208$

.0006 = Variation.

3. $.8143 = \frac{8143}{10000}$

8143	10000	1
7428	8143	4
715	1857	2
427	1430	1
288	427	1
278	288	2
	139	

		1	4	2	1	1	2
1	0	1	4	9	13	22	57
0	1	1	5	11	16	27	70
		1	4	2	1	1	2

$\frac{57}{70} = .81428$ *Ans.*

.00002 = Variation.

EXERCISE 57

1. While A makes 9 revolutions, B makes 10.675 revolutions. Therefore, A = 10.675 teeth and B = 9 teeth. By multiplying these by 1000 to clear of fractions, A = 10675 teeth and B = 9000 teeth. After being reduced by the principle of continued fractions, A = 51 teeth and B = 43 teeth. Thus,

9000	10675	1
8375	9000	5
625	1675	2
425	1250	1
200	425	2
200	400	8
	25	

		1	5	2	1	2	
1	0	1	5	11	16	43	
0	1	1	6	13	19	51	
		1	5	2	1	2	

When A = 51 teeth and B = 43 teeth, B will make $9 \times \frac{5}{4}\frac{1}{3}$ or 10.6744 revolutions while A makes 9 revolutions. .0006 = Variation.

2. While A makes 7.45 revolutions, B will make 7.99 revolutions; therefore, A = 7.99 teeth and B = 7.45 teeth. By multiplying these by 100 to clear of fractions, A = 799 teeth and B = 745 teeth. After being reduced by the principle of continued fractions, A = 74 teeth and B = 69 teeth. Thus,

799	745	1
745	702	13
54	43	1
43	33	3
11	10	1
10	10	10
1		

		1	13	1	3	1	
1	0	1	13	14	55	69	
0	1	1	14	15	59	74	
		1	13	1	3	1	

When A = 74 teeth and B = 69 teeth, B will make $7.45 \times \frac{74}{69}$ or 7.9898 revolutions while A makes 7.45 revolutions. .0002 = Variation.

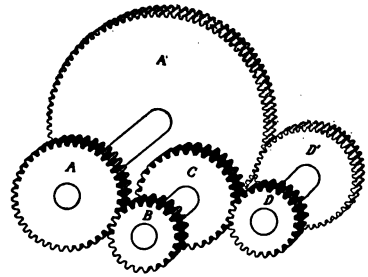
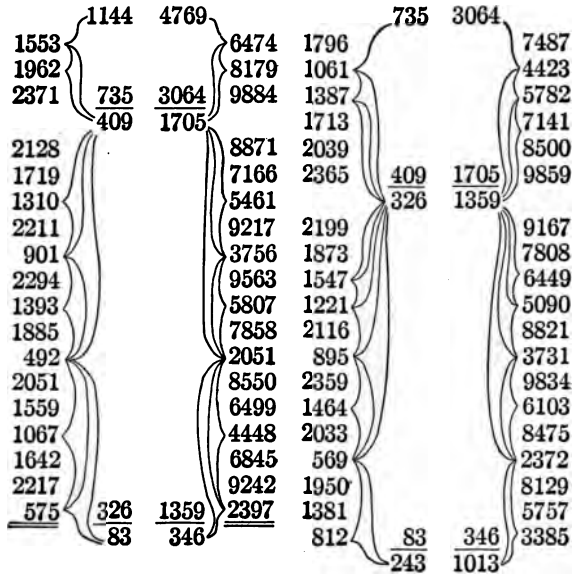


FIG. 449S

3. While the imaginary simple gear A′ makes 75 revolutions, the imaginary simple gear D′ makes 312.653 revolutions; therefore, A′ = 312.653 teeth and D′ = 75 teeth. By multiplying these by 1000 to clear of fractions, A′ = 312653 and D′ = 75000 teeth. After being reduced by the principle of continued fractions, A′ = 2397 teeth and B′ = 575 teeth. Thus,

312653	75000	4
300000	63265	5
12653	11735	1
11735	11016	12
918	719	1
719	597	3
199	122	1
122	77	1
77	45	1
45	32	1
32	13	2
26	12	2
6	1	6

												2 0
		4	5	1	12	1	3	1	1	1	1	1144
1	0	1	5	6	77	83	326	409	735	1144	1879	4769
0	1	4	21	25	321	346	1359	1705	3064	4769	7833	735
		4	5	1	12	1	3	1	1	1	1	3064
												2 1



$\frac{575}{2397}=\frac{23\times25}{47\times51}$. When A = 51 teeth, B = 23 teeth, C = 47 teeth, and D = 25 teeth, A will make $312.653\times\frac{23\times25}{51\times47}$ or 75.0003 revolutions, while D makes 312.653 revolutions. $\frac{575}{2397}$ is the first set that can be factored; therefore it is the closest set, .0003 = Variation. Many other sets may be obtained by simply uniting some of the smaller convergents, but the variation will be greater.

Thus, $\frac{77+83}{321+346}=\frac{160}{667}=\frac{10\times16}{23\times29}$.

$312.653\times\frac{10\times16}{23\times29}=74.9985$. .0015 = Variation.

EXERCISE 58

1. .1763″.
2. $.4311\div\frac{1}{5}$ or 2.1555 revolutions.
3. $.1027\div\frac{1}{10}$ or 1.027 revolutions.

4. While the spindle makes 1 revolution, the lead screw makes .5431 revolutions. By inverting these revolutions to obtain the number of teeth in the gears to be used, the gear on the spindle= .5431 of a tooth, and the gear on the lead screw=1 tooth. By multiplying these by 10000 to clear of fractions, the gear on the spindle=.5431×10000 or 5431 teeth, and the gear on the lead screw=1×10000 or 10000 teeth. These numbers are then reduced to suitable numbers for gears by the principle of continued fractions.

Thus,

10000	5431	1
5431	4569	1
4569	862	5
4310	777	3
259	85	

		1	1	5	3	3	2
1	0	1	1	6	19		44
0	1	1	2	11	35		81
		1	1	5	3	3	2

If the gear on the spindle had 44 teeth, and the gear on the lead screw had 81 teeth, the lead screw would make $\frac{44}{81}$ or .5432 revolution while the spindle made 1 revolution. The variation=.0001″.

5. $\frac{45}{32}\times\frac{1}{5}''=.28125''$.

6. To move the slide or tool .3153″, the lead screw must make .3153÷$\frac{1}{6}$ or 1.8918 revolutions; therefore, while the spindle makes 1 revolution, the lead screw must make 1.8918 revolutions. By inverting these revolutions to obtain the number of teeth in the gears to be used, the gear on the spindle=1.8918 teeth, and the gear on the lead screw=1 tooth. By multiplying these by 10000 to clear of fractions, the gear on the spindle=1.8918×10000 or 18918 teeth and the gear on the lead screw 1×10000 or 10000 teeth. These numbers are then reduced to suitable numbers for gears by the principle of continued fractions. Thus,

10000	18918	1
8918	10000	1
1082	8918	8
1048	8656	4
34	262	

		1	1	8	4
1	0	1	1	9	37
0	1	1	2	17	70
		1	1	8	4

If the gear on the spindle had 70 teeth, and the gear on the lead screw had 37 teeth, while the spindle made 1 revolution, the slide would move $\frac{70}{37}\times\frac{1}{6}''$ or .31531″. Variation=.00001″.

EXERCISE 59

1. While the spindle makes one revolution, the lead screw will make $\frac{50}{70}\times\frac{37}{90}$ or $\frac{37}{126}$ of a revolution. If one revolution of the lead screw moves the carriage $\frac{1}{6}$ of an inch, $\frac{37}{126}$ of a revolution will move it $\frac{37}{126}\times\frac{1}{6}''$ or .04894″. Therefore, the lathe will cut a .04894″ lead.

2. While the spindle makes 1 revolution, the imaginary simple gear D′ makes 1 revolution. While the lead screw makes 4.011 revolutions, the imaginary simple gear G′ makes 4.011 revolutions.

If we know the number of revolutions the imaginary simple gears D′ and G′ must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 76.) Thus, if D′ makes 1 revolution while G′ makes 4.011 revolutions, D′ must have 4.011 teeth and G′ 1 tooth. By multiplying these by 1000 to clear of fractions, D′=4011 teeth and G′=1000 teeth. The factors of these numbers are too large for compound gears; therefore, the numbers are reduced by the principle of continued fractions to other numbers from which suitable numbers for gears may be obtained. (See Articles 240 and 246.) Thus,

4011	1000	4
4000	990	90
11	10	1
10	10	10
1	0	∞

						2 ∞
		4	90	1	10	2091
1	0	1	90	91	1000	8387
0	1	4	361	365	4011	1909
		4	90	1	10	7657
						2 ∞

	2091	8387				1909	7657	
	1000	4011				1000	4011	
	1091	4376				909	3646	
2273			9117	1727		91	365	6927
1182			4741			818	3281	
2455			9847	1645				9478
1273	1000	4011	5106	2472		91	365	6197
	91	365				827	2916	

Since $\frac{4011}{1000}$ = approximately $\frac{5106}{1273}$, D′=5106 teeth, and G′=1273 teeth. After factoring (5106=69×74; 1273=19×67) and placing in proper compound order, D=69 teeth, E=19 teeth, F=74 teeth, and G=67 teeth.

CHECK: While the spindle makes one revolution, the lead screw will make $\frac{69\times74}{19\times67}$ or 4.01092 revolutions, which varies only .00008 from the true value.

3. While the spindle or the imaginary simple gear D′ makes 1 revolution, the carriage must move 3.9248″. To move the carriage 3.9248″, the lead screw or the imaginary simple gear G′ must make $3.9248\div\frac{1}{4}$ or 15.6992 revolutions; therefore, while the imaginary simple gear D′ makes 1 revolution, the imaginary simple gear G′ must make 15.6992 revolutions.

If we know the number of revolutions the imaginary simple gears D′ and G′ must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 76.) Thus, if D′ makes 1 revolution while G′ makes 15.6992 revolutions, D′ must have 15.6992 teeth and G′ 1 tooth. By multiplying these by 10000 to clear of fractions, D′ = 156992 teeth and G′ = 10000 teeth. The factors of these numbers are too large for compound gears; therefore, they are reduced by the principle of continued fractions to other numbers from which suitable factors can be obtained. (See Articles 240 and 246.) Thus,

10000	156992	15
6992	150000	1
3008	6992	2
2928	6016	3
80	976	12
80	960	5
0	16	∞

								∞ 0
		15	1	2	3	12	5	123
1	0	1	1	3	10	123	625	1931
0	1	15	16	47	157	1931	9812	502
		15	1	2	3	12	5	7881
								∞ 1

	502	7881	
	123	1931	
	379	5950	
635	123	1931	9969
	256	4019	
389			6107
522	123	1931	8195
	133	2088	

		123	1931
605			9498
482			7567
359			5636
595			9341
236			3705
585			9184
349			5479
462			7253
575	10	157	9027
	113	1774	

Since $\frac{156992}{10000}$ = approximately $\frac{2088}{133}$, D′ = 2088 teeth and G′ = 133 teeth. After factoring $\left(\frac{2088}{133}=\frac{87\times24}{7\times19}=\frac{87\times72}{21\times19}\right)$ and placing in

proper compound order, D=87 teeth, E=21 teeth, F=72 teeth, and G=19 teeth.

CHECK: While the spindle makes 1 revolution, the carriage will move $\frac{87\times72}{21\times19}\times\frac{1''}{4}$ or 3.92481″, which varies .00001″ from the true value.

4. While the spindle or gear D makes 1 revolution, the carriage must move 3.1429″. To move the carriage 3.1429″, the lead screw must make $3.1429\div\frac{1}{4}$ or 12.5716 revolutions. While the lead screw makes 12.5716 revolutions, F and E will make $12.5716\times\frac{24}{88}$ or 3.1429 revolutions; therefore, while the spindle or gear D makes 1 revolution, E must make 3.1429 revolutions.

If we know the number of revolutions the gears D and E must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 71.) Thus, if D makes 1 revolution while E makes 3.1429 revolutions, D must have 3.1429 teeth and E 1 tooth. By multiplying these by 10000 to clear of fractions, D=31429 teeth and E=10000 teeth. These numbers are too large; therefore, they are reduced by the principle of continued fractions. Thus,

10000	31429	3
8574	30000	6
1426	1429	1
	1426	
	3	

		3	6	1
1	0	1	6	7
0	1	3	19	22
		3	6	1

Since $\frac{3\,1\,4\,2\,9}{1\,0\,0\,0\,0}$ = approximately $\frac{2\,2}{7}=\frac{6\,6}{2\,1}$, D=66 teeth and E=21 teeth.

CHECK: While the spindle makes 1 revolution, the carriage will move $\frac{6\,6}{2\,1}\times\frac{9\,6}{2\,4}\times\frac{1}{4}''$ or 3.14285″, which varies only .00005″ from the true value.

EXERCISE 60

1. While the spindle makes 1 revolution, gear B will make $\frac{4\,8}{3\,2}$ revolutions. Then, 1 revolution of the spindle, or $\frac{4\,8}{3\,2}$ revolutions of gear B, will move the carriage $\frac{1}{7}$ of an inch. If $\frac{4\,8}{3\,2}$ revolutions of gear B moves the carriage $\frac{1}{7}$ of an inch, to move it .1537″, B must make $\frac{48}{32}\times\frac{.1537}{\frac{1}{7}}$ or 1.61385 revolutions. Therefore, while the

spindle, or gear A, makes one revolution, B must make 1.61385 revolutions.

If we know the number of revolutions the gears A and B must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 71.) Thus, if A makes 1 revolution while B makes 1.61385 revolutions, A must have 1.61385 teeth and B 1 tooth. By multiplying these by 100000 to clear of fractions, A = 161385 teeth, and B = 100000 teeth. These numbers are too large; therefore, they are reduced by the principle of continued fractions.

Thus,

100000	161385	1
61385	100000	1
38615	61385	1
22770	38615	1
15845	22770	1
13850	15845	2
1995	6925	3
1880	5985	2
	940	

		1	1	1	1	1	2	3	2 1
1	0	1	1	2	3	5	13	44	57
0	1	1	2	3	5	8	21	71	92
		1	1	1	1	1	2	3	2 1

Since $\frac{161385}{100000}$ = approximately $\frac{92}{57}$, A = 92 teeth, and B = 57 teeth.

CHECK: While the spindle makes 1 revolution, B will make $\frac{92}{57}$ revolutions. If $\frac{48}{32}$ revolutions of B moves the carriage $\frac{1}{7}$ of an inch, $\frac{92}{57}$ revolutions of B will move it $\frac{1}{7}'' \times \frac{\frac{92}{57}}{\frac{48}{32}}$ or .153717″, which varies .000017″ from the true value.

2. While the spindle makes one revolution, gear B will make $\frac{48}{32}$ revolutions. Then, 1 revolution of the spindle, or $\frac{48}{32}$ revolutions of B, will move the carriage $\frac{1}{7}$ of an inch. When A = 47 teeth and B = 33 tecth, B will make $\frac{47}{33}$ revolutions while the spindle makes 1 revolution. If $\frac{48}{32}$ revolutions of B moves the carriage $\frac{1}{7}$ of an inch, $\frac{47}{33}$ revolutions will move it $\frac{1}{7}'' \times \frac{\frac{47}{33}}{\frac{48}{32}}$ or .13564″.

EXERCISE 61

1. $40 \times \frac{48}{28} \times \frac{40}{50} = 48$ revolutions.

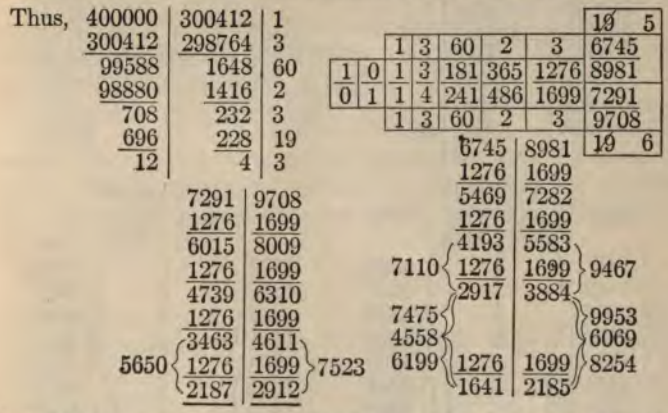
2. $\frac{11.05}{1} \times \frac{1}{4}'' = 2.7625''$.

3. $40 \times \frac{64}{57} \times \frac{62}{36} \times \frac{1}{4}'' = 19.3372''$.

EXERCISE 62

1. While the spiral makes 1 revolution, the table must move 7.5103 inches. To move the table 7.5103 inches the feed screw, or the imaginary simple gear G′, must make $7.5103 \div \frac{1}{4}$ or 30.0412 revolutions; therefore, to cut a 7.5103-inch lead the imaginary simple gear G′ must make 30.0412 revolutions while the spiral makes one revolution. While the imaginary simple gear D′ makes 40 revolutions, the spiral also makes 1 revolution; therefore, while D′ makes 40 revolutions, G′ must make 30.0412 revolutions.

If we know the number of revolutions the imaginary simple gears D′ and G′ must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 76.) Thus, if D′ makes 40 revolutions while G′ makes 30.0412 revolutions, D′ must have 30.0412 teeth and G′ 40 teeth. By multiplying these by 10000 to clear of fractions, D′ = 300412 teeth, and G′ = 400000 teeth. The factors of these numbers are too large for compound gears; therefore, they are reduced by the principle of continued fractions to other numbers from which suitable factors may be obtained.



Since $\frac{300412}{400000}$ = approximately $\frac{2187}{2912}$, D′ = 2187 teeth, and G′ = 2912 teeth. After factoring ($2187 = 81 \times 27$; $2912 = 91 \times 32$) and placing in proper compound order, D = 81 teeth, E = 91 teeth, F = 27 teeth and G = 32 teeth.

CHECK: While the spiral makes 1 revolution, the table will move $40 \times \frac{8}{91} \times \frac{27}{32} \times \frac{1}{4}''$ or 7.51302″, which varies only .00002″.

2. While the spiral makes 1 revolution, the table will move 50.0522 inches. To move the table 50.0522 inches the feed screw, or the imaginary simple gear G′, must make $50.0522 \div \frac{1}{4}$ or 200.2088 revolutions; therefore, to cut a 50.0522-inch lead the imaginary simple gear G′ must make 200.2088 revolutions while the spiral makes 1 revolution. While the imaginary simple gear D′ makes 40 revolutions, the spiral also will make 1 revolution; therefore, while D′ makes 40 revolutions, G′ must make 200.2088 revolutions.

If we know the number of revolutions the imaginary simple gears D′ and G′ must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 76.) Thus, if D′ makes 40 revolutions while G′ makes 200.2088 revolutions, D′ must have 200.2088 teeth and G′ 40 teeth. By multiplying these by 10000 to clear of fractions, D′=2002088 teeth, and G′=400000 teeth. The factors of these numbers are too large for compound gears; therefore, they are reduced by the principle of continued fractions to other numbers from which suitable factors may be obtained.

Thus,

400000	2002088	5
398808	2000000	191
1192	2088	1
896	1192	1
296	896	3
296	888	37
	8	

							37 1
		5	191	1	1	3	1724
1	0	1	191	192	383	1341	8629
0	1	5	956	961	1917	6712	958
		5	191	1	1	3	4795
							37 1

	958	4795	
1533	383	1917	7673
	575	2878	
1917			9595
1342			6717
767			3839
1726			8639
959			4800
1151			5761
1343			6722
1535			7683
1727			8644
1919			9605
	383	1917	
	192	961	

	1724	8629	
	1341	6712	
	383	1917	
1723			8624
1340			6707
957			4790
1531			7663
574			2873
1913			9575
1339			6702
765			3829
1721			8614
956			4785
1147			5741
1338			6697
1529	192	961	7653
	191	956	

Since $\frac{2002088}{400000}$ = approximately $\frac{961}{192}$, D′ = 961 teeth, and G′ = 192 teeth. After factoring and raising the factors $\left(\frac{961}{192}=\frac{31\times31}{12\times16}=\frac{93\times62}{36\times32}\right)$ and placing in proper compound order, D = 93 teeth, E = 36 teeth, F = 62 teeth, and G = 32 teeth.

CHECK: While the spiral makes 1 revolution, the table will move $40\times\frac{93}{36}\times\frac{62}{32}\times\frac{1}{4}''$ or 50.05203″, which varies only .00017″ from the true lead.

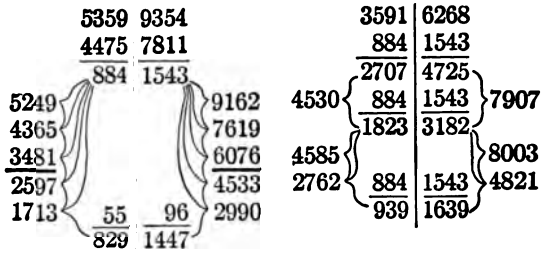
3. While the spiral makes 1 revolution, the table will move 5.7291 inches. To move the table 5.7291 inches the feed screw, or the imaginary simple gear G′, must make $5.7291\div\frac{1}{4}$ or 22.9164 revolutions; therefore, to cut a 5.7291-inch lead the imaginary simple gear G′ must make 22.9164 revolutions while the spiral makes 1 revolution. While the imaginary simple gear D′ makes 40 revolutions, the spiral also makes 1 revolution; therefore, while D′ makes 40 revolutions, G′ must make 22.9164 revolutions.

If we know the number of revolutions the imaginary simple gears D′ and G′ must make, the number of teeth in them are obtained by inverting the revolutions. (See Article 76.) Thus, if D′ makes 40 revolutions while G′ makes 22.9164 revolutions, D′ must have 22.9164 teeth and G′ 40 teeth. By multiplying these by 10000 to clear of fractions, D′ = 229164 teeth, and G′ = 400000 teeth. The factors of these numbers are too large for compound gears; therefore, they are reduced by the principle of continued fractions to other numbers from which suitable factors may be obtained.

Thus,

400000	229164	1
229164	170836	1
170836	58328	2
116656	54180	1
54180	4148	13
53924	4096	16
256	52	4
208	48	1
48	4	12
48		

		1	1	2	1	13	16	4	1	12 0
		1	1	2	1	13	16	4	1	3591
1	0	1	1	3	4	55	884	3591	4475	6268
0	1	1	2	5	7	96	1543	6268	7811	5359
		1	1	2	1	13	16	4	1	9354
										12 2



Since $\frac{229184}{400000}$ = approximately $\frac{3481}{6076}$, D′ = 3481 teeth, and G′ = 6076 teeth. After factoring (3481 = 59×59; 6076 = 62×98) and placing in proper compound order, D = 59 teeth, E = 62 teeth, F = 59 teeth, and G = 98 teeth.

CHECK: While the spiral makes 1 revolution, the table will move $40\times\frac{59}{62}\times\frac{59}{98}\times\frac{1}{4}$″ or 5.729099″, which varies only .000001″ from the true lead.

EXERCISE 63

1. $3+\frac{2(1.157)}{10}=3.2314$ inches.

2. Cosecant θ = (D.P.)(pitch diameter) = 3×10 = 30; therefore, $\theta = 1°\ 54'\ 37''$.

Lead of the hob $=\frac{\pi}{\text{D.P.}}(\text{secant }\theta)=\frac{3.1416}{10}(\text{secant } 1°\ 54'\ 37'')=$

$\frac{3.1416}{10}\times 1.0005 = .31432''$.

3. Lead of the gash = π(pitch diameter)(cotangent θ) = 3.1416×3″×29.9831 = 282.59″.

EXERCISE 64

1. While the spindle makes 1 revolution, B will make $\frac{96}{48}$ or 2 revolutions, and the carriage will move $\frac{1}{3}$ of an inch. If 2 revolutions of B moves the carriage $\frac{1}{3}$ of an inch, to move it .31432″ it must make $2\times\frac{.31432}{\frac{1}{3}}$ or 1.8859 revolutions. While the spindle, or A, makes 1 revolution, B must make 1.8859 revolutions to cause the carriage to move .31432″.

If we know the number of revolutions the gears A and B must make, the number of teeth in them is obtained by inverting the

revolutions. (See Article 71.) Thus, if A makes 1 revolution while B makes 1.8859 revolutions, A must have 1.8859 teeth and B 1 tooth. By multiplying these by 10000 to clear of fractions, A= 18859 teeth and B=10000 teeth. These numbers are too large for gears; therefore, they are reduced to other numbers by the principle of continued fractions.

Thus,

18859	10000	1
10000	8859	1
8859	1141	7
7987	872	1
872	269	3
807		

		1	1	7	1	3
1	0	1	1	8	9	35
0	1	1	2	15	17	66
		1	1	7	1	3

Since $\frac{10000}{18859}$=approximately $\frac{35}{66}$, A=66 teeth, and B=35 teeth.

CHECK: While the spindle makes 1 revolution, B will make $\frac{66}{35}$ revolutions. If 2 revolutions of B moves the carriage $\frac{1}{3}$ of an inch, $\frac{66}{35}$ revolutions will move it $\frac{1}{3}'' \times \frac{\frac{66}{35}}{2}$ or .31428″, which varies only .00004″ from the true value.

2. According to Articles 265 and 266, when the lead of the hob is .31428″, and the lead of the gash is 282.59″, the adjacent teeth along any flute will be out of line $\frac{1}{900 \cdot 16}$ of a complete revolution.

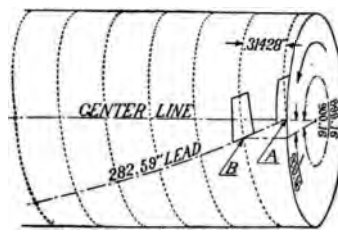


FIG. 462S(a)

Then, when tooth A is on the center line, the hob must make $1-\frac{1}{900 \cdot 16}$ or $\frac{899 \cdot 16}{900 \cdot 16}$ of a revolution to cause tooth B to come on the center line; therefore, when the hob makes $\frac{899 \cdot 16}{900 \cdot 16}$ of a revolution, the tool must reciprocate 6 times because there are 6 flutes in the hob. If 1 revolution of F causes the tool to reciprocate 4 times,

to cause the tool to reciprocate 6 times, F would have to make 6÷4 or 1.5 revolutions. (See Fig. 462S (b).)

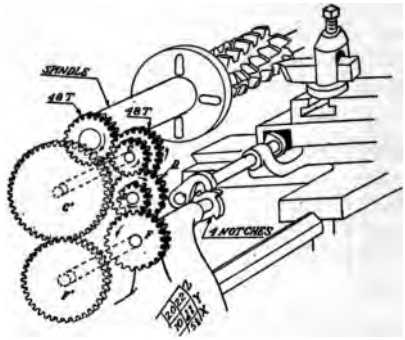


FIG. 462S (b)

If we know the number of revolutions the imaginary simple gears C′ and F′ must make, the number of teeth in them will be obtained by inverting the revolutions. (See Article 76.) Thus, if C′ makes $\frac{899\cdot16}{900\cdot16}$ of a revolution while F′ makes 1.5 revolutions, C′ must have 1.5 teeth and F′ $\frac{899\cdot16}{900\cdot16}$ teeth. By multiplying these by 90016 to clear of fractions, C′ = 1.5×90016 or 135024 teeth and F′ = $\frac{899\cdot16}{900\cdot16}$×90016 or 89916 teeth. The factors of these numbers are too large for compound gears; therefore, the numbers are reduced by the principle of continued fractions to other numbers for which suitable factors may be obtained.

Thus,

89916	135024	1
45108	89916	1
44808	45108	1
44700	44808	149
108	300	2
84	216	1
24	84	3
24	72	2
0	12	

		1	1	1	149	2	1	3	4196
1	0	1	1	2	299	600	899	3297	6301
0	1	1	2	3	449	901	1350	4951	5695
		1	1	1	149	2	1	3	8552
									2

5095	4196	6301
5994	3297	4951
	899	1350

7651
9001

	5695	8552
	3297	4951
	2398	3601
6295		
3897		
5396	899	1350
	1499	2251

9453
5852
8103

Since $\frac{1350}{899}\frac{24}{16}$ = approximately $\frac{1350}{899}$, C′ = 1350 teeth, and D′ = 899 teeth. After factoring (1350 = 50×27; 899 = 29×31) and placing in proper compound order, C = 50 teeth, D = 29 teeth, E = 27 teeth, and F = 31 teeth.

EXERCISE 65

1. Pitch diameter $= \frac{\text{N(secant } \theta)}{\text{D.P.}} = \frac{30(1.1034)}{5} = 6.6204$ inches.

2. Pitch diameter $= \frac{\text{N(secant } \theta)}{\text{D.P.}} = \frac{50(1.3054)}{10} = 6.527$ inches.

3. Lead of the spiral gear $= \text{N}\left(\frac{\pi}{\text{D.P.}}\right)(\text{cosecant } \theta) =$

$\frac{30\times 3.1416}{5}\times 2.3662 = 44.6019$ inches.

4. Lead of the spiral gear $= \text{N}\left(\frac{\pi}{\text{D.P.}}\right)(\text{cosecant } \theta) =$

$50\times\frac{3.1416}{5}\times 1.5557 = 24.4364$ inches.

EXERCISE 66

1. See Article 276. There are $20 \div .900$ or $22\frac{2}{9}$ turns of the spiral B within the distance of 1 turn of the spiral A. A straight line will cross $22\frac{2}{9}$ spaces between the turns of the spiral B in the distance equal to 1 turn of the spiral A; but the spiral A making 1 turn in the opposite hand from the spiral B crosses $22\frac{2}{9}+1$ or $23\frac{2}{9}$ spaces. If parallel lines are drawn from the intersecting points of these spirals to the end, they will divide the circle into $23\frac{2}{9}$ parts; therefore, $\text{X} = \frac{1}{23\frac{2}{9}}$ or $\frac{9}{209}$ of a complete circle.

2. $\text{Y} = 1 - \frac{9}{209} = \frac{200}{209}$ of a complete circle.

3. $\text{X} = \frac{1}{22\frac{2}{9}-1} = \frac{1}{21\frac{2}{9}} = \frac{9}{191}$ of a complete circle.

4. $\text{Y} = 1 + \frac{9}{191} = \frac{200}{191}$ of a complete circle.

5. See Article 277. If the part of a complete circle between each of the intersecting points of the spirals is $\frac{1}{12}$, the spiral line A must cross 12 spaces between the turns of the spiral B. When the hand of the spirals is the same, the one spiral crosses the spaces

between the turns of the other spiral once less than a straight line would cross; therefore, a straight line would cross 12+1 or 13 spaces between the turns of the spiral B. If a straight line crosses 13 spaces within the distance of 10 inches, the lead of the spiral B must be $10 \div 13$ or $\frac{10}{13}$ of an inch.

6. If $\frac{201}{200}$ is the relation of the intersecting points, the part of a complete circle between the intersecting points of the spirals must be $\frac{201}{200} - 1$ or $\frac{1}{200}$. The spiral line A crosses 200 spaces between the turns of the spiral B. The hand of the spirals are the same; therefore, a straight line will cross 200+1 or 201 spaces between the turns of the spiral B. Then the lead of the spiral $B = 102 \div 201$ or $\frac{34}{67}$ of an inch.

EXERCISE 67

1. SOLUTION OF THE INDEX GEARS.

Lead of the spiral gear $= N\left(\frac{\pi}{D.P.}\right)$ (cosecant θ) =

$$54 \times \frac{3.1416}{3} \times 2.5593 = 144.725''.$$

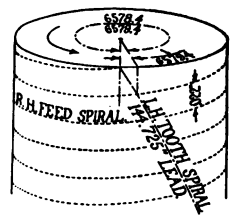


FIG. 486S

The feed spiral will be right-handed because a right-hand hob is being used. (See Article 274.) The hand of the tooth spiral and the hand of the feed spiral are opposite; therefore, according to Articles 275 and 276 the part of a complete circle between the intersecting points is $\frac{1}{6579.4}$. If a left-hand hob feeds down through the face of the work, the work must revolve as shown by the large arrow; and from this it can be seen that the work must make $1 - \frac{1}{6579.4}$ or $\frac{6578.4}{6579.4}$ of a revolution, while the hob makes 54 revolutions, since there are 54 teeth in the spiral gear.

While the hob makes 54 revolutions, the imaginary simple gear H′ will make 54. When the work makes $\frac{6578.4}{6579.4}$ of a revolution the imaginary simple gear E′ will make $\frac{6578.4}{6579.4}\times 90$ revolutions.

If we know the number of revolutions the simple gears H′ and E′ must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 76.) Thus, if H′ makes 54 revolutions while E′ makes $\frac{6578.4}{6579.4}\times 90$ revolutions, H′ must have $\frac{6578.4}{6579.4}\times 90$ teeth and E′ = 54 teeth. By multiplying these by $\frac{65794}{18}$ to clear of fractions, $H' = \frac{6578.4}{6579.4}\times 90\times\frac{65794}{18}$ or 328920 teeth, and $E' = 54\times\frac{65794}{18}\times 197382$ teeth. The factors of these numbers are too large for compound gears; therefore, the numbers are reduced by the principle of continued fractions to other numbers from which suitable factors can be obtained.

Thus,

328920	197382	1
197382	131538	1
131538	65844	1
65844	65694	1
65694	150	437
65550	144	1
144	6	24
144		

									24 3
			1	1	1	1	437	1	5261
1	0	1	1	2	3	1313	1316		8767
0	1	1	2	3	5	2188	2193		5267
		1	1	1	1	437	1		8777
									24 5

Since $\frac{197382}{328920}$ = approximately $\frac{1316}{2193}$, E′ = 1316 teeth, and H′ = 2193 teeth. After factoring (1316 = 28×47; 2193 = 43×51) and placing in proper compound order, E = 28 teeth, F = 43 teeth, G = 47 teeth, and H = 51 teeth.

CHECK: When E = 28 teeth, F = 43 teeth, G = 47 teeth, and H = 51 teeth, the work will make $54\times\frac{43}{28}\times\frac{51}{47}\times\frac{1}{90}$ or $\frac{6579}{6580}$ of a revolution while the hob makes 54 revolutions. Therefore, the intersecting points of the sucessive turns of the feed spiral are out of line $1-\frac{6579}{6580}$ or $\frac{1}{6580}$ of a complete circle along any tooth spiral being considered. From this it can be seen that the tooth spiral crosses 6580 spaces between the turns of the feed spiral in one complete revolution, while a straight line will cross 6580−1 or 6579 spaces in the same distance of the tooth spiral.

If one space of the feed spiral measured in a straight line equals .022″, 6579 spaces must equal 6579×.022″ or 144.738″, which varies only .009″ from the true lead. The variation for a gear one inch wide would be $\frac{.009}{144}$ or .00006″.

SOLUTION OF THE FEED GEARS.

When the feed gears A, B, C, and D all have the same number of teeth, the head carrying the hob will feed .125″; therefore, the feed gear ratio for a .020″ feed is $\frac{.125}{.020}=\frac{125}{20}=\frac{5\times25}{2\times10}=\frac{60\times50}{24\times20}$.

Then, A = 60 teeth, B = 24 teeth, C = 50 teeth, and D = 20 teeth.

2. When the feed gears A, B, C, and D all have the same number of teeth, the head carrying the hob will feed .125″; therefore, when A = 72 teeth, B = 30 teeth, C = 50 teeth, and D = 20 teeth, it will feed $.125''\times\frac{20\times30}{72\times50}$ or $.02\frac{1}{2}''$ or .0208333″.

When E = 36 teeth, F = 74 teeth, G = 24 teeth, and H = 39 teeth, the work will make $27\times\frac{39}{24}\times\frac{74}{36}\times\frac{1}{90}$ or $\frac{481}{480}$ revolutions while the hob makes 27 revolutions.

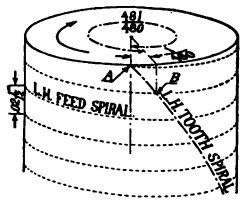


FIG. 487S

Since a left-hand hob is being used, the feed spiral must be left-handed. If a left-hand hob feeds down through the face of the work, the work must revolve as shown by the large arrow; and when traveling from A to B along the feed spiral, $\frac{481}{480}$ or $1\frac{1}{480}$ revolutions are required. Therefore, the intersecting points of the successive turns of the feed spiral are out of line $1\frac{1}{480}-1$ or $\frac{1}{480}$ of a complete circle along any tooth spiral being considered. From this it can be seen that the tooth spiral crosses 480 spaces between the turns of the feed spiral in one complete revolution; therefore, a straight line will cross 480+1 or 481 spaces in the same distance of the tooth spiral.

If one space between the feed spirals measures $.02\frac{1}{2}''$ or .0208333″, 481 spaces will measure 481×.0208333″ or 10.0208″; therefore, under conditions stated in the problem the lead will be 10.0208″.